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AIR FORCE SURVEYS IN GEOPHYSICS

No. 86

THE ARDC MODEL ATMOSPHERE, 1956

R. A. MINZNER
W.S. RIPLEY

DECEMBER 1956

GEOPHYSICS RESEARCH DIRECTORATE
AIR FORCE CAMBRIDGE RESEARCH CENTER
AIR RESEARCH AND DEVELOPMENT COMMAND

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ADDENDUM

The representation of the atmosphere contained between these covers is designated "THE ARDC MODEL ATMOSPHERE, 1956" since it is in this Command that these tables are officially accepted and directive in all design problems.

To an altitude of 300 kilometers the basic properties of this atmosphere are the result of the combined effort of the scientists and engineers listed in the preface where full acknowledgements are accorded. Without their help this representation would not have been possible.

PREFACE

The 1956 ARDC MODEL ATMOSPHERE, defined and tabulated to 542,248 meters or 1,850,870 feet in this Air Force Survey in Geophysics, has been prepared in partial fulfillment of ARDC Technical Requirement 140-56. This MODEL is to be used as the basis for engineering and design work performed within ARDC and by its contractors, insofar as the work requires the use of a model representing the average condition of atmospheric properties within the altitude limits of this MODEL.

This MODEL ATMOSPHERE is designed to be used for the same purposes as a standard atmosphere. For some of these purposes the MODEL should serve in the following ways:

1. As a reference atmosphere to be used in calculating flight performance of aircraft.

2. As the basis for calibrating barometric altimeters, where observed departures of atmospheric properties from the values of the MODEL provide the means for computing altimeter correction.

3. As the basis for ballistic tables where the observed departures of the atmospheric properties from the values of the MODEL provide the basis of corrections to be put into gunnery and bombing computers.

4. As a time average of the actual physical conditions existing at various altitudes for aircraft engineering and design purposes, and for use in solving geophysical problems.

It should be emphasized, particularly in regard to item 4, that this MODEL most probably will never completely match the actual atmosphere, and may only rarely approximate the average value at all altitudes simultaneously. While the properties at some altitude may exactly fit the values of the MODEL at any instant, the properties at other altitudes simultaneously may depart drastically from tabulated values. The greatest percentage departures probably occur at the higher altitudes. Maximum and minimum pressures at 120 km, for example, may differ by as much as a factor of 3. Neither this MODEL nor any other calculated model will accurately depict the total atmosphere at any particular moment.

The tables and graphs of this MODEL approximate the best average of available temperature, pressure, and density data, compiled and processed under Project 7603, "Atmospheric Standards." The tables are also consistent with the recently adopted Extension to the United States (ICAO) Standard Atmosphere[50,51] (1956) which was prepared concurrently under the same project. Both are consistent with the basic properties of the International Civil Aviation Organization (ICAO) Standard Atmosphere[26-28] adopted by the United States on November 20, 1952.

The tables of this MODEL partially duplicate the tables of the ICAO Standard Atmosphere, (in the altitude region of -5,000 to +20,000 geopotential meters), although the tables of this MODEL are given in larger increments. This partial duplication is desirable and necessary, not only for the sake of continuity, but because this MODEL includes values of seven additional altitude-dependent properties not found in the ICAO Standard: Acceleration of gravity, scale height, molecular weight, particle speed, number density, mean free path, and collision frequency.

The ARDC MODEL differs from the standard atmosphere not only because of the greater altitude of the former but because the MODEL is intended to be reviewed annually and modified at any time, if necessary, to reflect significant changes in thinking brought about by more reliable atmospheric data.

We wish to acknowledge the assistance of the several members of the Geophysics Research Directorate who participated in various ways in the preparation of this survey: Dr. R. Penndorf and Mr. M. Dubin for helpful suggestions and counsel, and Mr. L. R. Shedd for his expeditious handling of many details.

We also wish to thank the members of the Working Group on Extension to the Standard Atmosphere for their helpful suggestions and encouragement. This Working Group consisted of:

Dr. Fred L. Whipple, Chairman	Harvard University and Smithsonian Inst.
Dr. Charles J. Brasefield	Formerly at Signal Corps Engineering Lab.
Dr. William G. Brombacher	National Bureau of Standards
Dr. Austin R. Brown	Formerly at Ballistics Research Laboratory
*Mr. LeRoy Clem	Air Weather Service
Major R. F. Durbin	Formerly at Air Weather Service
Dr. Sigmund Fritz	U. S. Weather Bureau
**Dr. Boris Garfinkel	Ballistics Research Laboratory
Dr. Ralph J. Havens	Formerly at Naval Research Laboratory
***Dr. D. P. Johnson	National Bureau of Standards
****Dr. Hildegard K. Kallman	Rand Corporation
Dr. William W. Kellogg	Rand Corporation
Mr. Raymond A. Minzner	Air Force Cambridge Research Center
*****Dr. Homer E. Newell, Jr.	Naval Research Laboratory
Mr. William J. O'Sullivan	NACA Langley Aeronautical Laboratory
Mr. William A. Scholl	Wright Air Development Center
******Mr. William G. Stroud, Jr.	Signal Corps Engineering Laboratory
Mr. Norman Sissenwine Executive Secretary	Air Force Cambridge Research Center

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* Replacement for Major Durbin upon his departure from Air Weather Service.
** Replacement for Dr. Brown upon his departure from Ballistics Research Lab.
*** Substitute for Dr. Brombacher upon his retirement from National Bureau of Standards to status of consultant for the same organization.
**** Substitute for Dr. Kellogg.
***** Replacement for Dr. Havens upon his departure from Naval Research Lab.
****** Replacement for Dr. Brasefield upon his departure from Signal Corps Eng. Lab.

We are especially indebted to two subcommittees of this Working Group:

The first subcommittee, consisting of Dr. H. Newell, Dr. H. Kallman, and Mr. R. A. Minzner, formulated the general aspects of the temperature-altitude profile between 130 and 300 kilometers, and made recommendations concerning the degree of dissociation of O_2 and N_2 in this region.

The second subcommittee, consisting of Mr. L. P. Harrison, Mr. W. J. O'Sullivan, Mr. W. Scholl, and Mr. R. A. Minzner, studied some of the aspects of the following atmospheric properties: coefficient of viscosity, kinematic viscosity, and the speed of sound. This subcommittee recommended departures from the ICAO values of these properties and thereupon suggested values of constants, empirical expressions, and maximum altitude of tabulation for these properties.

We are particularly grateful to Dr. F. L. Whipple whose efficient chairmanship expedited the accomplishment of the Working Group, and to Mr. N. Sissenwine who in the capacity of Executive Secretary handled a flood of detail.

Finally we wish to thank Dr. H. Wexler of the U. S. Weather Bureau. Dr. Wexler served with Mr. Sissenwine as Co-chairman of the Parent Committee on Extension to the Standard Atmosphere, and though not an official member of the WGESA, was ever in the background to lend his advice and support wherever needed.

R. A. MINZNER

W. S. RIPLEY

Geophysics Research Directorate

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ABBREVIATIONS AND SYMBOLS

a	acceleration
b	subscript indicating base or reference level
°C	degrees, in thermodynamic Celsius scale
C_s	speed of sound
$(C_s)_o$	sea-level value of C_s
c_p	specific heat of dry air at constant pressure
c_v	specific heat of dry air at constant volume
cgs	centimeter-gram-second system of units
cm	centimeter
d	differential symbol
e	base of natural logarithms
°F	degrees, in thermodynamic Fahrenheit scale
F	force
f(H)	undefined function of H representing T_M
fps	foot-pound-second system of units
ft	foot
ft'	standard geopotential foot
G	dimensional constant in the geometric-geopotential relationship
g	effective value of acceleration of gravity
g_o	sea-level value of g
g_ϕ	sea-level value of g at latitude ϕ
gm	gram
gm-mol	gram mole

ABBREVIATIONS AND SYMBOLS (Contd.)

H	altitude in geopotential measure
H_o	sea-level value of H, (zero)
H_b	altitude at base of layer, or reference level in geopotential measure
Hg	mercury
H_s	scale height
H_s'	geopotential scale height
$(H_s)_o$	sea-level value of H_s
in	inch
i n mi	international nautical mile
°K	degrees, in thermodynamic Kelvin scale
kg	kilogram
kgf	kilogram force
kg-mol	kilogram mole
km	geometric kilometer
km'	standard geopotential kilometer
L	mean free path
L_o	sea-level value of L
L_M	molecular-scale-temperature gradient $\partial T_M / \partial H$
ℓ	length
lb	pound
lbf	pound force
ln	natural logarithm

ABBREVIATIONS AND SYMBOLS (Contd.)

log	logarithm
M	apparent molecular weight of air
M_o	sea-level value of M
M'	mass numerically equal to the molecular weight (a mole)
m	(geometric) meter
m'	standard geopotential meter
mb	millibar
mks	meter-kilogram-second system of units
m	mass
N	Avogadro's number (standard)
n	atmospheric number density
nt	newton
n_o	sea-level value of n
n_i	number density of a gas at temperature T_i and pressure P_o (Loschmid's number)
P	atmospheric pressure
P_o	sea-level value of P
pdl	poundal
P_b	value of P at base of layer or reference level
Q	constant, $\frac{GM_o}{R^*}$
°R	degrees, in thermodynamic Rankine scale
R*	universal gas constant
r	effective radius of earth (at 45° 32' 40" N. lat.)

ABBREVIATIONS AND SYMBOLS (Contd.)

r_ϕ	radius of earth at latitude ϕ
S	Sutherland's constant
sec	second
T	temperature (real kinetic) in the absolute thermodynamic scales
T_o	sea-level value of T
T_i	temperature of the ice point in the absolute thermodynamic scales
T_M	molecular-scale temperature in the absolute thermodynamic scales
$(T_M)_o$	sea-level value of T_M
$(T_M)_b$	value of T_M at base of layer or reference
t	temperature in nonabsolute thermodynamic scales, also signifies time
t_o	sea-level value of t
t_i	temperature of the melting point of ice at 1013.250 mb air pressure in the nonabsolute thermodynamic scales
t_M	molecular-scale temperature in the nonabsolute scales
$\overline{V}$	particle speed (arithmetic average)
$\overline{V}_o$	sea-level value of $\overline{V}$
v	volume of one mole of air at existing conditions of T and P
v_o	sea-level value of v
v_i	volume of one mole of air at a temperature T_i and pressure P_o (mol-volume)
Z	altitude in geometric measure
α	real temperature gradient $\partial T/\partial Z$

ABBREVIATIONS AND SYMBOLS (Contd.)

α'	real temperature gradient $\partial T/\partial H$
β	constant used in the empirical expression for the coefficient of viscosity
γ	ratio of specific heats, c_p/c_v
δ	partial differential symbol
η	kinematic viscosity
η_o	sea-level value of η
μ	coefficient of viscosity
μ_o	sea-level value of μ
ν	collision frequency
ν_o	sea-level value of ν
π	ratio of circumference to the diameter of a circle
ρ	atmospheric density
ρ_o	sea-level value of ρ
ρ_i	ice-point value of ρ
σ	effective collision diameter of a mean air molecule (standard)
ϕ	latitude of the earth
ω	specific weight
ω_o	sea-level value of ω

ABSTRACT

A realistic model of atmospheric properties based on reliable observations and current theories is presented.

Fifteen atmospheric properties are discussed and tabulated, thirteen to 500 km and two to only 90 km. The values of these properties are internally consistent through classical equations, and are dependent upon (1), a defined, linear, segmented, molecular-scale temperature function, (2) a molecular weight function, and (3) an acceleration of gravity function. Values of twelve physical constants required in the computations are adopted as exact. Internationally agreed-upon, exact transformation factors are employed in converting from Metric to English units. Both Metric and English tables are presented, and computational procedure is discussed. A thorough discussion of geopotential altitude, effective radius of the earth, and molecular-scale temperature is given. The relative virtues and validity of two methods for computing the acceleration of gravity are discussed. The concept and validity of the various properties as applied to high altitudes are considered briefly.

THE ARDC MODEL ATMOSPHERE

1956

(Tables and Graphs for Altitudes to 542,686 Meters or 1,850,870 Feet)

1. Introduction

1.1 Background and Early History of Standard Atmospheres

Standard atmospheres have been used for nearly a hundred years for altimetry purposes. The earliest of these were very simple and were based on an isothermal atmosphere. With the development of aircraft and precision artillery during the First World War, 1914-1918, the need for more extensive atmospheric tables for aeronautical and ballistic purposes became apparent. Atmospheric temperatures were measured at various locations in southern and western Europe. Several functions approximately fitting these temperature data were proposed and used in various countries for deriving an analytical expression for atmospheric pressure and density. No generally agreeable function was proposed, however, until 1919 when Toussaint[49] suggested a segmented straight-line function as the basis for an international standard. Toussaint's temperature function was defined by a value of 15 degrees Celsius (°C) at sea level, a constant gradient of -.0065°C per meter from sea level to 11,000 meters, (m), (yielding -56.5°C for 11,000 m), and a constant gradient of zero degrees per meter from 11,000 m to 20,000 m altitude.

1.2 First U. S. Aeronautical Standard Atmosphere

The Toussaint formula with minor variations has remained the basis for all major aeronautical standards prepared for the 0 - 20 km altitude region. These include the first United States Standard Atmosphere prepared by Gregg[21] in 1922, and the modification, extension, and amplification of the Gregg standard prepared by Diehl[14] in 1925. Neither of these agreed exactly with the Toussaint proposal, however: Gregg terminated his analytically derived atmosphere at 10 km altitude although he presented observed data to 20 km; Diehl extended the analytical atmosphere to 20 km but established the tropopause at an altitude of 10,769.23 m (65,000 ft) with a temperature of -55°C, instead of at 11,000 m and -56.5°C, as suggested by Toussaint. Thus Diehl's stratosphere, 10,769.23 m to 20,000 m, was warmer by 1.5°C than that used by Toussaint.

Brombacher[4,5] amplified the Gregg Standard Atmosphere in 1926 and again in 1935 by adding tables of altitude as a function of pressure for altimetry purposes.

1.3 First International Standard

In 1924 the International Committee on Air Navigation (ICAN)[29] prepared an international standard atmosphere based exactly on Toussaint's temperature-altitude function. This standard was adopted throughout most of Europe. It was never adopted formally by the United States, however, because of two small but basic differences between this and the Diehl-U. S. Standard.

In addition to using different altitudes and temperatures for the tropopause, the ICAN and U. S. Standard also used different values for the acceleration of gravity at sea level, 9.8 and 9.80665 respectively. These differences prevented United States and European agreement on a standard atmosphere until 1952 when a new international organization, ICAO, reached a compromise.

1.4 ICAO Standard Atmosphere[28] -- New U. S. Standard[26,27]

Between June 1950 and November 1952 the International Civil Aviation Organization (ICAO), of which the United States was a member, proposed and adopted a compromise standard atmosphere in which the United States standard sea-level value of gravity, and the ICAN values of tropopause altitude and tropopause temperature were employed. This ICAO Standard Atmosphere was formally adopted as the United States Standard Atmosphere by NACA vote on 20 November 1952.

1.5 High Altitude Models -- Warfield, Grimminger

The activities of ICAO emphasized international agreement and refinement of atmospheric tables within the altitude range of existing standards; i.e., sea level to 20,000 meters altitude. The ICAO did not concern itself with high altitude tables. The advances in aeronautics and ballistics during and since World War II resulted in demands for atmospheric tables to much greater altitudes. In 1947 these demands were met in part by Warfield's "Tentative Tables for the Properties of the Upper Atmosphere"[52] which depicted the atmosphere to 120,000 meters altitude and which were designed to be a continuous extension of the tables of the Diehl-U. S. Standard[14] at 20,000 m altitude. The Warfield tables were based on the best 1946 estimates of atmospheric temperature, and considered the variations of molecular weight of air and the acceleration of gravity with increasing altitude.

The 120 km altitude upper limit of the Warfield tables was inadequate, however, even before the publication of the report, and Grimminger[22] in 1948 published tables of atmospheric properties to altitudes of over 8,800 km. These tables were essentially in agreement with the Warfield tables up to 120 km and were based on the best 1947 theoretical and experimental data.

1.6 New Data from Rocket-Borne Experiments

Simultaneously with the preparation of the Warfield and Grimminger tables, a new research tool, the upper air sounding rocket, was beginning to be exploited. This new device permitted making measurements of the atmosphere by direct probing methods not previously possible. The new data compiled in 1952 as the Rocket Panel Atmosphere[45] indicated that pressures in the Warfield and Grimminger tables were 2 times higher than observed at 70 km, 5 times higher than observed at 90 km, and over 10 times higher than observed at 120 km. These discrepancies, plus the fact that the Warfield tables were not continuous with the newly adopted ICAO Standard, initiated the preparation of this extension of the ICAO Standard to high altitudes.

1.7 Extension to the Standard Atmosphere

In November 1953 the Geophysics Research Directorate, Air Force Cambridge Research Center, of ARDC, USAF, together with the U. S. Weather Bureau sponsored a three-day "Open Meeting on Extensions to the Standard Atmosphere."[17] Standard atmosphere requirements and scientific data supporting various models were presented. Brombacher[6] presented a Standard Atmosphere proposal which was not accepted because of an unrealistic stratosphere and because the constant gravity assumption employed was inconsistent with the ICAO Standard and this assumption introduced errors in the analysis. A Working Group on Extension to the Standard Atmosphere (WGESA) was appointed to recommend the temperature-altitude profile and other constants necessary for the preparation of the desired extension.

The discussions of the first meeting[18] of the Working Group dealt principally with the temperature-altitude profile in the 20 to 53 kilometer region. Temperatures were also recommended for the region between 53 and 83 km, although these were replaced by slightly different values at a later meeting. Recommendations were also made at this first meeting regarding the atmospheric properties to be included in the standard. Differences of opinion existed on the manner of accounting for variable gravity, and some conflicting recommendations resulted from this meeting.

The task of preparing the text and tables for the extension to the Standard Atmosphere was assigned to GRD (Geophysics Research Directorate). The recommendations were studied, and Minzner[40] prepared a paper, "Three Proposals for U. S. High Altitude Standard Atmosphere," which was presented at the second meeting[19] of the Working Group. Each of the three proposals suggested a different method for handling the acceleration of gravity and molecular weight as variables in the hydrostatic equation. Only one of these three proposals was consistent with the ICAO Standard Atmosphere and that one, using geopotential to account for variable gravity, and molecular-scale temperature to account for variable molecular weight, was adopted by the Working Group.

Preliminary tables of atmospheric properties to 130 km,[41] prepared at GRD, were tentatively adopted at this meeting. These tables were consistent with the temperature-altitude function to 83 km recommended by the Working Group and consistent with the temperatures of the Rocket Panel Atmosphere above this altitude. A subcommittee was appointed, however, to make recommendations concerning molecular weight and temperatures for extending the Standard Atmosphere to 300 km altitude.

This subcommittee met with several consultants and then agreed upon certain boundary conditions for oxygen and nitrogen dissociation, as well as for atmospheric temperature. Using these boundary conditions and all the available atmospheric pressure, temperature, and density data above balloon altitudes, two separate proposals were prepared, one at Rand Corporation[34,35] and the other at GRD.[42]

The Rand proposal assumed a density-altitude function and a molecular weight gradient arbitrarily related to this density function. From these, there was derived a nonlinear temperature-altitude profile with no discontinuous first or second derivatives.

The GRD proposal, in keeping with previous Working Group recommendations, assumed several constant gradients of molecular-scale temperature for as many altitude regions. These gradients were chosen to yield values of pressure and density consistent with the average of observed values of these properties below 160 km altitude, and consistent with current estimates of these properties at higher altitudes. Molecular weights[39] were computed from diffusion theory and the agreed-upon boundary conditions. The GRD proposal was adopted at the third and final meeting[20] of the Working Group.

A summary of the adjusted recommendations[47] resulting from the three meetings of the WGESA was prepared. A supplemental set of recommendations[43] on previously unresolved questions was also prepared. Within the framework of these recommendations, this ARDC MODEL ATMOSPHERE and the Extension to the U. S. Standard Atmosphere have been prepared.

2. Systems of Altitude Measure and Related Parameters

In accordance with agreements concerning publication of international aerological tables[30] and in keeping with the existing United States (ICAO) Standard Atmosphere, the basic altitude parameter of this MODEL is taken to be geopotential H, expressed in standard geopotential meters, m'. Supplemental to the existing (ICAO) United States Standard, this MODEL has been prepared with parallel tabulations in integral values of both geopotential and geometric altitude measure so that the values of tabulated properties are given for both integral geopotential and integral geometric kilometers.

The relationship between geopotential and geometric altitude depends directly upon the value of the acceleration of gravity at sea level at a particular altitude and upon the variation of the acceleration of gravity with altitude and latitude. The definition of the special unit of geopotential used in this MODEL is also related to the specific sea-level value of gravity, adopted by ICAO and used in this MODEL. Therefore, a digression is made to present a detailed discussion of the acceleration of gravity before geopotential is discussed further.

2.1 Acceleration of Gravity

2.1.1 Sea-level value

The sea-level value of the acceleration of gravity used in this MODEL is defined to be 9.80665 m sec^{-2} exact/ . This value was originally announced by Defforges and Lubanski[13] at the 1891 meetings of the International Committee on Weights and Measures as the best value for 45° latitude. Since then, it has been used by physicists and others as an arbitrary standard and was recently adopted as an international standard in the ICAO Standard Atmosphere. It has long been recognized, however, that this value of g is not correct for 45° latitude but rather is the value for 45° 32' 40" latitude.[15] This corrected latitude is the one to which all tables in this MODEL apply.

2.1.2 Altitude variation - classical expression

The variation of the acceleration of gravity with geometric altitude is classically expressed by the equation

$$g = g_\phi \left[\frac{r_\phi}{r_\phi + Z} \right]^2 , \qquad (1)$$

where

g = the acceleration of gravity of a point (in m sec^{-2}),

Z = the geometric altitude of the point (in m),

g_ϕ = the sea-level value of g at the latitude ϕ of the point (in m sec^{-2}), and

r_ϕ = the radius of the earth at latitude ϕ .

In its fundamental form this equation applies rigorously only for a nonrotating sphere composed of spherical shells of equal density. The earth, however, is definitely not spherical; furthermore, its rotation introduces centrifugal acceleration which varies with latitude and which increases with altitude. The sea-level value of the centrifugal acceleration at any selected latitude may be accounted for, in equation (1), by the proper choice of an effective value of g_ϕ. The increase of centrifugal acceleration with increasing altitude is not accounted for in the simple unadjusted inverse square law, which describes only the decreasing Newtonian component of the effective value of g. Hence, values of g computed from equation (1) become increasingly inaccurate as altitude increases. An adjustment of the value of r_ϕ to an effective

/ Basic constant

radius, however, was found to greatly improve the validity of that equation even at altitudes as great as 500 km.

2.1.3 Effective earth's radius

Harrison[23], using a suggestion by Lambert[37], developed an expression for an effective earth's radius as a function of latitude. This effective radius is derived in a manner consistent with the effective sea-level value of g at latitude ϕ , and consistent with the vertical gradient of g at the given latitude (neglecting local anomalies), assuming the International Ellipsoid represents the figure of the earth. The value of effective earth's radius at 45° 32' 40", computed from Harrison's equation (given in Appendix M) is

$$r = 6{,}356{,}766 \text{ meters}$$

which, for purposes of this MODEL, will be considered as an exact constant.

2.1.4 Computational equation

The exact form of the equation used to compute the acceleration of gravity and to relate geopotential to geometric altitude in this MODEL is

$$g = g_o \left[\frac{r}{r + Z}\right]^2 , \qquad (1a)$$

where

- g = the acceleration of gravity in meters per second squared, (m sec^{-2}) at altitude Z and at latitude 45° 32' 40", hereafter,
- g_o = 9.80665 m sec^{-2} (exact)⨍, the sea-level value of g at 45° 32' 40" latitude, and
- r = 6,356,766 m (exact)⨍, the effective earth's radius at latitude 45° 32' 40".

(For purposes of this MODEL, this equation is assumed to apply in free air below sea level as well as above sea level.)

2.1.5 Best available analytical expression

A more exact equation for g as a function of Z and ϕ in free air, based directly on the International Ellipsoid and the International Gravity Formula, was developed by Lambert[36,38] in the form of an infinite,

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⨍ Basic constant

alternating power series (see Appendix N). The values of g computed from equation (1a) are in good agreement with those computed from Lambert's more exact equation. For an altitude of 500 km the value of g from the two methods differs only by 3 parts in the fifth significant figure, or less than 1/1000 of 1 per cent. For lower altitudes the agreement is much better. Values of geopotential computed for specific values of Z on the basis of equation (1a) are also in good agreement with corresponding values of geopotential computed on the basis of the more exact equation for g. The percentage departures are similar. The more exact expression for g was not employed in this MODEL because of its much greater complexity. In the U. S. Standard Atmosphere, the tables will be recomputed by machine and will be based on the more exact equation.

2.2 Relation of Geopotential to Geometric Altitude

2.2.1 Basic definition of geopotential

The geopotential of a point is defined as the increase in potential energy per unit mass lifted from mean sea level to that point against the force of gravity.

2.2.2 Analytical development

The increase in potential energy of a body lifted against the force of gravity, from sea level, through a vertical distance to a given point is:

$$\Delta E = \int m g dZ, \tag{2}$$

where

ΔE = increase of potential energy over the sea-level value, in joules,

m = mass of the body in kilograms, kg.

The geopotential of that point $\Delta E/m$ is therefore:

$$\frac{\Delta E}{m} = \int g dZ. \tag{2a}$$

If geopotential is given a special designation, H, with special units, we have:

$$GH = \frac{\Delta E}{m} = \int g dZ, \tag{2b}$$

$$G dH = g dZ, \tag{2c}$$

or

$$H = \frac{1}{G} \int g dZ, \tag{2d}$$

where

H = geopotential (in unspecified units), and

G = a proportionality factor depending upon the units of H.

When H is in units of joules kg^{-1} or equivalently in $m^2 \ sec^{-2}$, G is nondimensional and unity. If H is expressed in some other units, standard geopotential meters for example, the value and dimensions of G must be correspondingly changed.

2.2.3 The standard geopotential meter[26-28]

The basic unit of geopotential employed in this MODEL is the standard geopotential meter where one standard geopotential meter, m', is defined to be an increment of potential energy per unit mass equal exactly to

9.80665 joules kg^{-1} (or $m^2 \ sec^{-1}$); i.e.,

$$1 \text{ m}' = 9.80665 \text{ m}^2 \text{ sec}^{-2} \text{ (exact).}^{*} \qquad (3)$$

It is evident from equation (2b) that if H is expressed in m', G is equal to 9.80665 $m^2 \ sec^{-2} \ m'^{-1}$. ## One standard geopotential meter is therefore the vertical distance through which one kilogram mass must be lifted against the force of gravity to increase its potential energy by 9.80665 joules. If a region existed where the value of the acceleration of gravity were constant at 9.80665 m sec^{-2} over an altitude interval of one geometric meter, in this region one geometric meter and one geopotential meter would then be exactly equal. This condition is very closely approximated at sea level at 45° 32' 40" latitude. Since g normally does decrease with altitude, however, even over a one meter interval, an altitude of one geometric meter at this latitude has a geopotential altitude of slightly less than 1 m', (see table in Section 2.2.5). Above sea level, at all points where the altitude gradient of g is continuously negative from sea level, the altitude in standard geopotential meters is always numerically less than the altitude in geometric meters, and the numerical difference increases with increasing altitude.

2.2.4 Standard geopotential kilometer and standard geopotential centimeter

The basic concept of the metric system of units leads directly to the conclusion that one geopotential kilometer, km', is equal to one thousand geopotential meters; i.e.,

$$1 \text{ km}' = 1 \times 10^3 \text{ m}'. \qquad (3a)$$

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* Basic conversion of units

Derived constant, inferred from transformation of units

Also, it follow that one geopotential centimeter, cm', is equal to one one-hundredth of a geopotential meter; i.e.,

$$1 \text{ cm}' = 1 \times 10^{-2} \text{ m}'. \tag{3b}$$

One cm' may also be defined in cgs units directly by analogy with equation (3),

$$1 \text{ cm}' = 980.665 \text{ ergs gm}^{-1} = 980.665 \text{ cm}^2 \text{ sec}^{-2} = .01 \text{ m}', \tag{3c}$$

where

980.665 is the numerical value of g_0 in the cgs units.

2.2.5 Conversion of standard geopotential meters to geometric meters

The replacement of g in equation (2b) by equation (1a) results in

$$H = \frac{g_0}{G} \int \left[\frac{r}{r + Z}\right]^2 dZ, \tag{4}$$

where

H = geopotential in standard geopotential meters, m',

Z = geometric altitude in m,

G = 9.80665 $\text{m}^2 \text{ sec}^{-2} \text{ m}'^{-1}$(exact)≠,

g_0 = 9.80665 m sec^{-2} (exact)≠,

r = 6,356,766 m (exact)≠.

Performing the indicated integration leads to

$$H = \left[\frac{g_0}{G}\right] \frac{rZ}{r + Z}, \tag{5}$$

or

$$Z = \frac{rH}{\left[\frac{g_0}{G}\right] r - H} \tag{6}$$

≠ Basic constant

The ratio g_o/G appearing in equations (4), (5), and (6) is numerically unity while its dimensions are m'/m. Hence while the ratio g_o/G may be ignored for numerical purposes, in this MODEL it must be retained in a dimensional analysis. (The definition of the standard geopotential meter was in fact chosen to make the ratio g_o/G numerically unity for the case when g_o = 9.80665 m sec^{-2}, the standard sea-level value of gravity in the ICAO Standard Atmosphere and in this MODEL.)

Using equation (5), the following tables of geopotential in m^2 sec^{-2}, as well as in standard geopotential meters, have been prepared for specified geometric altitudes.

Geometric Altitude Z	Geopotential $\Delta E/m$	H	Differences in Values of H
m	m^2 sec^{-2}	m' by equation (5)	m'
1×10^0	$9.806,648,45 \times 10^0$	$.999,999,839 \times 10^0$	.000,000,0
1×10^1	$9.806,634,56 \times 10^1$	$.999,998,423 \times 10^1$	.000,000,0
1×10^2	$9.806,495,72 \times 10^2$	$.999,984,265 \times 10^2$	.000,000,0
1×10^3	$9.805,107,53 \times 10^3$	$.999,842,719 \times 10^3$	.000,000,0
1×10^4	$9.791,247,11 \times 10^4$	$.998,429,339 \times 10^4$	.000,07
1×10^5	$9.654,768,23 \times 10^5$	$.984,512,367 \times 10^5$	.088
$.5 \times 10^6$	$4.545,771,23 \times 10^6$	$.463,539,663 \times 10^6$	9.9
1×10^6	$8.473,638,99 \times 10^6$	$.864,070,707 \times 10^7$	70.6

Equations (4) through (6) do not represent the only possible equations for converting geometric measure to geopotential measure. While equation (2d) is the fundamental and rigorously correct equation for converting geopotential measure to geometric measure, equations (4) through (6) are only as good as the expression for g introduced into equation (2d). A more precise expression for g is discussed in Appendix N. This expression is an alternating infinite-power series in terms of latitude and altitude. Evaluating this expression for latitude 45° 32' 40" and introducing it into equation (2d) yields another alternating power series as the expression for H in terms of Z. The departures of the result of equation (5) from the results of this more exact method are small. The differences in the values of H computed by both methods for $45^{\circ}\ 32'\ 40''$ latitude are given in the above table. For altitudes of 1×10^3 meters and below, the number of significant figures limits difference determinations. For altitudes above $.8 \times 10^6$ meters, the number of available terms in the series limits the difference determinations. From these results it is obvious, however, that for practical applications, at least, equations (4) and (5) are quite adequate. (See appendix P)

2.2.6 Other special units of geopotential

Two other special units of geopotential, neither of which is employed in this MODEL, preceded the standard geopotential meter. The geodynamic meter, the first of such units to be used, was defined by Bjerknes[3] to be equal to 10 joules kg^{-1}. Thus a geodynamic meter differed in magnitude from a geometric meter by about 2% at sea level.

The second special unit of geopotential to be introduced, and the one generally used by meteorologists, is the geopotential meter[23,32] equal to 9.8 joules kg^{-1} or 9.8 $m^2\ sec^{-2}$. This latter unit was defined on the basis of a sea-level value of g equal to 9.8 m sec^{-2}. The numerical differences between altitudes measured in geopotential meters and the same altitudes expressed in standard geopotential meters are small, of the order of 1/10 of 1 per cent, and in many instances may be neglected.

2.2.7 Analytical usage

Geopotential has its greatest appeal, for use in this MODEL, from an analytical point of view, because it is a parameter involving both g and Z, and hence its use reduced by one the number of variables in the differential form of the barometric equation relating the basic atmospheric properties of this MODEL. This reduction in the number of variables comes without requiring the erroneous assumption of constant acceleration of gravity, used in some of the earlier standards. (The constant gravity assumption would result in a computed pressure which, at 500 km, is 40 per cent lower than one finds when variations in gravity are accounted for.) This pressure discrepancy is equivalent to an altitude discrepancy of 42.6 km at 500 km. If variable gravity is retained in the hydrostatic equation explicitly, rather than being concealed in the geopotential altitude, the algebraic expression resulting from the integration of the hydrostatic equation is excessively complicated.

3. Basic Atmospheric Properties of the MODEL

The basic properties of this ARDC MODEL are those properties rigorously related by the hydrostatic equation and the equation of state (perfect gas law). These are pressure, density, and the ratio of temperature to molecular weight of air (which will be expressed in terms of molecular-scale temperature). Defining the altitude function of any one of these properties specifies the remainder of these basic properties in any model. In this MODEL, according to custom, the temperature function is the defining property.

3.1 Molecular-Scale Temperature and Its Development

3.1.1 Ratio of temperature to molecular weight, T/M

The property, T/M, is a composite of two variables which are conveniently handled as an entity because of the frequent occurrence of this ratio in atmospheric equations. In fact, the occurrence is so frequent and so fundamental that all so-called atmospheric-temperature measuring experiments successfully used in rockets to date, measure T/M, rather than T independently.

The combining of the two variables into a single parameter is of particular convenience in the computation of atmospheric tables to great altitudes because:

a. The values of T and M have not been independently measured above 90 km with any degree of reliability; and

b. The introduction of T/M, as a single function of H, into the differential form of the barometric equation greatly simplifies the integration and resulting algebraic computational equations over the case when two independent functional relationships are used.

Until recently, aerologists have not been concerned with relating pressure-altitude gradients or speed of sound etc., to the ratio T/M, since within the altitude region of their concern (below about 90 km), the molecular weight of air, M, is known to remain essentially constant at its sea-level value, M_0. For the same reason, the preparation of tables of atmospheric models and standards did not require the consideration of M as a variable; and hence the increased complexity of equations resulting from considering M a variable was not a problem. Defining the atmosphere in terms of T/M instead of in terms of T alone solves the problem of complexity but introduces the problem of consistency with existing standards. This consistency problem is solved by defining a new property, the molecular-scale temperature, such that it is a function of T/M and is equal to T at all altitudes where M is equal to M_0.

3.1.2 Molecular-scale temperature concept

The molecular-scale temperature, T_M, which Minzner[40,41] suggested as the basic parameter for the Standard Atmosphere, is a parameter which combines the ratio of two fundamental variables T/M with a constant in such a manner that T_M is equal to T wherever $M = M_0$, and simultaneously accounts for variations in M without specifying its functional variation. Molecular-scale temperature is that temperature derived from essentially all rocket experiments when variations in molecular weight from its sea-level value are unknown and hence neglected. Molecular-scale temperature is an amplification and redefinition of Whipple's T_{29} in the Rocket Panel Atmosphere.[45] Analytically T_M is defined by the following equation:

$$T_M = \left(\frac{T}{M}\right) M_o , \tag{7}$$

where

T = temperature (kinetic) in the absolute thermodynamic scales,

T_M = molecular-scale temperature in the absolute thermodynamic scales,

M = molecular weight (nondimensional),

M_o = sea-level value of molecular weight equal to 28.966 (nondimensional, exact).[26-28,31,47] (See section 5.1.)

The use of T_M in the ARDC MODEL retains consistency with the existing United States Standard Atmosphere, since over the altitude region of the Standard (0 to 20,000 m') as well as to considerably greater altitudes, the ratio of M_o/M is unity; and hence $T_M = T$ for these altitudes.

3.1.3 Form of altitude function of molecular-scale temperature

Molecular-scale temperature is the key or defining property of this MODEL, in that the specification of the variation of T_M with altitude simultaneously and completely establishes the altitude variation of more than half of the fifteen properties of this MODEL. (The determination of the remaining properties requires a definition of the altitude variation of molecular weight above 90 km in addition to the altitude variation of the molecular-scale temperature.)

In accordance with precedent[26-28] and by agreement of the Working Group on Extension to the Standard Atmosphere,[18] the temperature parameter of this MODEL is defined to be a continuous function of altitude consisting of a consecutive series of functions linear in geopotential H, whose first derivatives are discontinuous at the intersections of the linear segments. The use of such a function implies that the atmosphere is made up of a finite number of concentric layers, each layer characterized by a specific constant value of the slope of the temperature parameter with respect to altitude. This slope will hereinafter be referred to as the <u>gradient</u>. The following is the general form of each segment of the function:

$$T_M = (T_M)_b + L_M(H - H_b), \tag{8}$$

where

H = geopotential altitude in m',

T_M = the molecular-scale temperature in °K at altitude H,

L_M = the gradient of the molecular-scale temperature in terms of geopotential altitude; i.e., $\partial T_M / \partial H$, in °K m'^{-1}, constant for a particular layer,

H_b = geometric altitude in m' at the base of a particular layer characterized by a specific value of L_M, and

$(T_M)_b$ = the value of T_M at altitude H_b.

3.1.4 Kelvin or absolute temperature scale

In agreement with Resolution 164 of the 1947 meeting of the International Meteorological Organization,[31] and consistent with the ICAO Standard Atmosphere, the absolute temperature in degrees Kelvin of the melting point of ice subjected to atmospheric pressure of 1013.25 mb (or 101,325. newtons m^{-2}) is taken* to be T_i = 273.16°K. Temperatures on the absolute Kelvin scale are related to temperatures on the Celsius scale[44] by the relationship:

$$T(^{\circ}K) = T_i + t(^{\circ}C), \qquad (9)$$

where

T_i = ice-point temperature, 273.16°K (exact)≠,

$t(^{\circ}C)$ = temperature in the thermodynamic Celsius scale.

The magnitude of Kelvin degree and the Celsius degree are equal and hence temperature gradients are numerically the same in both systems.**

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* The Tenth General Conference on Weights and Measures[12,48] has adopted 273.15°K for t_i but this value will not be used in this MODEL.

** For relations between the two metric and two English temperature scales commonly used in scientific and engineering fields refer to Appendix C.

≠ Basic constant

3.1.5 Specific altitude function of molecular-scale temperature

In accordance with the ICAO Standard Atmosphere, $(T_M)_o$, the sea-level value of T_M, is taken to be 15°C (exact) or 288.16°K (exact) by equation (9). This sea-level temperature plus the values of L_M, and the extent of the respectively associated layers completely define the profile of molecular-scale temperature with respect to altitude. The following are the values of L_M and their respectively associated altitude layers employed in this MODEL.

Table of Molecular-Scale Temperature Gradients Versus Altitude ≠

L_M in °K m'$^{-1}$	Atmospheric Layers in m'
-0.0065 exact	-5,000 to 0
-0.0065 exact	0 to 11,000
0.0 exact	11,000 to 25,000
+0.003 exact	25,000 to 47,000
0.0 exact	47,000 to 53,000
-0.0039 exact	53,000 to 75,000
0.0 exact	75,000 to 90,000
+0.0035 exact	90,000 to 126,000
+0.0100 exact	126,000 to 175,000
+0.0058 exact	175,000 to 500,000

These values of L_M, together with equation (8), imply ten specific functions of H to define T_M over the desired altitude intervals. This molecular-scale temperature profile results in the following values of molecular-scale temperature $(T_M)_b$ associated with the base of the respective layers, H_b:

Base Altitudes and the Respective Base Values of Molecular-Scale Temperatures

H_b in m'	$(T_M)_b$ in °K
0	288.16
11,000	216.66
25,000	216.66
47,000	282.66
53,000	282.66
75,000	196.86
90,000	196.86
126,000	322.86
175,000	812.86

≠ Entire table consists of basic constants.

3.1.6 Basis for selecting the temperature-altitude function

The temperature-altitude function of this MODEL was selected to be in exact agreement with the present ICAO Standard Atmosphere which extends from -5,000 m' to 20,000 m'. (The temperature-altitude function is also in agreement with the recently adopted Extension of the Standard Atmosphere to 300,000 m' which was prepared concurrently with this MODEL.) The values of the function between 20,000 m' and 53,000 m' were suggested by Whipple and adopted at the First Meeting[18] of the WGESA. Between 53,000 m' and 500,000 m', the temperature-altitude function is that presented by Minzner[20,42] and adopted to 300,000 m' for the Standard Atmosphere at the Third Meeting of the WGESA.

The linearized temperature-altitude function of this MODEL follows approximately along the average of observed temperatures up to about 90 or 100 km, the highest altitude for which "direct" temperature observations have been reliably made. The pressures and densities inferred by this linearized temperature-altitude function at the various altitudes agree very well with the average of all measured pressures and densities up to 160 km, the maximum altitude of such observations. Agreement between the inferred pressures or densities and the average of observed values was, in fact, the primary criterion for choosing the temperature-altitude function between 70 and 160 km.

Above 160 km, only theoretical approaches are presently available for estimating temperatures, pressures, or densities. Between 160 and 300 km, this MODEL represents an approximate mean value of the recent theoretical estimates of these properties.

For the region above 300 km, there are two basic theories on which to base a temperature-altitude profile. This MODEL follows that theory which results in the higher atmospheric densities at 500 km.

One of these theories, fostered principally by Bates,[1,2] assumes an upward conduction of energy from layers of high solar energy absorptivity, between 100 and 250 km. The proponents of this theory generally deduce an essentially isothermal atmosphere at a temperature between 850° and 1100°K extending upward from 250 or 300 km.

A second theory, proposed by Chapman,[8-10] suggests that the earth is bathed in the solar corona which extends outward from the sun beyond the earth's orbit around the sun. Some of the energy of the very high-temperature (high-velocity) particles comprising the corona, through which the earth is said to move in its orbin, is conducted downward toward the earth's surface. Thus a temperature of the order of 2 x 10^5 °K, a few earth's radii away from the earth, drops to the order of 1000°K at 300 km altitude as the conducted energy is shared by increasing numbers of particles. This theory, therefore, implies a positive real-temperature gradient which Chapman suggests might be of the order of 2.5°K per kilometer, in the 300 to 500 km region. This value corresponds closely with the molecular-scale temperature gradient of 5.8°K/km used in that region of this MODEL.

Neither theory has any strong experimental support at present. The positive temperature-altitude gradient above 300 km was selected for this MODEL, however, because it inferred a higher atmospheric density at 500 km than is inferred by an isothermal atmosphere above 300 km. Higher densities in the vicinity of 500 km altitude are conservative from the point of view of satellite design.

3.2 Pressure

3.2.1 Development of the general pressure-altitude equation

Atmospheric pressure is expressed as a function of altitude through the hydrostatic equation,

$$dP = -g\,\rho\, dZ, \tag{10}$$

where

P = atmospheric pressure in newtons m^{-2},

g = acceleration of gravity in m sec^{-2},

ρ = atmospheric density in kg m^{-3}, and

Z = altitude in m.

The density, ρ, may be eliminated by replacing it with its equivalent in terms of pressure and temperature in the form of the perfect gas law,

$$\rho = \frac{PM}{R^*T}, \tag{11}$$

where

T = atmospheric temperature in °K, and

R^* = universal gas constant; i.e., 8.31439×10^3 joules $(°K)^{-1}$ kg^{-1} (exact). ≠ 11,16,46,47

The value of R^* was chosen to be in agreement with recent determinations of its value and consistent with the ICAO Standard Atmosphere.

The substitution of equation (11) into equation (10) plus some manipulation, leads to the differential form of the barometric equation,

≠ Basic constant

$$d \ln P = \frac{-g M}{R^{*} T} dZ \; . \tag{12}$$

It is to be noted that the pressure is now expressed as a function of T/M. The introduction of molecular-scale temperature from equation (7) and geo-potential from equation (2c) changes equation (12) in five variables to the following equation in only three variables:

$$d \ln P = \frac{-G M_{o}}{R^{*}} \frac{dH}{T_{M}} \; . \tag{13}$$

Equation (13) in turn leads to

$$\ln \frac{P}{P_{b}} = - Q \int_{H_{b}}^{H} \frac{dH}{f(H)} \; , \tag{14}$$

where

P_b = pressure at altitude H_b,

Q = $G\ M_o/R^*$, a constant equal to 0.034,164,794,2 $°K\ m'^{-1}$ ##

f(H) = a functional representation of T_M.

3.2.2 Pressure-altitude equations for linear temperature functions

For purposes of this MODEL, f(H) is defined by equation(8). Thus the integration of equation (14) yields two different forms of the barometric equation, depending on whether L_M of equation (8) is equal to zero or equal to a non-zero constant:

For $L_M = 0$,

$$P = P_{b} \text{ exponential } \frac{-Q(H - H_{b})}{(T_{M})_{b}} \; ; \tag{15}$$

For L_M not equal to zero,

$$P = P_{b} \left[\frac{(T_{M})_{b}}{(T_{M})_{b} + L_{M}(H - H_{b})} \right]^{\frac{Q}{L_{M}}} \; ; \tag{16}$$

Derived constant

where

$(T_M)_b$ = the value of molecular-scale temperature in °K at the base of a layer characterized by a constant value of L_M,

L_M = the value of T_M/H in °K m'$^{-1}$ for a particular altitude region.

The forms of equations (15) and (16) are such that pressure may be computed in any units merely by introducing P_b in terms of the desired units. For numerical computation purposes equation (15) is more usable in the form

$$P = \frac{P_b}{\text{antilog}_{10} \dfrac{\log_{10} e^Q}{(T_M)_b} (H - H_b)}, \tag{17}$$

where

$\text{Log}_{10}\, e$ = .434,294,482///, the modulus of common logarithms.

3.2.3 Sea-level value of pressure

Pressures at all altitudes computed from equation (15) or (16) depend directly on the sea-level value of pressure. In keeping with the ICAO Standard Atmosphere[26-28] and implicit in the Resolution of the Proceedings of the International Committee on Weights and Measures,[44] the sea-level value of pressure, P_o, is taken to be 101,325 newtons m^{-2} or 1,013.25 mb./ This pressure corresponds to the pressure exerted by a column of mercury 760 mm high having a density of 13.595,1... gm cm^{-3} and subject to a gravitational acceleration of 9.80665 m sec^{-2}.

3.2.4 Base pressures for various layers

With P_o used for P_b in equation (16) and using suitable values of $(T_M)_b$ and L_M, the value of P is computed for 11,000 m', the top of the troposphere, the first atmospheric layer above sea level. This value of P, designated by P_{11}, in turn becomes the value of P_b for use in computing the pressure within and at the top of the next layer. In this way the values of P_b for each successive layer are determined. The value adopted in this MODEL for P_o, i.e., 1,013.250 mb or 101,325.0 newtons m^{-2} (exact) is identical to that adopted by ICAO and other prominent groups.[31,46]

/ Basic constant

/// Numerical constant

3.2.5 Specific computational equations

The specific equations for computing pressure for each of ten atmospheric layers (determined by ten molecular-scale temperature functions) are as follows:

For $-5{,}000.0 \text{ m}' \leq H \leq 0.0 \text{ m}'$,

$$P = P_o \left[\frac{288.160 - 6.500{,}00 \times 10^{-3} H}{288.160} \right]^{5.256{,}122{,}18} \tag{16a}$$

where

P_o = atmospheric pressure at sea level, defined to be 101,325.0 newtons m^{-2}, or 1,013.25 mb (exact).⨍

For $0.0 \text{ m}' \leq H \leq 11{,}000 \text{ m}'$,

$$P = \frac{P_o}{\left[\dfrac{288.160}{288.160 - 6.500{,}00 \times 10^{-3} H} \right]^{5.256{,}122{,}18}} . \tag{16b}$$

For $11{,}000 \text{ m}' \leq H \leq 25{,}000 \text{ m}'$,

$$P = \frac{P_{11}}{\text{antilog}_{10} \left[(0.068{,}483{,}253{,}0 \times 10^{-3})(H - 11{,}000.0) \right]} , \tag{17a}$$

where

P_{11} = the pressure at 11 km' computed from equation (16b).

For $25{,}000 \text{ m}' \leq H \leq 47{,}000 \text{ m}'$,

$$P = \frac{P_{25}}{\left[\dfrac{141.660 + 3.000{,}00 \times 10^{-3} H}{216.660} \right]^{11.388{,}264{,}73}} , \tag{16c}$$

⨍ Basic constant

where

P_{25} = the pressure at 25 km' computed from equation (17a).

For 47,000 m' $\leqq$ H $\leqq$ 53,000 m',

$$P = \frac{P_{47}}{\text{antilog}_{10}\left[(0.052{,}492{,}682{,}3 \times 10^{-3})(H - 47{,}000.0)\right]}, \tag{17b}$$

where

P_{47} = the pressure at 47 km' computed from equation (16c).

For 53,000 m' $\leqq$ H $\leqq$ 75,000 m',

$$P = \frac{P_{53}}{\left[\dfrac{282.660}{489.360 - 3.900{,}00 \times 10^{-3} H}\right]^{8.760{,}203{,}64}}, \tag{16d}$$

where

P_{53} = pressure at 53 km' computed from equation (17b).

For 75,000 m' $\leqq$ H $\leqq$ 90,000 m',

$$P = \frac{P_{75}}{\text{antilog}_{10}\left[(0.075{,}371{,}236{,}4 \times 10^{-3})(H - 75{,}000.0)\right]} \tag{17c}$$

where

P_{75} = the pressure at 75 km' computed from equation (16d).

For 90,000 m' $\leqq$ H $\leqq$ 126,000 m',

$$P = \frac{P_{90}}{\left[\dfrac{3.500,00 \times 10^{-3}H - 118.140}{196.860}\right]^{9.761,369,77}}, \qquad (16e)$$

where

P_{90} = the pressure at 90 km' computed from equation (17c).

For 126,000 m' $\leqq$ H $\leqq$ 175,000 m',

$$P = \frac{P_{126}}{\left[\dfrac{10.000,0 \times 10^{-3}H - 937.140}{322.860}\right]^{3.416,479,42}}, \qquad (16f)$$

where

P_{126} = the pressure at 126 km' computed from equation (16e).

For 175,000 m' $\leqq$ H $\leqq$ 500,000m',

$$P = \frac{P_{175}}{\left[\dfrac{5.800,00 \times 10^{-3}H - 202.140}{812.860}\right]^{5.890,481,75}}, \qquad (16g)$$

where

P_{175} = the pressure at 175 km' computed from equation (16f).

3.3 Density

3.3.1 Computational equation

Atmospheric density at altitude H is readily computed from the perfect gas law, equation (11), implicit in the barometric equation. With the introduction of the molecular-scale temperature concept, equation (11) for density in kg m^{-3} becomes,

$$\rho = \frac{M_o}{R^*} \frac{P}{T_M} = 3.483,839,46 \times 10^{-3} \frac{P}{T_M} . \tag{18}$$

where

P = atmospheric pressure in newtons m^{-2} (or mb x 10^2), expressed by equations (16a - 16g) and (17a - 17c),

T_M = molecular scale temperature in °K expressed by equation (8) with its various values of L_M.

The computational equation for ρ is left in terms of P and T_M instead of in terms of H, for to convert to the latter would require ten different functions, as in the case of T_M and P. The computational equations of all other properties of this MODEL will be similarly expressed in terms of P or T_M, rather than in terms of H.

3.3.2 Sea-level value - ratio equation

Evaluating equation (18) at sea level yields the sea-level value of density:

$$\rho_o = \frac{M_o}{R^*} \cdot \frac{P_o}{(T_M)_o} = 1.225,013,998 \text{ kg m}^{-3}, \text{ ♯♯} \tag{18a}$$

where

P_o = sea-level value of P, 101,325.0 newtons m^{-2} (exact)♯, and

$(T_M)_o$ = sea-level value of T_M, 288.16°K (exact).♯

Dividing equation (18) by equation (18a) yields

$$\frac{\rho}{\rho_o} = \frac{P}{P_o} \cdot \frac{(T_M)_o}{T_M} . \tag{18b}$$

3.4 Validity of the Basic Properties

The three basic properties of this atmospheric MODEL are rigorously self-consistent through the perfect gas law and the hydrostatic equation, which accounts for the variations of the effective acceleration of gravity with altitude,

---

♯ Basic constant

♯♯ Derived constant

through the use of geopotential. The user of these tables is warned that the validity of the hydrostatic equation as well as some of the other classical equations, in their simple forms, may decrease considerably at great altitudes.[53] The uncertainties at high altitudes in most equations relating the various atmospheric properties, however, are perhaps small compared with the present uncertainties at these altitudes in the defining property of this MODEL, T/M.

4. Secondary Properties Defined as Functions of T/M

This section is devoted to all those atmospheric properties of the ARDC MODEL ATMOSPHERE, except P and ρ, which are classically defined as functions of the ratio T/M and which are, therefore, conveniently redefined in terms of molecular-scale temperature without otherwise involving M or T explicitly. (Some of the properties of this group depend also upon the acceleration of gravity.) Properties which depend also upon P or ρ, or combinations of these, are implicitly in this group. The properties of this group tabulated in this MODEL are scale height, speed of sound, air-particle speed (arithmetic average), and specific weight.

4.1 Scale Height

4.1.1 Definition

If both sides of equation (12) are divided by dZ, we have

$$\frac{d \ln P}{dZ} = \frac{-gM}{R^*T} \quad . \tag{12a}$$

A dimensional analysis of the quantities in the right-hand side of this equation show that the net dimensions are reciprocal meters. The reciprocal of the right-hand side of equation (12a), by virtue of its dimensions has been given the name "scale height." Thus scale height as tabulated in this MODEL is defined as

$$H_s = \frac{R^*T}{gM} , \tag{19}$$

where

H_s = scale height in m (not m'),

g = acceleration of gravity in m sec^{-2},

and R^*T and M have their usual significance.

4.1.2 Concepts

Using equation (19), equation (12a) may now be rewritten as

$$\frac{d \ln P}{dZ} = \frac{-1}{H_s} , \qquad (12b)$$

and scale height is seen to be the negative reciprocal of the slope of the ln P versus Z curve.

The geometric-altitude-pressure equation for an isothermal atmospheric layer may be manipulated to show that when gravity is considered to be constant, the scale height at any altitude represents the vertical distance above the reference altitude at which the atmospheric pressure has dropped to a value of 1/e of its value at the reference altitude. This concept for scale height is often erroneously thought to apply to an atmosphere in which temperature and gravity vary. A check of pressures and scale heights in the troposphere of this MODEL shows the scale height at sea level to be 8.4344 km. The pressure, however, has dropped to 1/e of its sea-level value at an altitude of 7.68 km, where the scale height is 7.0 km. Since this concept of scale height is developed from the equation for an isothermal constant-gravity atmosphere, the concept will not hold for other conditions.

From the same basic, isothermal, pressure-altitude equation one may demonstrate that the scale height at any altitude is the length to which the total of a unit cross-section column of the atmosphere above that point would be compressed, if subjected to the pressure and gravity of that altitude. That is, the reduced thickness of the residual, isothermal, constant-gravity atmosphere above a given altitude, when subjected to the pressure of that altitude, is equal to the scale height. Again this concept does not apply rigorously anywhere in this MODEL since the atmosphere is not indefinitely isothermal above any point, neither is the gravity constant.

4.1.3 Definition of geopotential scale height

The limitations imposed by constant gravity in the latter two concepts of scale height can be eliminated through the use of a <u>geopotential scale height.</u> If both sides of equation (13) are divided by dH, we obtain

$$\frac{d \ln P}{dH} = \frac{-G\, M_o}{R^* T_M} , \qquad (13a)$$

A dimensional analysis of the right-hand side of this equation shows the net dimensions to be reciprocal geopotential meters. Thus the reciprocal of this equation serves to define geopotential scale height:

$$H_s' = \frac{R^* T_M}{G M_0} , \qquad (13b)$$

where

H_s' = geopotential scale height in m', and

G = $9.80665 \ m^2 \ sec^{-2} \ m'^{-1}$.

4.1.4 Concept of geopotential scale height

The combining of equations (13a) and (13b) yields

$$\frac{d \ln P}{dH} = \frac{-1}{H_s'} , \qquad (13c)$$

and the geopotential scale height is seen to be the negative reciprocal of the slope of the ln P versus H curve.

The manipulation of equation (15) (for a variable-gravity, isothermal atmosphere) leads to the conclusion that for a variable-gravity, isothermal atmosphere, the geopotential scale height at any altitude represents the increment in geopotential above the reference altitude at which the atmospheric pressure has dropped to a value of 1/e of its value at the reference altitude. This concept does apply rigorously to isothermal regions of this MODEL. Equation (15) also leads to the conclusion that the geopotential scale height at any altitude is the reduced thickness in geopotential of the residual, isothermal, variable-gravity atmosphere above a given altitude when subjected to the pressure of that altitude. Even though this concept accounts for variable gravity, it still is not rigorously applicable to the MODEL since no indefinite isothermal atmosphere to great altitudes is speculated in this MODEL.

The geopotential scale height at any altitude is readily transformed to a geometric length by adding the geopotential scale height to the reference geopotential altitude and converting the resulting geopotential measure to geometric altitude, by means of equation (6). Then the reference geopotential altitude is converted to geometric altitude with the same equation. Finally, the smaller geometric altitude is subtracted from the larger. The difference is the equivalent geometric length for the geopotential scale height at the reference altitude.

While geopotential scale height is obviously the preferable parameter from the point of view of using the several concepts in a variable-gravity atmosphere, only geometric scale height from equation (19) will be tabulated in this edition of the ARDC MODEL.

4.1.5 Computational equation for (geometric) scale height

Introducing T_M from equation (7) into equation (19) leads to the computational equation for H_s:

$$H_s = \frac{R^* T_M}{M_0 g} = 287.039{,}632{,}6 \left[\frac{T_M}{g}\right] . \qquad (19a)$$

4.1.6 Sea-level value and ratio equation

The sea-level value of H_s is obtained by evaluating equation (19a) at sea level, such that

$$(H_s)_0 = \frac{R^* (T_M)_0}{M_0 g_0} = 8.434{,}413{,}43 \times 10^3 \text{ m } \not\not \qquad (19b)$$

where

$(H_s)_0$ = sea-level value of H_s,

$(T_M)_0$ = sea-level value of T_M, 288.16°K (exact), $\not$

g_0 = sea-level value of g, 9.806,65 m sec^{-2} (exact) $\not$

Dividing equation (19a) by (19b) yields

$$\frac{H_s}{(H_s)_0} = \frac{T_M}{(T_M)_0} \frac{g_0}{g} , \qquad (19c)$$

which is an alternate form for computing values of H_s.

4.1.7 Validity

Because the analytical expression for scale height is implicit in the barometric equation, as is evident from equation (12), the validity of the value of H_s at various altitudes depends directly on the validity of the barometric equation. (Scale height from this consideration might also be considered one of the basic properties along with pressure and density.) The use

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$\not$ Basic constant

$\not\not$ Derived constant

of the tabulated values of scale height, however, in connection with several commonly accepted concepts of scale height is to be avoided except for rough approximations.

4.2 Speed of Sound

4.2.1 Defining equation

The square of the speed of sound propagation is defined in this MODEL to be

$$C_s{}^2 = \frac{\gamma P}{\rho}, \tag{20}$$

where

C_s = speed of sound in m sec^{-1},

P = pressure in newtons m^{-2},

ρ = density in kg m^{-3}, and

γ = ratio of specific heat of air at constant pressure to the specific heat of air at constant volume, defined to be 1.4 (dimensionless, exact.≠)

4.2.2 Computational equation

Eliminating ρ between equations (18) and (20) and extracting the square root results in:

$$C_s = \left[\frac{\gamma R^*}{M_o} T_M\right]^{\frac{1}{2}} = 20.046,333,47\ (T_M)^{\frac{1}{2}}. \tag{20a}$$

4.2.3 Sea-level value and ratio equation

Evaluating equation (20a) at sea level yields

$$(C_s)_o = \left[\frac{\gamma R^*}{M_o} \cdot (T_M)_o\right]^{\frac{1}{2}} = 340.292,046 \text{ m sec}^{-1}, \; ≠≠ \tag{20b}$$

where

$(C_s)_o$ = sea-level value of C_s.

≠ Basic constant

≠≠ Derived constant

Dividing equation (20a) by equation (20b) reduces the number of constants so that:

$$\frac{C_s}{(C_s)_o} = \left[\frac{T_M}{(T_M)_o}\right]^{\frac{1}{2}} . \qquad (20c)$$

4.2.4 Validity

These equations for computing the velocity of sound apply only when the sound wave is a small perturbation on the ambient condition. Harrison[24] has shown that even when this condition is met, the above definition for the velocity of sound is not quite correct for two reasons: First, γ is not really a constant, but rather, varies with pressure and temperature over a small region around the value 1.4; second, the form of the above relationship is not completely correct, since even if the best value of γ is used for a given set of conditions, computed values of C_s differ slightly from experimentally determined values. In spite of these discrepancies, however, the stated relationships are adopted in accordance with Subcommittee recommendations[43] which are in conformity with established aerodynamic practice but at variance with the present United States Standard Atmosphere.

The limitations of the concept of velocity of sound due to extreme attenuation are also of concern. This situation exists for high frequencies at sea-level pressures and applies to successively lower frequencies as atmospheric pressure decreases, or as mean free path increases. For this reason the concept of speed of sound progressively loses its meaning at high altitudes, except for frequencies approaching zero and for very short distances. To call attention to this limitation, it was agreed to terminate at 90 km' the tabulation of the velocity of sound, in the Extension to the United States Standard Atmosphere. In conformity with this agreement, tabulations in this MODEL are also similarly terminated. Because of the relationship between sound velocity and air particle speed (Section 4.3), sound velocities for altitudes above 90 km' may readily be obtained for use with suitable caution.

4.3 Air Particle Speed (Arithmetic Average)

4.3.1 Concept

The mean air particle speed is the arithmetic average of the distribution of speeds of all air particles within a given elemental volume. This quantity has significance provided that the volume considered contains a sufficiently large number of particles so that their velocities follow a Maxwellian distribution, and provided that variations of ρ and T/M in any direction are negligible within the volume element.

4.3.2 Defining equation

Arithmetic average of air particle speed is defined to be:

$$\bar{V} = \left[\frac{8\,R^*}{\pi} \frac{T}{M} \right]^{\frac{1}{2}}, \tag{21}$$

where

$\bar{V}$ = air particle speed (arithmetic average) in m sec^{-1},

π = 3.141,592,654 (dimensionless).///

4.3.3 Computational equation

The introduction of T_M from equation (7) into equation (21) yields the computation equation for $\bar{V}$:

$$\bar{V} = \left[\frac{8\,R^*}{\pi M_o} T_M \right]^{\frac{1}{2}} = 27.035,909,86\ (T_M)^{\frac{1}{2}} \tag{21a}$$

4.3.4 Sea-level value and ratio equation

Evaluating equation (21a) at sea level leads to

$$\bar{V}_o = \left[\frac{8\,R^*}{\pi M_o} (T_M)_o \right]^{\frac{1}{2}} = 458.942,035 \text{ m sec}^{-1} \quad // \tag{21b}$$

where

$\bar{V}_o$ = sea-level value of $\bar{V}$.

Equation (21a) divided by equation (21b) yields

$$\frac{\bar{V}}{\bar{V}_o} = \left[\frac{T_M}{(T_M)_o} \right]^{\frac{1}{2}} \tag{21c}$$

4.3.5 Validity

On considering the restrictions applied to the volume element for which we desire the value of $\bar{V}$, it is evident that these restrictions come

// Derived constant

/// Numerical constant

into conflict with each other at high altitudes and the validity of the concept of $\bar{\bar{V}}$ decreases with altitude. It is uncertain whether or not the concept retains reasonable significance at altitudes as great as 500 km. Nevertheless, as in the case of pressures and densities, etc., values have been tabulated to this altitude, on the basis that with suitable caution, such values are better than no values.

4.3.6 Relationship to sound velocity

From a comparison of equation (20c) and equation (21c) it is evident that

$$\frac{C_s}{(C_s)_o} = \frac{\bar{V}}{\bar{V}_o} . \qquad (22)$$

Since values of $\bar{V}/\bar{V}_o$ are tabulated to 500 km', values of $C_s/(C_s)_o$ and hence values of C_s are readily available to the same altitude, even though their significance is extremely questionable.

4.4 Specific Weight

4.4.1 Concept

The specific weight ω of a body of uniform density at any particular point in space is the weight per unit volume of that body at that point. The weight per unit volume is equal to the mass per unit volume times the acceleration of gravity, which in turn is equal to the density of the body times the acceleration of gravity, g. Since g is assumed to vary in this MODEL in accordance with equation (1a), the specific weight of a body will vary proportionately.

The density of the air mass also varies with altitude and hence ω is dependent upon two variables, ρ and g. This is at variance with the procedure in the ICAO Standard Atmosphere in which specific weight is defined to vary only with ρ .

4.4.2 Defining and computational equation

In this MODEL specific weight is defined by

$$\omega = \rho g,$$

where

ω = specific weight in kg m^{-2} sec^{-2} or newtons m^{-3} (at any point),

ρ = density in kg m^{-3} (at the point),

g = acceleration of gravity in m sec^{-2} (at the point).

Eliminating ρ by means of equation (18) results in

$$\omega = \frac{g\ M_o\ P}{R^* T_M} = 3.483,839,46 \times 10^{-3} \frac{g\,P}{T_M} \quad . \tag{23a}$$

4.4.3 Sea-level value and ratio equation

The evaluation of equation (23) and (23a) at sea level yields

$$\omega_o = \rho_o\ g_o = \frac{M_o P_o g_o}{R^*(T_M)_o} = 12.013,283,5 \text{ kg m}^{-2}\text{sec}^{-2}, \ /\!/\!\!\!/ \tag{23b}$$

where

ω_o = sea-level value of ω ,

ρ_o = sea-level value of ρ , 1.225,014,00 kg m^{-3}, ≠≠

g_o = sea-level value of g, 9.806,65 (exact). ≠

Dividing equations (23) and (23a) by the appropriate portions of equation (23b) results in:

$$\frac{\omega}{\omega_o} = \frac{\rho}{\rho_o}\ \frac{g}{g_o} = \frac{P}{P_o}\ \frac{(T_M)_o}{T_M}\ \frac{g}{g_o} \ . \tag{23c}$$

Introducing H_s from equation (19a) into the right-hand member of equation (23c) leads to:

$$\omega = \frac{P\,M}{R^* T} \cdot g = \frac{P}{H_s} \quad . \tag{23d}$$

4.4.4 Validity

The validity of the values of ω depends only upon the validity of the values of g and ρ which have already been discussed.

≠ Basic constant

≠≠ Derived constant

5. Other Secondary Properties

The last group of properties of this ARDC MODEL ATMOSPHERE includes all those properties considered in this MODEL which are defined by functions of T and M, in forms different from T/M, so that these functions cannot be redefined in terms of molecular-scale temperature without the additional use of either M or T in its independent form. This group includes molar volume, number density, mean free path, collision frequency, coefficient of viscosity, and kinematic viscosity, as well as temperature and molecular weight. Either molecular weight or temperature must now be defined in terms of altitude before any of these remaining secondary properties can be computed. The molecular weight is the one specifically defined in this MODEL.

5.1 Molecular Weight

5.1.1 General definition

Molecular weight is defined to be dimensionless. On the chemical scale* molecular weight (of a compound) is defined to be 16 times the ratio of the average mass of a molecule of the compound to the average mass of an oxygen atom, where both the oxygen and the compound are assumed to have their natural distribution of isotopes, and where average is to be construed as the arithmetic mean.

5.1.2 Concept applied to air

The definition of molecular weight includes the concept of a mixture of the several isotopes of an atomic species and the resulting mixture of similar molecules of different masses. Therefore, it is not unreasonable to extend the definition of molecular weight to include mixtures of different kinds of molecules as in the atmosphere. Such an extension of the basic definition is employed in this MODEL in establishing the concept of the molecular weight of air.

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* The definitions of atomic or molecular weights on the physical scale are more specific than the equivalent definitions on the chemical scale, in that on the physical scale, the ratios are established with reference to the mass of an atom of a specific oxygen isotope, O^{16}. Because the mass of an O^{16} atom is less than the mass of an average oxygen atom, the atomic or molecular weights on the physical scale are greater than on the chemical scale by approximately the ratio 32.0087/32.0000 . When the physical scale is used for expressing molecular weight, values of the universal gas constant, R^*, and other constants must be proportionately changed.

5.1.3 Molecular weight of air and mole defined

Molecular weight of air, M, is defined as 16 times the ratio of the arithmetic mean mass of a single molecule of the air mixture to the arithmetic mean mass of a single atom of oxygen in a natural mixture of the several oxygen isotopes.

A kilogram mole of air is defined as a quantity of air having a mass in kilograms numerically equal to the molecular weight of the air.

5.1.4 Sea-level and low-altitude value of molecular weight of air

The value of M at sea level is determined from an assumed distribution of the several atmospheric constituents at sea level. In accordance with the ICAO agreements the atmosphere of this ARDC MODEL is assumed to be dry and to have the following composition at sea level and at all altitudes up to and including 20 km'. This model has assumed a continuation of this composition up to 90 km'.

Constituent Gas	Mol. Fraction Per Cent	Molecular Weight (O = 16.000)
Nitrogen (N_2)	78.09	28.016
Oxygen (O_2)	20.95	32.0000
Argon (A)	0.93	39.944
Carbon dioxide (CO_2)	0.03	44.010
Neon (Ne)	1.8×10^{-3}	20.183
Helium (He)	5.24×10^{-4}	4.003
Krypton (Kr)	1.0×10^{-4}	83.7
Hydrogen (H_2)	5.0×10^{-5}	2.0160
Xenon (Xe)	8.0×10^{-6}	131.3
Ozone (O_3)	1.0×10^{-6}	48.0000
Radon (Rn)	6.0×10^{-18}	222.

The above data yield a value of 28.966 (nondimensional) for the molecular weight of air. In this MODEL the molecular weight of air at sea level, and for a considerable altitude above and below sea level, is defined as a constant. Thus

for $-5,000\ m' \leq H \leq 90,000\ m'$,

$$M = 28.966\ . \tag{24}$$

5.1.5 Molecular weight of air at high altitudes and validity of the values

Atmospheric composition at high altitudes is thought to vary considerably from that near sea level. The variation in composition may result from dissociation of various molecules of the atmosphere as well as from diffusive separation of molecules of various masses in a gravitational field. While several theories describing these phenomena exist, there are only a few data to support or disprove these theories. The choice of 90,000 m' as the top of the region of constant composition is quite arbitrary but is as good as any other current choice.

It is thought that the dissociation of O_2 is the principal factor in producing a change in molecular weight between 90,000 and 175,000 m'. Rocket measurements of O_2 concentration obtained by Byram, Chubb, and Friedman provide partial support to this contension. Diffusive separation and the dissociation of N_2 is thought to dominate the variation of molecular weight of the mixture of atmospheric gases above 175,000 m' .

Miller[39] combined these theories, assumptions, and data with scale height gradients of this MODEL and computed molecular weights for specific altitudes between 90,000 and 500,000 m'. A plot of these data versus altitude suggested the possibility of approximating the graph with two analytical functions. Campen of GRD developed the desired functions in the form of the following two equilateral hyperbolae which for this MODEL define molecular weight from 90 to 500 km.

For $90,000\ m' \leq H \leq 175,000\ m'$,

$$M = \frac{23.160,126,7\ H - 1,757,856.05}{H - 78,726.25}\ . \tag{24a}$$

For $175,000\ m' \leq H \leq 500,000\ m'$,

$$M = \frac{13.139,119,0\ H + 514,492.02}{H - 56,969.89}\ . \tag{24b}$$

For purposes of defining other atmospheric properties, it is convenient to establish the following relationships:

$$M = |M'| \quad , \text{ and} \tag{24c}$$

$$\frac{M}{M_0} = \frac{M'}{M'_0} \quad , \tag{24d}$$

where

M' is a kilogram mole of air, a mass in kg numerically equal to the molecular weight, and

M'_0 is the sea-level value of M'.

Using equation (5), relating geopotential and geometric altitude, equations (24), (24a) and (24b) are converted to the following in terms of Z:

For $-4{,}996.070{,}27 \text{ m} \leq Z \leq 91{,}292.532{,}7 \text{ m}$,

$$M = 28.966. \tag{25}$$

For $91{,}292.532{,}7 \text{ m} \leq Z \leq 179{,}954.085 \text{ m}$,

$$M = \frac{23.170{,}552{,}5\ Z - 1{,}779{,}899.46}{Z - 79{,}713.475{,}7} \ . \tag{25a}$$

For $179{,}954.085 \text{ m} \leq Z \leq 542{,}685.673$,

$$M = \frac{13.339{,}605{,}8\ Z + 519{,}144.64}{Z - 57{,}485.075{,}2} \ . \tag{25b}$$

These equations yield results within ± 1% of Miller's values at all altitudes except for a small region around 105 km where the analytical results are about 3% higher than Miller's values.

5.2 Mol Volume

5.2.1 Concept and definition

Density of the air at any altitude is expressed as the mass per unit volume at that altitude. If the mass is that of a mole of air, the related volume is that of a mole of air. Thus the mol volume of air is given by

$$v = \frac{M'}{\rho}, \tag{26}$$

where

v = the volume (in m^3) of a mole of atmospheric gas at a particular altitude,

ρ = the density (in kg m^{-3}) of air at the same altitude, and

M' = the kilogram molecular weight, the mass in kg of a kilogram mole of air having the composition of this altitude. (This mass is numerically equal to the molecular weight defined by equations (24), (24a), and (24b).)

5.2.2 Computational equation

Eliminating ρ between equations (18) and (26) yields a computational* expression for v in terms of basic properties and constants:

$$v = \frac{R^* M' T_M}{M_o P} = 287.039,632,6 \; \frac{M' T_M}{P} \; , \tag{26a}$$

where

R^* = universal gas constant, 8.314,39 x 10^3 joules $(°K)^{-1}$ kg^{-1} (exact)$\not$,

M_o = sea-level value of molecular weight, 28.966 (dimensionless, exact)$\not$

T_M = molecular scale temperature, in °K, at the altitude in question, and

P = atmospheric pressure in newtons m^{-2} (or mb x 10^2).

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* Values of v are not tabulated for various altitudes in this edition of the MODEL but the equations are developed for use in the expressions for number density and implicitly mean free path. It will be noted from a comparison of equations (26c) and (28c) that $v/v_o = L/L_o$. Thus values of v for any altitude are readily available from these tables.

$\not$ Basic constant

5.2.3 Sea-level value and ratio equation

Equations (26) and (26a) evaluated at sea level yield:

$$v_o = \frac{M'_o}{\rho_o} = \frac{R^* M'_o (T_M)_o}{M_o P_o} = 23.645,444,1 \text{ m}^3, \quad /\!/ \qquad (26b)$$

where

v_o = the sea-level value of v,

M'_o = a mole of air at sea level, 28.966 kg (exact)/ /,

ρ_o = sea-level value of ρ , 1.225,013,998 kg m^{-3}, / /

$(T_M)_o$ = the sea-level value of T_M, 288.16°K (exact)/, and

P_o = the sea-level value of P, 101,325.0 newtons m^{-2}(exact)/.

From equations (24d), (26), (26a), and (26b) it is obvious that

$$\frac{v}{v_o} = \frac{M'}{M'_o} \cdot \frac{\rho_o}{\rho} = \frac{M}{M_o} \cdot \frac{T_M}{(T_M)_o} \cdot \frac{P_o}{P} . \qquad (26c)$$

5.2.4 Ice-point value

The (standard) ice-point value* of the volume of a mole of gas is considered to be one of the basic physical constants. This value may be computed by evaluating equation (27) at the ice point, i.e., at a temperature of 273.16° K and a pressure of 101,325.0 newtons m^{-2} (1013.250 mb),

$$v_i = \frac{M'_o}{\rho_i} = \frac{R^* M'_o (T_M)_i}{M_o P_o} = 22.414,594,3 \text{ m}^3, \quad /\!/ \qquad (26d)$$

/ Basic constant

/ / Derived constant

* These conditions referred to as standard conditions by chemists are not to be confused with the standard sea-level values of the standard atmosphere where the $T_o = (T_M)_o = 288.16$.

where

v_i = the ice-point value of v, and

$(T_M)_i$ = the ice-point value of T_M = 273.16° K (exact)≠,

ρ_i = the ice-point value of ρ , 1.292,283,037 from the left-hand members of equation (26d).

The above value of v_i for a kilogram mole is in keeping with 22.4146 m^3, the value currently accepted outside of the realm of this standard. (The latter is equivalent to 22,414.6 cm^3 for a gram mole.)

5.2.5 Validity

The validity of the concept of molar volume at great altitudes becomes vague because the volume becomes so large that density and molecular weight cannot be assumed to remain constant throughout the volume and hence the specified volume will most probably not contain exactly one mole of atmospheric gases.

5.3 Number Density

5.3.1 Concept and definition

The number density of air is defined to be the number of atmospheric particles per unit volume, considering only neutral or ionized atoms or molecules. (Electrons and other subatomic particles are ignored.) The number of particles contained in a mole of air is by definition Avogadro's number. Thus Avogadro's number divided by the mol volume yields number density, i.e. :

$$n = \frac{N}{v} , \tag{27}$$

where

n = atmospheric-particle, number density, at a specified altitude, in m^{-3},

v = mol volume at that altitude in m^3, and

N = Avogadro's number, $6.023,80 \times 10^{26}$ (dimensionless, exact)≠ [16,46]

A more recent value of N might have been used but that would not be consistent with the current values adopted by the National Research Council.[46]

≠ Basic constant

5.3.2 Computational equation

Introducing equation (26a) into equation (27) leads to that computational form of the expression for number density in terms of basic properties and constants:

$$n = \frac{N M_0 P}{R^* M' T_M} = 2.098,595,21 \times 10^{24} \frac{P}{M' T_M} \quad . \tag{27a}$$

5.3.3 Sea-level value and ratio equation

Upon evaluation of equation (27) and (27a) at sea level, one obtains:

$$n_0 = \frac{N}{v_0} = \frac{N M_0 P_0}{R^* M'_0 (T_M)_0} = 2.547,552,07 \times 10^{25} \text{ m}^{-3}, \tag{27b}$$

where

n_0 = the sea-level value of n,

v_0 = the sea-level value of v.

The manipulation of equations (27),(27a), and (27b) and reference to equations (26c) and (24d) show the following relationships to exist:

$$\frac{n}{n_0} = \frac{v_0}{v} = \frac{\rho}{\rho_0} \cdot \frac{M_0}{M} = \frac{M_0}{M} \cdot \frac{(T_M)_0}{T_M} \cdot \frac{P}{P_0} \quad . \tag{27c}$$

5.3.4 Validity

In the form of equation (27) the validity of n would be open to considerable question at high altitudes. In terms of equation (27a), however, where all the parameters are defined at a point or within a volume considerably smaller than v, the validity of n is probably limited principally by the validity of the values of T_M and M.

5.4 Mean Free Path

5.4.1 Concept and definition

Mean free path is the mean value of the distances traveled by each of the molecules of a given volume between successive collisions with other molecules of that volume, provided that a sufficiently large number of molecules are contained within the volume. It is usually considered necessary that the volume be the cube of a length many orders of magnitude greater than the mean free path. From kinetic theory and assuming a gas of uniform temperature and density, the following expression for mean free path is developed:

$$L = \frac{1}{\sqrt{2}\,\pi\sigma^2 n}, \qquad (28)$$

where

L = mean free path in m at a particular altitude,

n = number density in m^{-3} at the same altitude,

π = a numerical constant, 3.141,592,654 ≁

σ = average effective collision diameter, taken to be exactly 3.65×10^{-10} m for this MODEL. ≁

This value of σ is an arbitrarily adopted average of several published values.

5.4.2 Computational equation

Eliminating n between equation (27a) and equation (28) yields:

$$L = \frac{R^* M' T_M}{\sqrt{2}\,\pi\sigma^2 N M_o P} = 8.050,460,475 \times 10^{-5} \frac{M' T_M}{P}. \qquad (28a)$$

5.4.3 Sea-level value and ratio equation

The evaluation of equations (28) and (28a) at sea level results in:

$$L_o = \frac{1}{\sqrt{2}\,\pi\sigma^2 n_o} = \frac{R^* M'_o (T_M)_o}{\sqrt{2}\,\pi\sigma^2 N M_o P_o} = 6.631,722,3 \times 10^{-8} \text{ m}, \qquad (28b)$$

where

L_o = sea-level value of L,

n_o = sea-level value of number density, $2.547,552,07 \times 10^{25}$ m^{-3}.

≁ Basic constant

Equation (28a) divided by the right-hand member of equation (28b) and the use of equation (24d) leads to the following ratio equation:

$$\frac{L}{L_o} = \frac{M}{M_o} \cdot \frac{(T_M)_o}{T_M} \cdot \frac{P}{P_o} \quad . \tag{28c}$$

A comparison of equations (26c), (27c), and (28c) shows that:

$$\frac{L}{L_o} = \frac{v}{v_o} = \frac{n_o}{n} = \frac{\rho_o}{\rho} \cdot \frac{M}{M_o} = \frac{M}{M_o} \cdot \frac{(T_M)_o}{T_M} \cdot \frac{P}{P_o} \quad . \tag{28d}$$

5.4.4 Validity

Equation (28) for mean free path is based on the concept that temperature and density are uniform throughout a volume equal to the cube of a length many orders of magnitude greater than the mean free path. At 90,000 m' the mean free path is 2.5 cm. A length two orders of magnitude greater than L would be 2.5 meters and a cube of this dimension is perhaps approaching the smallest size cube which contains a sufficient number of molecules at this altitude to rigorously apply the derivation of equation (28). Temperatures and densities within this volume may certainly be considered constant. At higher altitudes, however, this may no longer be true for the necessary size cube.

In this MODEL, the value of L from equation (28) becomes 1 meter at 114,000 m'. A cube of length two orders of magnitude larger, a 100-meter cube, would have a change in density from top to bottom of about 1%. This amount is considerably more than should be tolerated for the conditions of rigorous validity of the equation for L. At an altitude of 210,000 m', the value of L is 1 kilometer; while at 390,000 m', the value of L is 100 kilometers. Certainly at these altitudes the density is not uniform throughout a sufficiently large cube and the distance through which a molecule will travel between successive collisions depends on its direction of motion. The value of L from equation (28) for a given altitude requires that conditions along the path of the molecule remain equal to those at the particular altitude. At high altitudes this condition can only be met for those molecules moving in a horizontal direction. For molecules moving vertically downward, the distance traveled between collisions will be less than L, because the motion is into a region of exponentially increasing density. For molecules moving vertically upward, the distance traveled between collisions will be greater than L because the motion is into a region of exponentially decreasing density. Some kind of average of these directional mean free path lengths, considering all possible directions, is suggested as a more general concept of mean free path at these altitudes. An unpublished study at GRD shows that the horizontal mean free path, obtained from equation (28), yields values which agree well with this newly suggested mean free path concept to altitudes of about 220,000 m'. Above this altitude, equation (28) should only apply to a horizontal mean free path.

5.5 Collision Frequency

5.5.1 Concept and definition

The average velocity of the molecules or atoms within any given volume of air, divided by the mean free path of the molecules within that volume yields the mean collision frequency of the molecules of that volume. That is, any particular molecule in that volume will collide successively with other molecules at a mean rate given by the collision frequency. Analytically collision frequency is defined by

$$\nu = \frac{\bar{V}}{L}, \tag{29}$$

where

ν = the collision frequency in sec^{-1},

$\bar{V}$ = the average particle velocity in m sec^{-1}, and

L = the mean free path in m.

5.5.2 Computational equation

Equation (21a) for $\bar{V}$ divided by equation (28a) for L leads to:

$$\nu = 4\sigma^2 N \cdot \left[\frac{\pi M_o}{R^*}\right]^{\frac{1}{2}} \cdot \frac{P}{M'(T_M)^{\frac{1}{2}}} = 3.358,306,019 \times 10^7 \frac{P}{M'(T_M)^{\frac{1}{2}}}. \tag{29a}$$

5.5.3 Sea-level value and ratio equation

From the evaluation of equations (29) or (29a) at sea level one obtains:

$$\nu_o = \frac{\bar{V}_o}{L_o} = 4\sigma^2 N \cdot \left[\frac{\pi M_o}{R^*}\right]^{\frac{1}{2}} \cdot \frac{P_o}{M'_o (T_{M_o})^{\frac{1}{2}}} = 6.920,404,9 \times 10^9 \ sec^{-1}, \tag{29b}$$

where

$\bar{V}_o$ = 458.942,034 m sec^{-1},

L_o = 6.631,722,29 x 10^{-8} m.

Equations (29), (29a) and (29b) permit the following ratio expressions:

$$\frac{\nu}{\nu_o} = \frac{\bar{V}}{\bar{V}_o} \cdot \frac{L_o}{L} = \frac{P}{P_o} \cdot \frac{M_o}{M} \cdot \left[\frac{(T_M)_o}{T_M} \right]^{\frac{1}{2}} . \qquad (29c)$$

5.5.4 Validity

The validity of the value of ν is limited principally by the validity of L. Even with the broader concept of L suggested in Section 5.4.4, the value of L should not apply without restrictions above 220 to 250 km. Similarly, values of ν must not be used without caution above this altitude.

5.6 Temperature (Real Kinetic)

5.6.1 Concept and validity

Temperature in this MODEL is a measure of the kinetic energy of the molecules and atoms comprising the atmosphere at any specified altitude. Tabulated values most probably will not indicate the temperature of any body suspended in or passing through the region.

The determination of the value of atmospheric temperature, T, at any given altitude, from conventional measuring techniques requires a knowledge of molecular weight M of the air at that altitude. Without this knowledge of molecular weight, the measurement yields only the value of T/M. Because values of M have not been measured at high altitudes, the so-called temperature measurements from rockets yield only the ratio T/M. This ratio, however, was shown to relate the basic atmospheric properties of pressure, density, specific weight, scale height, particle speed and sound speed. The altitude function of this ratio, T/M, in the form of molecular scale temperature, T_M, defines the altitude functions of these properties.

With the establishment of the independent assumption regarding the altitude function of molecular weight in Section 5.1, it is now possible to specify values of T with the same degree of reliability as exists in the values of M. These values of T will then permit the determination of the coefficient of viscosity and kinematic viscosity from empirical expressions involving T.

5.6.2 Computational equation

The computational equation for real temperature follows directly from the definition of molecular-scale temperature in equation (7). Thus,

$$T = T_M \frac{M}{M_o} = .034,523,234,1 \; M \cdot T_M , \qquad (30)$$

where

T = temperature (real kinetic, absolute scale) at any specified altitude, and

T_M = molecular scale temperature (absolute scale) at that altitude.

5.6.3 Sea-level value and ratio equation

Equation (30) evaluated at sea level yields:

$$T_o = (T_M)_o \frac{M_o}{M_o} = (T_M)_o = 288.16^\circ \text{ K (exact)},^{\neq\neq} \qquad (30a)$$

where

T_o = sea-level value of T, and

$(T_M)_o$ = sea-level value of T_M defined to be 288.16° K (exact)$^{\neq}$.

From the division of equation (30) by (30a), one obtains:

$$\frac{T}{T_o} = \frac{T_M}{(T_M)_o} \cdot \frac{M}{M_o} \qquad (30b)$$

5.7 Coefficient of Viscosity

5.7.1 Concept

Viscosity of a fluid (or gas) is a kind of internal friction which resists the relative motion between adjacent regions of a fluid. If two very large parallel plates surrounded by a gas (at normal pressures) are moving relative to each other so that their separation remains constant, experiments show that the layer of gas directly at the surface of each plate is at rest with respect to that plate. It has also been shown that each layer of gas exerts a

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$\neq$ Basic constant

$\neq\neq$ Derived constant

drag on the neighboring layers so that there exists a velocity gradient normal to the surface of the plates. If the plates are sufficiently close, the velocity gradient is constant. The relative motion of the plates is resisted by a drag force proportional to the product of the area of the plates times the normal velocity gradient of the fluid. The proportionality factor in this relationship is known as the coefficient of viscosity μ. This proportionality factor has been found to vary with the temperature of the gas, but to be independent of the gas pressure within limited ranges of pressure. Various people have contributed to the development of a theoretical expression for μ from kinetic theory and Chapman[7] has recently derived cumbersome formulas which accurately represent the dependence of μ on the temperature, at least over the range of 100—1500° K. Because of the complexity of the Chapman equations, however, the values for coefficient of viscosity in this MODEL are computed from the well-known empirical Sutherland's equation, with coefficients as used by the National Bureau of Standards.[25]

5.7.2 Computational equation

Sutherland's empirical equation for computing viscosity is

$$\mu = \frac{\beta T^{3/2}}{T + S} , \tag{31}$$

where

μ = viscosity in kg sec^{-1} m^{-1}
(1 kg sec^{-1} m^{-1} = 10 poise),

β = 1.458 x 10^{-6} kg sec^{-1} m^{-1} (°K)$^{-\frac{1}{2}}$ (exact),$\nmid$

S = 110.4° K (exact),$\nmid$

T = temperature in °K .

5.7.3 Sea-level value and ratio equation

The sea-level value of μ is

$$\left.\begin{aligned} \mu_o = \frac{\beta T_o^{3/2}}{T_o + S} &= 1.789{,}428{,}53 \times 10^{-5} \text{ kg m}^{-1} \text{ sec}^{-1} \\ &= 1.789{,}428{,}53 \times 10^{-4} \text{ poise,} \end{aligned}\right\} \tag{31a}$$

where

μ_o = the sea-level value of μ ,

T_o = the sea-level value of T.

$\nmid$ Basic constant

Equation (31) divided by equation (31a) yields the ratio equation:

$$\frac{\mu}{\mu_o} = \left[\frac{T}{T_o}\right]^{3/2} \left[\frac{T_o + S}{T + S}\right] . \qquad (31b)$$

5.7.4 Validity

The users of this MODEL are cautioned that the value of the coefficient of viscosity determined by equation (31) is open to question for conditions of very high and very low values of pressure and density. While equation (31) suggests that the coefficient of viscosity is independent of pressure and depends only on temperature, the measurement of μ with an oscillating disk viscometer indicates this situation to be true only within certain limits of pressure, of the order of 2 to .1 atmospheres.

As the pressure decreases below .1 atmosphere, a point is reached where μ begins to fall off with further decrease in pressure in a manner which depends upon the size of the viscometer. This change in the dependence of μ first occurs when the mean free path of air molecules becomes some small fraction of a linear dimension characteristic of the apparatus or other body. Such a dimension in the case of the viscometer would be the distance between plates.

As the pressure is decreased still further, a point is reached when the mean free path becomes equal to or greater than this characteristic dimension. At this point the viscous stress (drag force per unit area) becomes directly proportional to the quadruple product of density of the gas, velocity of the moving plates or other body, one-fourth the mean speed of the molecules, and a function indicating the reflective properties of the surfaces. This situation characterizes the "free-molecule region" of the gas.

For pressures in between the free-molecule region and the region characterized by viscosity independent of pressure, there exists for any particular viscometer a transition region where the coefficient of viscosity is neither independent of pressure nor directly proportional to it, and the relationship is rather difficult to treat theoretically. Studies indicate, however, that as the dimensions of the viscometer are made larger, both the high and low pressure boundaries of the transition region are moved to smaller values of pressure. Thus by greatly increasing the size and plate separation of the viscometer, the pressure region for which equation (31) yields satisfactory values of μ is extended to very low values of pressure.

It may well be that this procedure can be extended until the characteristic dimension becomes so great that appreciable differences in density or temperature exist over a vertical distance equal to this dimension. At this point, equation (31) would begin to become inaccurate regardless of further increase in viscometer size. By dividing atmospheric density by the density gradient at various altitudes, it may be shown that 0.1 per cent variation in density occurs over a vertical distance of 5 to 10 meters at all altitudes below 130 km. Viscometers with plate separations of 10 meters would be expected to yield values of μ consistent with equation(31) for pressures as low as those found at 90 kilometers altitude.

Thus values of μ tabulated in this MODEL only from-5,000 m' to 90,000 m' are probably reliable for suitable conditions over this entire range of altitudes, but only when these conditions include body dimensions which are sufficiently large. For altitudes above 40 km, each case ought to be examined with caution before using the tabulated values of μ.

5.8 Kinematic Viscosity

5.8.1 Definition and computational equation

Kinematic viscosity is defined as the ratio of the coefficient of viscosity of a gas to the density of the gas. Analytically it is expressed as:

$$\eta = \frac{\mu}{\rho}, \tag{32}$$

where

η = kinematic viscosity of air in $m^2\ sec^{-1}$,

μ = coefficient of viscosity of air in kg $sec^{-1}\ m^{-1}$, and

ρ = atmospheric density in kg m^{-3}.

Because of the empirical nature of the expression for μ and since no other atmospheric properties of this MODEL depend upon η, the expression for η has not been transformed to an expression in terms of the three properties, pressure, molecular-scale temperature, and molecular weight. Computations of η have been made directly from equation (32).

5.8.2 Sea-level value and ratio equation

Equation (32) evaluated at sea-level yields:

$$\eta_o = \frac{\mu_o}{\rho_o} = 1.460,741,29 \times 10^{-5} \text{ m}^2 \text{ sec}^{-1}, \tag{32a}$$

where

η_o = sea-level value of η,

μ_o = sea-level value of μ,
$1.789,428,53 \times 10^{-5}$ kg m^{-1} sec^{-1}, ++

ρ_o = sea-level value of ρ,
$1.225,013,998$ kg m^{-3}. ++

From the division of equation (32) by equation (32a) and from equations (7), (18b), and (31b), one obtains:

$$\frac{\eta}{\eta_o} = \frac{\mu}{\mu_o} \cdot \frac{\rho_o}{\rho} = \frac{P}{P_o} \cdot \frac{M}{M_o} \cdot \left[\frac{T}{T_o}\right]^{\frac{1}{2}} \left[\frac{T_o + S}{T + S}\right]. \tag{32b}$$

5.8.3 Validity

The validity of the tabulated values of η is no better than the validity of either μ or ρ. Within the altitude range of tabulation of η, values of μ are the more uncertain and the use of values of η should be subject to the same restrictions applied to the use of μ.

++ Derived constant

5.9 Summary of Ratio Equations

Because of the common relationship of molecular-scale temperature or real temperature and molecular weight to all the properties of this MODEL, the ratio of these properties to their sea-level values are all interrelated in the following multiple equation:

$$\frac{T_M}{(T_M)_o} \cdot \frac{P_o}{P} = \frac{\rho_o}{\rho} = \frac{H_S}{(H_S)_o} \; \frac{g}{g_o} \cdot \frac{P_o}{P} = \left[\frac{C_S}{(C_S)_o}\right]^2 \cdot \frac{P_o}{P} = \left[\frac{\bar{V}}{\bar{V}_o}\right]^2 \cdot \frac{P_o}{P} =$$

$$\frac{\omega_o}{\omega} \; \frac{g}{g_o} = \frac{v}{v_o} \; \frac{M_o}{M} = \frac{n_o}{n} \; \frac{M_o}{M} = \frac{L}{L_o} \; \frac{M_o}{M} = \frac{\nu_o}{\nu} \; \frac{\bar{V}}{\bar{V}_o} \; \frac{M_o}{M} = \frac{T}{T_o} \cdot \frac{M_o}{M} \cdot \frac{P_o}{P} =$$

$$\frac{\mu}{\mu_o} \; \frac{\eta_o}{\eta} \; . \qquad (33)$$

6. Metric Gravitational System of Units

6.1 Unconventional Form

In this MODEL, as in the ICAO Standard Atmosphere, the system of units employing the dimensions of the Type I gravitational system is not strictly a gravitational system; rather, it is a form of absolute system employing the names of gravitational units, (see Appendix J). In order that there be no confusion between the kilogram force as used in this MODEL and the kilogram force as used in a pure gravitational system of units, the following development is presented.

6.2 Basic Concepts

All properties in this MODEL may be expressed in terms of mass m, length ℓ, time t, and temperature T. The metric absolute system of mechanical units, which has been employed throughout the discussion to this point, uses the kilogram as the unit of mass, the meter as the unit of length, and the second as the unit of time. The unit of acceleration a, therefore, has the dimension of m sec^{-2}, while the unit of force F, expressed by Newton's second law as $F = m a$, has the dimensions of kg m sec^{-2} and has been named the "newton."

The metric gravitational system of units is based on the kilogram force kgf, meter, and second. These units through Newton's law imply a unit of mass equal to the unit of force divided by the unit of acceleration, and having the dimensions of kgf sec^2 m^{-1}, for which there is no specific, commonly used name. The English counterpart of this unit of mass is the slug or lbf sec^2 ft^{-1}.

In its fundamental concept, the kilogram force is the force which gravity exerts on a kilogram mass at the particular altitude and latitude under consideration, and the relationship between the absolute and the gravitational system of units thus depends upon the location. For any fixed latitude, as applied to this MODEL, the variations of gravity with altitude could be used to rigorously relate the kilogram mass and the kilogram force at various altitudes.

6.3 Modified Definition of the Kilogram Force

The drafters of the ICAO Standard Atmosphere, on which this MODEL is based, have chosen not to follow the fundamental concept of the gravitational system of units. They have in effect defined the kilogram force as the force which gravity exerts on a kilogram mass at a location where g is equal to g_0, i.e., at sea level and at 45° 32' 40" latitude. This definition makes the kilogram force an absolute unit, and makes the resulting system of units an absolute system, employing only the dimensions of a gravitational system. The system might therefore be called an absolute-force, gravitational system of units. In equation form, the definition of this absolute kilogram force in terms of the kilogram mass is:

$$1 \text{ kgf} = 9.80665 \text{ m sec}^{-2} \times 1 \text{ kg}, \tag{34}$$

or conversely,

$$1 \text{ kg} = \frac{1}{9.80665} \text{ kgf sec}^2 \text{ m}^{-1}. \tag{35}$$

The dimensions of the right-hand side of equation (35) are those previously associated with mass in the metric gravitational system. Thus it appears that the metric units of mass in this absolute-force, gravitational system is always exactly 9.80665 times as great as the kilogram mass.

6.4 Conversion from Absolute System

Since units of length, time, and temperature are the same in both absolute and gravitational systems of units, only those properties of the MODEL which inherently involve the dimensions of mass have different magnitudes in the two systems. Thus solving equation (35) for unity provides the necessary factor for converting in either direction between the absolute system and the absolute-force gravitational system of units:

$$1 = 9.80665 \text{ m sec}^{-2} \text{ kg kgf}^{-1} (\text{exact}). \tag{36}$$

The factor required for converting from the absolute system to the pure gravitational system of units varies according to the geographic location and is expressed by:

$$1 = g \text{ kg kgf}^{-1} \tag{36a}$$

where g is the acceleration of gravity in m sec^{-2} at the particular altitude and latitude in question.

6.5 Properties Requiring Conversion

A dimensional analysis of the various properties of this MODEL in terms of mass, length, and time indicates that only pressure, density, specific weight, and coefficient of viscosity involve the dimensions of mass. Hence, only these properties are expressed differently in the two systems of units. For each of these properties the conversion from the metric, absolute system to the metric, absolute-force, gravitational system at any altitude is accomplished by dividing the magnitude and dimensions of the property in the former system by the right-hand side of equation (36), (which is equal to unity).

6.6 Converted Sea-Level Values

The sea-level values of atmospheric pressure, density, specific weight, and coefficient of viscosity in units of the metric, absolute-force, gravitational system are obtained by dividing the defined value of P_o in $newtons^{-2}$ and the right-hand members of each of equations (18a), (23b), and (31a) respectively by the right-hand side of equation (36). Thus:

$$P_o = \frac{101{,}325.\ \text{nt m}^{-2}}{9.80665\ \text{m sec}^{-2}\ \text{kg kgf}^{-1}} = 10{,}332.2745\ \text{kgf m}^{-2}, \qquad (37)$$

$$\rho_o = \frac{1.225{,}013{,}998\ \text{kg m}^{-3}}{9.80665\ \text{m sec}^{-2}\ \text{kg kgf}^{-1}} = .124{,}916{,}663\ \text{kgf sec}^2\ \text{m}^{-4}, \qquad (38)$$

$$\omega_o = \frac{12.013{,}283{,}5\ \text{kg m}^{-2}\ \text{sec}^{-2}}{9.80665\ \text{m sec}^{-2}\ \text{kg kgf}^{-1}} = 1.225{,}013{,}998\ \text{kgf m}^{-3}, \qquad (39)$$

$$\mu_o = \frac{1.789{,}428{,}53 \times 10^{-5}\ \text{kg m}^{-1}\ \text{sec}^{-1}}{9.80665\ \text{m sec}^{-2}\ \text{kg kgf}^{-1}} = 1.824{,}709{,}28 \times 10^{-6}\ \text{kgf sec m}^{-2} \qquad (40)$$

6.7 Conversion for All Altitudes

The ratios P/P_o, ρ/ρ_o, ω/ω_o, and μ/μ_o in the absolute system of units, when multiplied by the respective sea-level values given above, yield the values of P, ρ, ω, and μ in the absolute-force, gravitational system of units.*

7. Preparation of the Metric Tables

7.1 Computation of the Tables

The acceleration of gravity, molecular-scale temperature, pressure, and molecular weight are the only properties which were computed directly as functions of H alone, g in terms of a single function for all altitudes, T_M and P in terms of ten different functions for ten altitude regions respectively, and M in terms of three different functions for three altitude regions respectively. The remaining properties were computed from expressions in terms of g, T_M, P, and M, or in terms of T derived from T_M and M. To have computed each of the properties in terms of H alone would have required the development of ten functions for each property, each function applying to a specific altitude region.** Such a procedure would have been unwieldy, and would not have added to the accuracy or validity of the tables. Even the stated computational equations for each of the properties, while serving well for isolated calculations, do not necessarily represent the best approach for development of the tables.

From the multiple equation (33) it is evident that if the ratios of certain basic atmospheric properties to their sea-level values are determined, the remaining ratios are readily computed from products or quotients of not more than two previously determined ratios. The tabulated ratios, when multiplied by the sea-level values of the respective properties in any desired absolute system of units, then yield the required absolute tables.***

7.2 Detailed Computational Procedure

The following procedure is suggested as one of the better methods for use in any expansion or revision of these tables by desk calculator

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* For conversion to the pure gravitational system, these values in the absolute-force, gravitational system of units would have to be multiplied by g_0/g .

** A single function of altitude, closely approximating the densities of this MODEL, particularly above 100 km, was developed by L. Jacchia[33] of the Astrophysical Observatory, Smithsonian Institute and is presented in Appendix L.

*** The tabulation of properties in the absolute-force, gravitational system employed in this MODEL is also made in this manner, although this procedure would not apply to the pure gravitational units.

techniques:

A. List all integral multiples of the desired increment of geometric altitude for which atmospheric properties are to be computed and determine the corresponding values of geopotential altitude to nine significant figures by means of equation (5).

B. List all integral multiples of the same increment of geopotential altitude for which atmospheric properties are to be computed and determine the corresponding values of geometric altitude to nine significant figures.

C. Combine the entries of lists compiled in steps A and B into a single list arranged in numerically ascending values of geopotential.

D. Compute values of g/g_o to nine significant figures for all tabulated values of H by means of equation (1a).

E. Compute values of T_M in ^oK to nine significant figures for all tabulated values of H, using equation (8) and the values of L_M tabulated in Section 3.1.5 .

F. Compute values of $T_M/(T_M)_o$ to nine significant figures for all tabulated values of H, using the defined value of $(T_M)_o$, 288.16^{o}K.

G. Compute values of $\left[T_M/(T_M)_o\right]^{\frac{1}{2}}$ to nine significant figures for all tabulated values of H.

H. Compute values of P/P_o to nine significant figures for all tabulated values of H from equations (17a) through (17c), as each applies to its respective altitude range.

I. Compute value of M to nine significant figures for all tabulated values of H, using equations (24), (24a), and (24b) as each applies to its respective altitude region.

J. Compute values of M/M_o to nine significant figures, using the defined value of M_o, 28.966 .

K. Compute values of T in ^oK to nine significant figures, and T/T_o for all tabulated values of H above 90,000 m', using equations (30) and (30b), in terms of previously determined quantities. (Below 90,000 m', $T = T_M$, and $T/T_o = T_M/(T_M)_o$; hence T and T/T_o need not be computed for this altitude region.)

L. Compute values of $(T/T_o)^{3/2}$ to nine significant figures for all tabulated values of H up to and including 90,000 m' only. For this altitude region,

$$(T/T_o)^{3/2} = \left[T_M/(T_M)_o\right] \cdot \left[T_M/(T_M)_o\right]^{\frac{1}{2}} .$$

M. Compute values of $\dfrac{T_o + S}{T + S}$ to nine significant figures for all tabulated values of H up to and including 90,000 m' only, using S = 110.4 °K from equation (31).

N. Using the previously established ratios and the following equations, compute to nine significant figures the values of the eleven ratios of atmospheric properties to their respective sea-level values, for all tabulated values of H, except in the case of $C_s/(C_s)_o$, μ/μ_o, and η/η_o, which are computed only to 90,000 m' inclusively:

$$\frac{\rho}{\rho_o} = \frac{(T_M)_o}{T_M} \cdot \frac{P}{P_o} \tag{18b}$$

$$\frac{H_s}{(H_s)_o} = \frac{T_M}{(T_M)_o} \cdot \frac{g_o}{g} \tag{19c}$$

$$\frac{C_s}{(C_s)_o} = \left[\frac{T_M}{(T_M)_o}\right]^{\frac{1}{2}} \tag{20c}$$

$$\frac{\bar{\bar{V}}}{\bar{\bar{V}}_o} = \left[\frac{T_M}{(T_M)_o}\right]^{\frac{1}{2}} \tag{21c}$$

$$\frac{\omega}{\omega_o} = \frac{\rho}{\rho_o} \cdot \frac{g}{g_o} \tag{23c}$$

$$\frac{v}{v_o} = \frac{M}{M_o} \cdot \frac{\rho_o}{\rho} \tag{26c}$$

$$\frac{n}{n_o} = \frac{\rho}{\rho_o} \cdot \frac{M_o}{M} \qquad (27c)$$

$$\frac{L}{L_o} = \frac{n_o}{n} \qquad (28d)$$

$$\frac{\nu}{\nu_o} = \frac{\bar{V}}{\bar{V}_o} \cdot \frac{L_o}{L} \qquad (29c)$$

$$\frac{\mu}{\mu_o} = \left[\frac{T_o + S}{T + S} \cdot \frac{T}{T_o} \right]^{3/2} \qquad (31b)$$

$$\frac{\eta}{\eta_o} = \frac{\rho_o}{\rho} \cdot \frac{\mu}{\mu_o} \qquad (32b)$$

O. Compute the mks values of g, P, ρ, H_s, C_s, $\bar{V}$, ω, v, n, L, ν, μ, and η to nine significant figures in the mks absolute units by multiplying the tabulated values of g/g_o, P/P_o and the tabulated values of each of the eleven ratios listed under step N respectively, by the following corresponding, sea-level values, as they are basically defined or as they are derived by the several equations, using the mks system of units.

g_o = 9.80665 m sec^{-2}, defined (Section 2.1.1)

* $(T_M)_o$ = 288.16°K, defined (Section 3.1.5)

P_o = 101,325 newtons m^{-2}, defined (Section 3.2.3)

P_o = .76 m Hg, defined (Section 3.2.3)

ρ_o = 1.225,013,998 kg m^{-3} from equation (18a)

$(H_s)_o$ = 8.434,413,43 m " " (19b)

$(C_s)_o$ = 340.292,046 m sec^{-1} " " (20b)

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* These properties are listed here only for completeness and are not used in step O of the computational procedure since values of T_M, M, and T have already been tabulated.

	$V_o = 458.942,035$ m sec^{-1}	from	equation	(21b)
	$\omega_o = 12.013,283,5$ kg m^{-2} sec^{-2}	"	"	(23b)
*	$M_o = 29.966$, defined	"	"	(24)
	$v_o = 23.645,444,1$ m^3	"	"	(26b)
	$n_o = 2.547,552,07 \times 10^{25}$ m^{-3}	"	"	(27b)
	$L_o = 6.631,722,3 \times 10^{-8}$ m	"	"	(28b)
	$\nu_o = 6.920,404,9 \times 10^9$ sec^{-1}	"	"	(29b)
*	$T_o = 288.16$	"	"	(30a)
	$\mu_o = 1.789,428,53 \times 10^{-5}$ kg m^{-1} sec^{-1}	"	"	(31a)
	$\eta_o = 1.460,741,29 \times 10^{-5}$ m^2 sec^{-1}	"	"	(32a)

P. Compute the values of P, ρ, ω, and μ in the mks, absolute-force, gravitational units** to nine significant figures by dividing the tabulated mks absolute values of these four properties by 9.80665 m sec^{-2} kg kgf^{-1} (exact) from equation (36). In principle this procedure is equivalent to multiplying the tabulated values of P/P_o, ρ/ρ_o, ω/ω_o, and μ/μ_o by the following sea-level values in gravitational units:

$P_o = 10,332.274,5$ kgf m^{-2},	from	equation	(37)
$\rho_o = .124,916,663$ kgf sec^2 m^{-4},	"	"	(38)
$\omega_o = 1.225,013,998$ kgf m^{-3},	"	"	(39)
$\mu_o = 1.824,709,28 \times 10^{-6}$ kgf sec m^{-2},	"	"	(40)

Q. Independently repeat the entire procedure of steps A through P, compare the two results, and account for any discrepancies.

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* See footnote on page 56.

** The remaining atmospheric properties of this MODEL are numerically and dimensionally equal in both mks systems tabulated.

R. Tabulate the corrected results to any desired number of significant figures less than nine, with values of the ratios always given to one more significant figure than the values of the property itself.

7.3 Tabulations Presented

Of the sixteen properties discussed, only one, the mol-volume, is not tabulated for other than sea-level values. In the present edition of the metric tables, the values of pressure, density, specific weight, and coefficient of viscosity are given only in the absolute system of units.

7.4 Significant Figures

The number of significant figures to which these tables might be computed is limited only by the capabilities of the machine. The constants, the defining properties, and the functional relationships are all specified as being exact, and thus they do not limit the number of significant figures of the tables. Such a procedure makes for internal consistency to any degree desired. The choice of the number of significant figures tabulated in this MODEL resulted from arbitrary decisions and does not in the slightest amount indicate the validity of the values in depicting the actual atmosphere.

The sea-level values of the various properties are given to eight or nine significant figures depending on whether the first significant figure is greater than or less than 5. Tabulated values of geopotential and geometric altitude are listed to the nearest meter or standard geopotential meter. Tabulated values of g are given in six significant figures* and values of T_M to five significant figures for all altitudes. The values of the remaining properties are given to five significant figures from -5,000 m' to +75,000 m'. Above 75,000 m', the values of these properties are given to only four significant figures. The ratios of the various properties to their respective sea-level values are given to one more significant figure than the corresponding value of the property.

7.5 Accuracy of Tabulations

The metric tables were prepared with the aid of desk calculators from the equations developed above. The values of the atmospheric properties discussed in Sections 3 and 4 were computed independently by two people and any discrepancies in results were resolved. Any errors which may appear in the tabulated values of these properties will be due to inaccurate copying. The tables of properties in Section 5 have been computed only once and here some possibility of computational error exists.

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* A comparison with a more accurate method for computing g indicates that the sixth significant figure is not meaningful for indicating the actual effective gravity above about 40 km.

8. Preparation of the English Tables

8.1 Conversion of Basic Units

The English tables of THE ARDC MODEL ATMOSPHERE are given in terms of the foot (ft), pound (lb), second (sec), and degree Rankine (^{o}R), each of which is defined exactly in terms of the corresponding units employed in the metric tables. The second, of course, is common to both the English and metric systems of measurement. The foot and the pound are defined as follows:

$$1 \text{ ft} = 0.3048 \text{ m (exact)}^{*} \tag{41}$$

$$1 \text{ lb} = 0.453,592,3 \text{ kg (exact).}^{**} \tag{42}$$

The magnitude of the degree Rankine in terms of the degree Kelvin is derived from the defined relationship of the two temperature scales:

$$T(^{o}R) = 1.8\ T(^{o}K) \text{ (Ref. 60)} \tag{43}$$

where $T(^{o}R)$ is the absolute temperature in the thermodynamic Rankine scale.

From equation (43) one infers that

$$1^{o}K = 1.8^{o}R \text{ (exact),} \tag{43a}$$

and from equations (41), (42), and (43a) respectively, one determines the following three conversion factors:

$$1 = 0.3048 \text{ m ft}^{-1} \text{ (exact)} \tag{41a}$$

$$1 = 0.453,592,3 \text{ kg lb}^{-1} \text{ (exact)} \tag{42a}$$

$$1 = 1.8^{o}R\ (^{o}K)^{-1} \text{(exact).} \tag{43b}$$

These three factors are sufficient to convert values of all atmospheric properties in the mks ^{o}K absolute system of units to the correct values in the fps ^{o}R absolute system of units.

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* "The round value has been accepted by the U.S. National Bureau of Standards and the Commonwealth Standards Laboratory as the common basis on which the American and British representation of the 'foot' should be unified when necessary legal provision is forthcoming." 26-28

** "This value is based on an informal understanding between the National Bureau of Standards (Washington, D.C.) and the National Physical Laboratory (Teddington, England) that this rounded quantity would be convenient if the English-speaking nations could arrive at a uniform basis of conversion from the metric to the English system of units." 26-28

8.2 Other Necessary Conversions

8.2.1 English absolute to English gravitational units

As in the metric system of units, the English gravitational system employed in this MODEL is not a pure gravitational system where the unit of force varies with the location in accordance with the value of g. Rather, the unit of force, the pound force (lbf) is taken to be that force which gravity exerts on a pound mass (lb) at a point where g has the standard sea-level value of this MODEL, g_o. The definition of the pound force in equation form is

$$1 \text{ lbf} = g_o \times 1 \text{ lb}. \tag{44}$$

Dividing the defined metric value of g_o by the conversion factor of equation (41a) yields

$$g_o = \frac{9.80665}{.3048} \text{ ft sec}^{-2} \tag{45}$$

$$= 32.174{,}048{,}55 \text{ ft sec}^{-2}. \tag{45a}$$

Thus,

$$1 \text{ lbf} = \frac{9.80665}{.3048} \text{ ft sec}^{-2}\text{lb}. \tag{44a}$$

Since force has the dimension of lbf, and acceleration is in ft sec^{-2} by Newton's second law, mass must have the dimensions of lbf sec^2 ft^{-1}. This unit is called the slug. Solving equation (44a) for 1 lbf sec^2 ft^{-1}, one obtains:

$$1 \text{ slug} = 1 \text{ lbf sec}^2 \text{ ft}^{-1} = \frac{9.80665}{.3048} \text{ lb}. \tag{45}$$

Thus we find that the slug, the unit of mass in the English (absolute-force) gravitational system of units is exactly 9.80665/.3048 times as large as 1 lb (mass). The factor for converting back and forth between the two Englsih systems of units employed in this MODEL is therefore:

$$1 = \frac{9.80665}{.3048} \text{ ft sec}^{-2} \text{ lb lbf}^{-1} \tag{46}$$

or

$$1 = \frac{9.80665}{.3048} \text{ lb slug}^{-1} \tag{46a}$$

8.2.2 Metric gravitational to English gravitational units

The combining of equations (35), (42a), and (45) yields the following direct relationship between the metric and English gravitational units of mass:

$$1 \text{ slug} = 1 \text{ (lbf sec}^2 \text{ ft}^{-1}) = \frac{.453,592,3}{.3048} \text{ (kgf sec}^2 \text{ m}^{-1}). \quad (47)$$

Dividing the two right-hand members of equation (47) respectively by the corresponding parts of equation (41a) yields

$$1 \text{ lbf} = .453,592,3 \text{ kgf}. \quad (48)$$

This equation provides the factor for converting directly between the two gravitational systems of this MODEL:

$$1 = .453,592,3 \text{ kgf lbf}^{-1}. \quad (49)$$

8.2.3 Rankine-to-Fahrenheit scale and Kelvin-to Fahrenheit scale conversions

The relationship of the thermodynamic Fahrenheit temperature scale to the thermodynamic Rankine scale is established by the following definition:

$$t\ (^{\circ}F) - t_i(^{\circ}F) = T\ (^{\circ}R) - T_i(^{\circ}R), \quad (50)$$

where $t_i(^{\circ}F)$ is defined to be 32°F (exact)⌿, the ice-point temperature.

Using the definition of T_i in °K (see Section 3.1.4) and equation (43), one obtains

$$T_i(^{\circ}R) = 1.8 \times 273.16 = 491.688^{\circ}R. \quad (51)$$

Introducing equations (43) and (51) into equation (50) yields

$$t\ (^{\circ}F) = 1.8\ (T^{\circ}K - 273.16) + 32. \quad (52)$$

8.2.4 Standard geopotential meter to standard geopotential foot

From equation (41) it follows directly that

1 std. geopotential foot (ft') = 0.3048 x 1 std. geopotential meter m'. (53)

Thus the factor for converting m' to ft' and vice versa becomes:

$$1 = 0.3048 \text{ m' ft'}^{-1} \text{(exact)}. \quad (53a)$$

⌿ Basic constant

8.2.5 Geometric meter to nautical mile

The defined conversion* from meters to the international nautical mile (i n mi) in this MODEL is:

$$1 \text{ (i n mi)} = 1{,}852 \text{ meters (exact).} \tag{54}$$

The conversion factor is therefore:

$$1 = 1{,}852 \text{ m (i n mi)}^{-1} \tag{54a}$$

8.3 Sea-Level Values of Atmospheric Properties in English Units

By means of equation (43a) for T_M or by the proper application of equations (41a), (42a), and (43b) to the mks, absolute, sea-level values of the various other atmospheric properties listed under computational procedure, step 0 of Section 7.2, the following sea-level values in English absolute units** are derived. The English absolute values of P_o, ρ_o, ω_o, and μ_o, when divided by the conversion factor given in equation (46) yield the sea-level values of these properties in the English (absolute-force) gravitational system.***

$$g_o = 32.174{,}048{,}55 \text{ ft sec}^{-2}, \text{ from equation (45a)}$$

$$(T_M)_o = 1.8(288.16^\circ K) = 518.688^\circ R \tag{55}$$

$$P_o = \frac{101{,}325 \times .3048}{.453{,}592{,}3} = 68{,}087.266{,}9 \text{ lb ft}^{-1} \text{ sec}^{-2} \text{ or poundals ft}^{-2} \tag{56}$$

$$P_o = \frac{101{,}325 \times (.3048)^2}{.453{,}592{,}3 \times 9.80665} = 2{,}116.216{,}95 \text{ lbf ft}^{-2} \tag{56a}$$

or

$$P_o = \frac{.76 \times 12}{.3048} = 29.921{,}259{,}84 \text{ in Hg} \tag{56b}$$

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* United States Department of Defense Directive 2045.1, 17 June 1954, directed the adoption of the <u>international nautical mile</u> (equal to 1852 meters) as a standard value with the Department of Defense effective 1 July 1954.

** See Appendix J.

*** All remaining properties are numerically and dimensionally the same in both systems.

$$\rho_o = \frac{1.225,013,998 \times (.3048)^3}{.453,592,3} = .076,475,137,4 \text{ lb ft}^{-3} \quad (57)$$

$$\rho_o = \frac{1.225,013,998 \times (.3048)^4}{.453,592,3 \times 9.80665} = 2.376,919,99 \times 10^{-3} \text{ lbf sec}^2 \text{ ft}^{-4} \text{ or slugs ft}^{-3} \quad (57a)$$

$$(H_s)_o = \frac{8,434.413,43}{.3048} = 2.767,196,007 \times 10^4 \text{ ft} \quad (58)$$

$$(C_s)_o = \frac{340.292,046}{.3048} = 1.116,443,720 \times 10^3 \text{ ft sec}^{-1} \quad (59)$$

$$\bar{V}_o = \frac{458.942,035}{.3048} = 1.505,715,337 \times 10^3 \text{ ft sec}^{-1} \quad (60)$$

$$\omega_o = \frac{12.013,283,5 \times (.3048)^2}{.453,592,3} = 2.460,514,77 \text{ lb ft}^{-2} \text{ sec}^{-2} \quad (61)$$

$$\omega_o = \frac{12.013,283,5 \times (.3048)^3}{.453,592,3 \times 9.80665} = 7.647,513,72 \times 10^{-2} \text{ lbf ft}^{-3} \quad (61a)$$

$$M_o = 28.966 \text{ (nondimensional) (unchanged)} \quad (62)$$

$$v_o = \frac{23.645,444,08}{(.3048)^3} = 835.030,977 \text{ ft}^3 \quad (63)$$

$$n_o = 2.547,552,07 \times (.3048)^3 \times 10^{25} = 7.213,864,115 \times 10^{23} \text{ ft}^{-3} \quad (64)$$

$$L_o = \frac{6.631,722,29 \times 10^{-8}}{.3048} = 2.175,761,906 \times 10^{-7} \text{ ft} \quad (65)$$

$$\nu_o = 6.920,404,91 \times 10^9 \text{ sec}^{-1} \text{ (unchanged)} \quad (66)$$

$$\mu_o = \frac{1.789,428,53 \times .3048 \times 10^{-5}}{.453,592,3} = 1.202,440,640 \times 10^{-5} \text{ lb ft}^{-1} \text{sec}^{-1} \quad (67)$$

$$\mu_o = \frac{1.789,428,53 \times (.3048)^2 \times 10^{-5}}{.453,592,3 \times 9.80665} = 3.737,299,76 \times 10^{-7} \text{ lbf sec ft}^{-2} \qquad (67a)$$

$$\eta_o = \frac{1.460,741,29 \times 10^{-5}}{(.3048)^2} = 1.572,328,83 \times 10^{-4} \text{ ft}^2 \text{ sec}^{-1} \qquad (68)$$

It is to be noted that only three exactly defined numerical constants were employed in all the above conversions. Hence the English values may be reliably carried to any number of significant figures consistent with the metric absolute value.

8.4 Calculation of the English Tables

8.4.1 Functions employed

This MODEL ATMOSPHERE is defined exactly in terms of various gradients of molecular-scale temperature in $^{\circ}$K m'$^{-1}$ between specific exact values of altitude expressed in m', and in terms of constants defined exactly in metric units. These definitions cannot be converted exactly to English units. Thus it is preferable to compute English tables from exactly the same equations used for the metric tables, after first making the necessary conversion of the English altitudes to metric altitudes, and then obtaining the English values of the various properties by another conversion.

8.4.2 Altitude increments

The argument of the English tables, similar to the metric tables, is given in consecutive integral multiples of a fixed altitude increment in both geometric feet and standard geopotential feet, i.e.,

n x 2500 ft and n x 2500 ft',

where n = -6, -5, -4, -3, -2, -1, 0, +1,2,3 etc. to 24. From -15,000 ft' to 60,000 ft' the increment is 2500 ft or ft'; from 60,000 ft' to 300,000 ft', the increment is 10,000 ft or ft'; from 300,000 ft' to 500,000 ft', the increment is 25,000 ft or ft'; from 500,000 ft' to 1,000,000 ft', the increment is 50,000 ft or ft'; and from 1,000,000 ft' to 1,700,000 ft', the increment is 100,000 ft or ft'.

8.4.3 Altitude conversions

In order to use identically the same equations for converting between geopotential and geometric altitude for the English tables as was used in the metric tables, these conversions must be made in metric units. Thus, to convert the tabulated integral multiple values of ft to m', multiply the altitudes in ft by exactly .3048 m ft^{-1}, from equation (41a), to obtain the equivalent in meters, and then convert the results to m' by using equation (5). This value of m' is then converted to the equivalent in ft' by dividing by exactly .3048 m' ft'$^{-1}$

from equation (53a). Starting with tabulated, integral, multiple values of ft', the conversion to m' is directly by means of equation (53a). This value of m' is then converted to m by means of equation (6), and the corresponding value of ft is then obtained by means of equation (41a). Since the conversion factors cited and the constants of equations (5) and (6) are all defined to be exact, the conversions may be carried to any desired number of significant figures.

8.4.4 Computational procedure

Having arranged in sequence the values of m' for each English altitude to be tabulated, the computation of the tables proceeds exactly as indicated in Section 7.2, steps D through N, but stopping short of O.

Compute the values of T_M and T in °C to nine significant figures from the Kelvin values by means of equation (9). Compute the values of T_M and T in °F to nine significant figures from the Kelvin values by using equation (50).

Compute the values of the remaining properties in English units from the multiplication of the ratios of the various properties determined in step N by their respective sea-level values in the desired English absolute and absolute-force units.

8.4.5 Tabulated values

In this edition of the MODEL, only half of the properties discussed are contained in the English tables. The properties tabulated are those designated by g, P, ρ, C_s, M, T, μ, and η. It should be noted that ρ and μ are given only in Type I, absolute-force, gravitational units, while P is given not only in this system (lbf ft^{-2}) but also in mb and in inches of Hg. Temperatures in the English tables are given in °C, °F, and °R.

These tables were prepared from a single computation using desk calculators; as the values have not been checked by independent calculations, some chance of error exists.

Above 60,000 ft the altitude increments of the English tables are considerably larger than the increments of the metric tables.

9. Conclusions and Recommendations

The tables included in this report are based on the totality of the available, reputable, atmospheric data from observations of the upper atmosphere to 160 km, and above this altitude, on estimates and theories acceptable at the time of this writing, 1956. The Geophysics Research Directorate, AFCRC, ARDC, believes that these tables provide the best representation of the properties of the upper atmosphere consistent with a segmented, linear, temperature-altitude function.

It is recommended that these tables be used as the basis for all aircraft and missile design work within ARDC and by its contractors.

Section 10

METRIC TABLES

OF THE

ARDC MODEL ATMOSPHERE, 1956

NOTE: Superscripts appearing in the following tables indicate the power of ten by which each tabulated value should be multiplied.

METRIC TABLE I

TEMPERATURES AND MOLECULAR WEIGHT AS FUNCTIONS OF GEOMETRIC AND GEOPOTENTIAL ALTITUDE

ALTITUDE		TEMPERATURE MOLECULAR SCALE		TEMPERATURE REAL KINETIC		MOLECULAR WEIGHT	
Z,m	H,m'	T_M, °K	T_M/T_{Mo}	T, °K	T/T_o	M	M/M_o
-5,000	-5,003.9	320.69	1.11287			28.966	1.00000
-4,966.1	-5,000	320.66	1.11278				
-4,000	-4,002.5	314.18	1.09028				
-3,997.5	-4,000	314.16	1.09023				
-3,000	-3,001.4	307.67	1.06770				
-2,998.6	-3,000	307.66	1.06767				
-2,000	-2,000.6	301.16	1.04513				
-1,999.4	-2,000	301.16	1.04511				
-1,000	-1,000.2	294.66	1.02256				
- 999.8	-1,000	294.66	1.02256				
0	0	288.16	1.000000	same as T_M for altitudes up to 90 km'	same as T_M/T_{Mo} for altitudes up to 90 km'	constant at 28.966 for altitudes up to 90 km'	1.00000 for altitudes up to 90 km'
1,000	999.8	281.66	.974443				
1,000.2	1,000	281.66	.974443				
2,000	1,999.4	275.16	.954886				
2,000.6	2,000	275.16	.954886				
3,000	2,998.6	268.67	.932364				
3,001.4	3,000	268.66	.932329				
4,000	3,997.5	262.18	.909842				
4,002.5	4,000	262.16	.909772				
5,000	4,996.1	255.69	.885237				
5,003.9	5,000	255.66	.887215				
6,000	5,994.3	249.20	.864797				
6,005.7	6,000	249.16	.864659				
7,000	6,992.3	242.71	.842275				
7,007.7	7,000	242.66	.842102				
8,000	7,989.9	236.23	.819788				
8,010.7	8,000	236.16	.819545				
9,000	8,987.3	229.74	.797265				
9,012.8	9,000	229.66	.796988				
10,000	9,984.3	223.26	.774778				
10,016	10,000	223.16	.774431				
11,000	10,981	216.78	.752290				
11,019	11,000	216.66	.751874				
12,000	11,977	216.66	.751874				
12,023	12,000	216.66	.751874				
13,000	12,973	216.66	.751874				
13,027	13,000	216.66	.751874				
14,000	13,979	216.66	.751874				
14,031	14,000	216.66	.751874			28.966	1.00000

ALTITUDE		TEMPERATURE MOLECULAR SCALE		TEMPERATURE REAL KINETIC		MOLECULAR WEIGHT	
Z,m	H,m'	T_M,°K	T_M/T_{Mo}	T,°K	T/T_o	M	M/M_o
15,000	14,965	216.66	.751874			28.966	1.00000
15,035	15,000	216.66	.751874				
16,000	15,960	216.66	.751874				
16,040	16,000	216.66	.751874				
17,000	16,955	216.66	.751874				
17,046	17,000	216.66	.751874				
18,000	17,949	216.66	.751874				
18,051	18,000	216.66	.751874				
19,000	18,943	216.66	.751874				
19,057	19,000	216.66	.751874				
20,000	19,937	216.66	.751874				
20,063	20,000	216.66	.751874				
21,000	20,931	216.66	.751874				
21,070	21,000	216.66	.751874				
22,000	21,924	216.66	.751874				
22,076	22,000	216.66	.751874				
23,000	22,917	216.66	.751874				
23,084	23,000	216.66	.751874				
24,000	23,910	216.66	.751874				
24,091	24,000	216.66	.751874				
25,000	24,902	216.66	.751874	same as T_M for altitudes up to 90 km'	same as T_M/T_{Mo} for altitudes up to 90 km'	constant at 28.966 for altitudes up to 90 km'	1.00000 for altitudes up to 90 km'
25,099	25,000	216.66	.751874				
26,000	25,894	219.34	.761182				
26,107	26,000	219.66	.762285				
27,000	26,886	222.32	.771507				
27,115	27,000	222.66	.772696				
28,000	27,877	225.29	.781828				
28,124	28,000	225.66	.783107				
29,000	28,868	228.26	.792146				
29,133	29,000	228.66	.793517				
30,000	29,859	231.24	.802461				
30,142	30,000	231.66	.803928				
31,000	30,850	234.21	.812773				
31,152	31,000	234.66	.814339				
32,000	31,840	237.18	.823081				
32,162	32,000	237.66	.824750				
33,000	32,830	240.15	.833387				
33,172	33,000	240.66	.835161				
34,000	33,819	243.12	.843689				
34,183	34,000	243.66	.845572			28.966	1.00000

METRIC TABLE I CONTINUED

ALTITUDE		TEMPERATURE MOLECULAR SCALE		TEMPERATURE REAL KINETIC		MOLECULAR WEIGHT	
Z,m	H,m'	T_M, °K	T_M/T_{Mo}	T, °K	T/T_o	M	M/M_o
35,000	34,808	246.09	.853988			28.966	1.00000
35,194	35,000	246.66	.855983				
36,000	35,797	249.05	.864283				
36,205	36,000	249.66	.866394				
37,000	36,786	252.02	.874575				
37,217	37,000	252.66	.876805				
38,000	37,774	254.98	.884865				
38,229	38,000	255.66	.887215				
39,000	38,762	257.95	.895151				
39,241	39,000	258.66	.897626				
40,000	39,750	260.91	.905433				
40,253	40,000	261.66	.908037				
41,000	40,737	263.87	.915713				
41,266	41,000	264.66	.918448				
42,000	41,724	266.83	.925989				
42,279	42,000	267.66	.928859				
43,000	42,711	269.79	.936262				
43,293	43,000	270.66	.939270				
44,000	43,698	272.75	.946532				
44,307	44,000	273.66	.949681				
45,000	44,684	275.71	.956798	same as T_M for altitudes up to 90 km'	same as T_M/T_{Mo} for altitudes up to 90 km'	constant at 28.966 for altitudes up to 90 km'	1.00000 for altitudes up to 90 km'
45,321	45,000	276.66	.960092				
46,000	45,670	278.67	.967062				
46,335	46,000	279.66	.970503				
47,000	46,655	281.63	.977322				
47,350	47,000	282.66	.980913				
48,000	47,640	282.66	.980913				
48,365	48,000	282.66	.980913				
49,000	48,625	282.66	.980913				
49,381	49,000	282.66	.980913				
50,000	49,610	282.66	.980913				
50,396	50,000	282.66	.980913				
51,000	50,594	282.66	.980913				
51,412	51,000	282.66	.980913				
52,000	51,578	282.66	.980913				
52,429	52,000	282.66	.980913				
53,000	52,562	282.66	.980913				
53,446	53,000	282.66	.980913				
54,000	53,545	280.53	.973535				
54,463	54,000	278.76	.967379			28.966	1.00000

METRIC TABLE I CONTINUED

ALTITUDE		TEMPERATURE MOLECULAR SCALE		TEMPERATURE REAL KINETIC		MOLECULAR WEIGHT	
Z,m	H,m'	T_M, °K	T_M/T_{Mo}	T, °K	T/T_o	M	M/M_o
55,000	54,528	276.70	.960230			28.966	1.00000
55,480	55,000	274.86	.953845				
56,000	55,511	272.87	.946929				
56,498	56,000	270.96	.940311				
57,000	56,493	269.04	.933633				
57,516	57,000	267.06	.926777				
58,000	57,476	265.21	.920340				
58,534	58,000	263.16	.913243				
59,000	58,457	261.38	.907052				
59,553	59,000	259.26	.899709				
60,000	59,439	257.55	.893767	same as T_M for altitudes up to 90 km'	same as T_M/T_{Mo} for altitudes up to 90 km'	constant at 28.966 for altitudes up to 90 km'	1.00000 for altitudes up to 90 km'
60,572	60,000	255.36	.886174				
61,000	60,420	253.72	.880487				
61,591	61,000	251.46	.872640				
62,000	61,401	249.90	.867211				
62,611	62,000	247.56	.859106				
63,000	62,382	246.07	.853939				
63,631	63,000	243.66	.845572				
64,000	63,362	242.25	.840672				
64,651	64,000	239.76	.832038				
65,000	64,342	238.43	.827408				
65,672	65,000	235.86	.818504				
66,000	65,322	234.61	.814148				
66,692	66,000	231.96	.804969				
67,000	66,301	230.79	.800893				
67,714	67,000	228.06	.791435				
68,000	67,280	226.97	.787642				
68,735	68,000	224.16	.777901				
69,000	68,259	223.15	.774395				
69,757	69,000	220.26	.764367				
70,000	69,238	219.33	.761152				
70,779	70,000	216.36	.750833				
71,000	70,216	215.52	.747913				
71,802	71,000	212.46	.737299				
72,000	71,194	211.70	.734678				
72,825	72,000	208.56	.723765				
73,000	72,171	207.89	.721448				
73,848	73,000	204.66	.710230				
74,000	73,148	204.08	.708221				
74,872	74,000	200.76	.696696			28.966	1.00000

METRIC TABLE I CONTINUED

ALTITUDE		TEMPERATURE MOLECULAR SCALE		TEMPERATURE REAL KINETIC		MOLECULAR WEIGHT	
Z,m	H,m'	T_M, °K	T_M/T_{Mo}	T, °K	T/T_o	M	M/M_o
75,000	74,125	200.27	.694999			28.966	1.00000
75,895	75,000	196.86	.683162	same as T_M for altitudes up to 90 km'	same as T_M/T_{Mo} for altitudes up to 90 km'	constant at 28.966 for altitudes up to 90 km'	1.00000 for altitudes up to 90 km'
76,000	75,102	196.86	.683162				
76,920	76,000	196.86	.683162				
77,000	76,078	196.86	.683162				
77,944	77,000	196.86	.683162				
78,000	77,055	196.86	.683162				
78,969	78,000	196.86	.683162				
79,000	78,030	196.86	.683162				
79,994	79,000	196.86	.683162				
80,000	79,006	196.86	.683162				
81,000	79,981	196.86	.683162				
81,020	80,000	196.86	.683162				
82,000	80,956	196.86	.683162				
82,045	81,000	196.86	.683162				
83,000	81,930	196.86	.683162				
83,072	82,000	196.86	.683162				
84,000	82,904	196.86	.683162				
84,098	83,000	196.86	.683162				
85,000	83,878	196.86	.683162				
85,125	84,000	196.86	.683162				
86,000	84,852	196.86	.683162				
86,152	85,000	196.86	.683162				
87,000	85,825	196.86	.683162				
87,179	86,000	196.86	.683162				
88,000	86,798	196.86	.683162				
88,207	87,000	196.86	.683162				
89,000	87,771	196.86	.683162				
89,235	88,000	196.86	.683162				
90,000	88,744	196.86	.683162				
90,264	89,000	196.86	.683162				
91,000	89,716	196.86	.683162				
91,293	90,000	196.86	.683162	196.9	.68316	28.96	1.00000
92,000	90,688	199.27	.691526	197.0	.68355	28.63	.98848
92,322	91,000	200.36	.695308	197.1	.68395	28.49	.98367
93,000	91,659	202.67	.703325	197.5	.68523	28.22	.97429
93,351	92,000	203.86	.707454	197.7	.68609	28.09	.96980
94,000	92,630	206.07	.715109	198.3	.68799	27.87	.96208
94,381	93,000	207.36	.719600	198.6	.68929	27.75	.95787

METRIC TABLE I CONTINUED

ALTITUDE		TEMPERATURE MOLECULAR SCALE		TEMPERATURE REAL KINETIC		MOLECULAR WEIGHT	
Z,m	H,m'	T_M, °K	T_M/T_{Mo}	T, °K	T/T_o	M	M/M_o
95,000	93,601	209.46	.726902	199.3	.69163	27.56	.95147
95,411	94,000	210.86	.731746	199.8	.69334	27.45	.94751
96,000	94,572	212.86	.738691	200.6	.69597	27.29	.94217
96,441	95,000	214.36	.743892	201.2	.69808	27.18	.93842
97,000	95,542	216.26	.750477	202.0	.70090	27.05	.93394
97,472	96,000	217.86	.756038	202.7	.70340	26.95	.93038
98,000	96,512	219.65	.762258	203.5	.70632	26.84	.92661
98,503	97,000	221.36	.768184	204.4	.70920	26.74	.92322
99,000	97,482	223.05	.774037	205.2	.71215	26.65	.92004
99,534	98,000	224.86	.780330	206.2	.71541	26.56	.91680
100,000	98,451	226.44	.785811	207.0	.71833	26.48	.91412
100,566	99,000	228.36	.792476	208.0	.72196	26.39	.91102
101,000	99,420	229.83	.797582	208.9	.72481	26.32	.90876
101,598	100,000	231.86	.804622	210.0	.72881	26.24	.90578
102,000	100,389	233.22	.809349	210.8	.73155	26.18	.90387
102,631	101,000	235.36	.816768	212.1	.73592	26.10	.90101
103,000	101,358	236.61	.821113	212.8	.73852	26.05	.89941
103,663	102,000	238.86	.828914	214.2	.74325	25.97	.89665
104,000	102,326	240.00	.832873	214.9	.74568	25.93	.89531
104,696	103,000	242.36	.841061	216.3	.75078	25.86	.89265
105,000	103,294	243.39	.844629	217.0	.75302	25.82	.89154
105,730	104,000	245.86	.853207	218.6	.75848	25.75	.88897
106,000	104,261	246.78	.856382	219.1	.76051	25.72	.88806
106,764	105,000	249.36	.865353	220.8	.76633	25.65	.88557
107,000	105,229	250.16	.868131	221.3	.76814	25.63	.88483
107,798	106,000	252.86	.877499	223.1	.77432	25.56	.88241
108,000	106,196	253.55	.879876	223.6	.77590	25.54	.88182
108,832	107,000	256.36	.889645	225.5	.78243	25.48	.87948
109,000	107,162	256.93	.891618	225.8	.78376	25.46	.87903
109,867	108,000	259.86	.901791	227.8	.79065	25.40	.87675
110,000	108,129	260.31	.903356	228.1	.79172	25.39	.87642
110,902	109,000	263.36	.913937	230.2	.79897	25.32	.87420
111,000	109,095	263.69	.915091	230.5	.79976	25.32	.87397
111,937	110,000	266.86	.926083	232.7	.80738	25.25	.87182
112,000	110,061	267.07	.926822	232.8	.80789	25.25	.87168
112,973	111,000	270.36	.938229	235.1	.81586	25.19	.86958
113,000	111,026	270.45	.938549	235.2	.81609	25.19	.86952
114,000	111,992	273.83	.950273	237.5	.82435	25.13	.86749
114,009	112,000	273.86	.950375	237.6	.82443	25.13	.86747

METRIC TABLE I CONTINUED

ALTITUDE		TEMPERATURE MOLECULAR SCALE		TEMPERATURE REAL KINETIC		MOLECULAR WEIGHT	
Z,m	H,m'	T_M, °K	T_M/T_{Mo}	T, °K	T/T_o	M	M/M_o
115,000	112,957	277.21	.961993	239.9	.83268	25.07	.86558
115,045	113,000	277.36	.962521	240.1	.83618	25.07	.86549
116,000	113,921	280.58	.973709	242.4	.84106	25.02	.86377
116,082	114,000	280.86	.974667	242.6	.84175	25.02	.86362
117,000	114,885	283.96	.985422	244.8	.84949	24.97	.86206
117,119	115,000	284.36	.986813	245.1	.85049	24.96	.86186
118,000	115,850	287.33	.997131	247.2	.85796	24.92	.86043
118,156	116,000	287.86	.998959	247.6	.85929	24.92	.86019
119,000	116,813	290.71	1.00884	249.7	.86648	24.87	.85889
119,194	117,000	291.36	1.01110	250.2	.86814	24.87	.85860
120,000	117,777	294.08	1.02054	252.2	.87504	24.84	.85743
120,232	118,000	294.86	1.02325	252.7	.87703	24.83	.85710
121,000	118,740	297.45	1.03224	254.6	.88363	24.80	.85604
121,270	119,000	298.36	1.03540	255.3	.88596	24.79	.85567
122,000	119,703	300.82	1.04393	257.1	.89226	24.76	.85471
122,309	120,000	301.86	1.04754	257.9	.89493	24.75	.85431
123,000	120,665	304.19	1.05562	259.6	.90091	24.72	.85344
123,348	121,000	305.36	1.05969	260.5	.90393	24.71	.85302
124,000	121,627	307.56	1.06731	262.1	.90960	24.69	.85223
124,387	122,000	308.86	1.07184	263.1	.91297	24.67	.85178
125,000	122,589	310.92	1.07899	264.6	.91831	24.65	.85108
125,427	123,000	312.36	1.08398	265.7	.92204	24.64	.85060
126,000	123,551	314.29	1.09067	267.1	.92704	24.62	.84997
126,467	124,000	315.86	1.09613	268.3	.93113	24.61	.84947
127,000	124,512	317.65	1.10235	269.7	.93580	24.59	.84892
127,507	125,000	319.36	1.10827	270.9	.94025	24.57	.84840
128,000	125,473	321.02	1.11402	272.2	.94458	24.56	.84790
128,548	126,000	322.86	1.12042	273.6	.94940	24.54	.84736
129,000	126,434	327.20	1.13549	277.1	.96168	24.53	.84693
129,589	127,000	332.86	1.15512	281.7	.97766	24.52	.84637
130,000	127,395	336.81	1.16882	284.9	.98881	24.51	.84599
130,630	128,000	342.86	1.18983	289.9	1.0059	24.49	.84542
131,000	128,355	346.41	1.20214	292.7	1.0159	24.48	.84509
131,672	129,000	352.86	1.22453	298.0	1.0341	24.46	.84451
132,000	129,315	356.01	1.23545	300.6	1.0430	24.45	.84423
132,774	130,000	362.86	1.25923	306.1	1.0623	24.44	.84363
135,000	132,193	384.79	1.33532	323.9	1.1241	24.38	.84183
137,929	135,000	412.86	1.43275	346.7	1.2031	24.32	.83972

METRIC TABLE I CONTINUED

ALTITUDE		TEMPERATURE MOLECULAR SCALE		TEMPERATURE REAL KINETIC		MOLECULAR WEIGHT	
Z,m	H,m'	T_M, °K	T_M/T_{Mo}	T, °K	T/T_o	M	M/M_o
140,000	136,983	432.69	1.50157	362.7	1.2588	24.28	.83835
143,153	140,000	462.86	1.60626	387.2	1.3435	24.23	.83644
145,000	141,766	480.52	1.66755	401.4	1.3931	24.20	.83541
148,385	145,000	512.86	1.77978	427.6	1.4837	24.15	.83366
150,000	146,542	528.28	1.83329	440.0	1.5269	24.13	.83288
153,625	150,000	562.86	1.95329	467.9	1.6237	24.08	.83127
155,000	151,311	575.97	1.99877	478.5	1.6604	24.06	.83069
158,874	155,000	612.86	2.12680	508.2	1.7635	24.02	.82919
160,000	156,072	623.58	2.16399	516.8	1.7935	24.01	.82878
164,131	160,000	662.86	2.30032	548.4	1.9032	23.97	.82737
165,000	160,826	671.12	2.32897	555.1	1.9263	23.96	.82709
169,397	165,000	712.86	2.47383	588.6	2.0428	23.92	.82575
170,000	165,572	718.58	2.49369	593.2	2.0587	23.91	.82558
174,671	170,000	762.86	2.64735	628.8	2.1823	23.88	.82432
175,000	170,311	765.97	2.65815	631.3	2.1909	23.87	.82424
179,954	175,000	812.86	2.82086	669.0	2.3217	23.84	.82303
180,000	175,043	813.11	2.82174	669.1	2.3220	23.84	.82290
185,000	179,768	840.52	2.91684	679.7	2.3588	23.42	.80869
185,245	180,000	841.86	2.92150	680.2	2.3606	23.41	.80802
190,000	184,486	867.88	3.01179	690.4	2.3960	23.04	.79555
190,545	185,000	870.86	3.02214	691.6	2.4001	23.00	.79418
195,000	189,196	895.20	3.10660	701.3	2.4336	22.69	.78337
195,854	190,000	899.86	3.12278	703.1	2.4401	22.63	.78138
200,000	193,899	922.48	3.20127	712.2	2.4715	22.36	.77204
201,171	195,000	928.86	3.22342	714.8	2.4804	22.29	.76951
205,000	198,595	949.71	3.29579	723.2	2.5097	22.06	.76149
206,497	200,000	957.86	3.32406	726.5	2.5212	21.97	.75846
210,000	203,284	976.91	3.39016	734.3	2.5481	21.77	.75162
211,831	205,000	986.86	3.42469	738.3	2.5622	21.67	.74816
215,000	207,966	1004.1	3.48440	745.4	2.5867	21.50	.74238
217,175	210,000	1015.9	3.52533	750.3	2.6036	21.39	.73854
220,000	212,641	1031.2	3.57849	756.6	2.6256	21.25	.73371
222,526	215,000	1044.9	3.62597	762.3	2.6452	21.13	.72953
225,000	217,308	1058.2	3.67243	767.8	2.6645	21.02	.72555
227,887	220,000	1073.9	3.72661	774.3	2.6871	20.89	.72106
230,000	221,969	1085.3	3.76624	779.1	2.7037	20.79	.71787
233,256	225,000	1102.9	3.82725	786.5	2.7292	20.66	.71310
235,000	226,622	1112.3	3.85990	790.4	2.7429	20.58	.71062
238,634	230,000	1131.9	3.92789	798.6	2.7715	20.44	.70561

METRIC TABLE I CONTINUED

ALTITUDE		TEMPERATURE MOLECULAR SCALE		TEMPERATURE REAL KINETIC		MOLECULAR WEIGHT	
Z, m	H, m'	T_M, °K	T_M/T_{M_0}	T, °K	T/T_0	M	M/M_0
240,000	231,268	1139.2	3.95342	801.7	2.7823	20.39	.70377
244,021	235,000	1160.9	4.02853	810.9	2.8140	20.23	.69853
245,000	235,908	1166.1	4.04680	813.1	2.8218	20.20	.69729
249,417	240,000	1189.9	4.12916	823.2	2.8567	20.04	.69184
250,000	240,540	1193.0	4.14003	824.5	2.8613	20.02	.69114
254,821	245,000	1218.9	4.22980	835.5	2.8995	19.86	.68550
255,000	245,165	1219.8	4.23313	835.9	2.9010	19.85	.68530
260,000	249,784	1246.6	4.32608	847.4	2.9407	19.69	.67975
260,235	250,000	1247.9	4.33044	847.9	2.9425	19.68	.67950
265,000	254,395	1273.4	4.41890	858.8	2.9804	19.54	.67447
265,657	255,000	1276.9	4.43108	860.3	2.9856	19.52	.67379
270,000	258,999	1300.0	4.51157	870.3	3.0202	19.39	.66943
271,088	260,000	1305.9	4.53172	872.8	3.0289	19.36	.66837
275,000	263,597	1326.7	4.60411	881.8	3.0600	19.25	.66463
276,528	265,000	1334.9	4.63236	885.3	3.0722	19.21	.66321
280,000	268,187	1353.3	4.69650	893.3	3.0999	19.12	.66005
281,977	270,000	1363.9	4.73300	897.8	3.1157	19.07	.65829
285,000	272,771	1379.9	4.78876	904.8	3.1398	18.99	.65566
287,435	275,000	1392.9	4.83363	910.4	3.1592	18.93	.65359
290,000	277,347	1406.5	4.88088	916.3	3.1797	18.87	.65146
292,902	280,000	1421.9	4.93427	922.9	3.2029	18.80	.64911
295,000	281,917	1433.0	4.97286	927.8	3.2197	18.75	.64744
298,377	285,000	1450.9	5.03491	935.5	3.2466	18.68	.64482
300,000	286,480	1459.4	5.06470	939.3	3.2596	18.64	.64359
303,862	290,000	1479.9	5.13555	948.2	3.2905	18.56	.64072
305,000	291,036	1485.9	5.15640	950.8	3.2995	18.54	.63989
309,356	295,000	1508.9	5.23619	960.8	3.3344	18.45	.63679
310,000	295,585	1512.3	5.24797	962.3	3.3395	18.43	.63634
314,859	300,000	1537.9	5.33683	973.5	3.3783	18.34	.63302
320,000	304,663	1564.9	5.43069	985.3	3.4194	18.24	.62964
325,893	310,000	1595.9	5.53810	988.9	3.4665	18.13	.62593
330,000	313,714	1617.4	5.61286	1008	3.4993	18.06	.62344
336,963	320,000	1653.9	5.73938	1024	3.5549	17.94	.61938
340,000	322,738	1669.7	5.79449	1031	3.5791	17.89	.61767
348,069	330,000	1711.9	5.94066	1050	3.6435	17.77	.61331

METRIC TABLE I CONTINUED

ALTITUDE		TEMPERATURE MOLECULAR SCALE		TEMPERATURE REAL KINETIC		MOLECULAR WEIGHT	
Z,m	H,m'	T_M, °K	T_M/T_{M_0}	T, °K	T/T_0	M	M/M_0
350,000	331,735	1721.9	5.97558	1,054	3.6588	17.74	.61230
359,213	340,000	1769.9	6.14194	1,075	3.7322	17.60	.60767
360,000	340,705	1773.9	6.15613	1,077	3.7385	17.59	.60728
370,000	349,648	1825.8	6.33614	1,100	3.8181	17.45	.60259
370,394	350,000	1827.9	6.34321	1,101	3.8212	17.45	.60241
380,000	358,565	1877.5	6.51561	1,123	3.8975	17.33	.59818
381,612	360,000	1885.9	6.54449	1,127	3.9103	17.31	.59750
390,000	367,456	1929.1	6.69456	1,146	3.9768	17.21	.59404
392,867	370,000	1943.9	6.74577	1,153	3.9996	17.17	.59290
400,000	376,320	1980.5	6.87297	1,169	4.0560	17.09	.59014
404,160	380,000	2001.9	6.94704	1,178	4.0889	17.05	.58859
410,000	385,158	2031.8	7.05086	1,192	4.1351	16.99	.58647
415,491	390,000	2059.9	7.14832	1,204	4.1784	16.93	.58454
420,000	393,970	2082.9	7.22823	1,214	4.2140	16.89	.58299
426,860	400,000	2117.9	7.34960	1,230	4.2680	16.82	.58072
430,000	402,756	2133.8	7.40507	1,237	4.2928	16.79	.57971
438,267	410,000	2175.9	7.55087	1,256	4.3577	16.72	.57712
440,000	411,516	2184.7	7.58139	1,260	4.3713	16.70	.57659
449,713	420,000	2233.9	7.75215	1,282	4.4475	16.62	.57372
450,000	420,250	2235.3	7.75719	1,282	4.4498	16.62	.57363
460,000	428,959	2285.8	7.93247	1,305	4.5280	16.53	.57082
461,197	430,000	2291.9	7.95343	1,307	4.5374	16.52	.57050
470,000	437,642	2336.2	8.10725	1,327	4.6061	16.46	.56815
472,721	440,000	2349.9	8.15471	1,333	4.6273	16.44	.56744
480,000	446,300	2386.4	8.28151	1,350	4.6840	16.38	.56560
484,283	450,000	2407.9	8.35598	1,359	4.7173	16.35	.56455
490,000	454,932	2436.5	8.45526	1,372	4.7618	16.31	.56317
495,884	460,000	2465.9	8.55726	1,385	4.8074	16.27	.56179
500,000	463,540	2486.4	8.62851	1,394	4.8393	16.25	.56085
507,525	470,000	2523.9	8.75854	1,411	4.8976	16.20	.55918
510,000	472,122	2536.2	8.80125	1,417	4.9167	16.18	.55864
519,205	480,000	2581.9	8.95981	1,437	4.9877	16.12	.55668
520,000	480,679	2585.8	8.97348	1,439	4.9939	16.12	.55651
530,000	489,212	2635.3	9.14522	1,461	5.0709	16.06	.55448
530,925	490,000	2639.9	9.16109	1,463	5.0780	16.06	.55430
540,000	497,719	2684.6	9.31646	1,484	5.1489	16.01	.55266
542,686	500,000	2697.9	9.36237	1,489	5.1683	15.99	.55203

METRIC TABLE II

PRESSURE, DENSITY AND ACCELERATION OF GRAVITY AS FUNCTIONS OF GEOMETRIC AND GEOPOTENTIAL ALTITUDE

ALTITUDE		PRESSURE		DENSITY		ACCELERATION OF GRAVITY	
Z,m	H,m'	P,mb	P/P_o	ρ,kg/m³	ρ/ρ_o	g,m/sec²	g/go
-5,000	-5,003.9	1.7776^{+3}	1.75438	1.9312	1.57644	9.82210	1.001575
-4,996.1	-5,000	1.7769	1.75365	1.9305	1.57591	9.82209	1.001574
-4,000	-4,002.5	1.5960	1.57515	1.7698	1.44472	9.81901	1.001260
-3,997.5	-4,000	1.5956	1.57469	1.7694	1.44437	9.81900	1.001259
-3,000	-3,001.4	1.4297	1.41104	1.6189	1.32157	9.81592	1.000945
-2,998.6	-3,000	1.4295	1.41082	1.6187	1.32140	9.81591	1.000944
-2,000	-2,000.6	1.2778	1.26112	1.4782	1.20667	9.81283	1.000630
-1,999.4	-2,000	1.2777	1.26103	1.4781	1.20660	9.81282	1.000629
-1,000	-1,000.2	1.1393	1.12441	1.3470	1.09960	9.80774	1.000315
- 998.8	-1,000	1.1393	1.12439	1.3470	1.09958	9.80774	1.000315
0	0	1.01325^{+3}	1.00000	1.2250	1.00000	9.80665	1.000000
1,000	999.8	8.9876^{+2}	8.87008^{-1}	1.1117	9.07475^{-1}	9.80356	.9996854
1,000.2	1,000	8.9875	8.86994	1.1117	9.07464	9.80356	.9996854
2,000	1,999.4	7.9501	7.84615	1.0066	8.21671	9.80048	.9993710
2,000.6	2,000	7.9495	7.84556	1.0065	8.21622	9.80048	.9993708
3,000	2,998.6	7.0121	6.92039	9.0926^{-1}	7.42243	9.79740	.9990568
3,001.4	3,000	7.0108	6.91917	9.0913	7.42137	9.79740	.9990563
4,000	3,997.5	6.1660	6.08537	8.1935	6.68847	9.79432	.9987427
4,002.5	4,000	6.1640	6.08339	8.1913	6.68671	9.79431	.9987419
5,000	4,996.1	5.4048^{+2}	5.33413^{-1}	7.3643^{-1}	6.01161^{-1}	9.79124	.9984287
5,003.9	5,000	5.4020	5.33133	7.3612	6.00906	9.79123	.9984275
6,000	5,994.3	4.7217	4.65998	6.6011	5.38859	9.78816	.9981149
6,005.7	6,000	4.7181	4.65635	6.5969	5.38519	9.78815	.9981131
7,000	6,992.3	4.1105	4.05676	5.9002	4.81643	9.78509	.9978013
7,007.7	7,000	4.1060	4.05233	5.8950	4.81216	9.78506	.9977988
8,000	7,989.9	3.5651	3.51851	5.2578	4.29206	9.78201	.9974877
8,010.1	8,000	3.5599	3.51339	5.2516	4.28701	9.78198	.9974846
9,000	8,987.3	3.0800	3.03977	4.6706	3.81270	9.77894	.9971744
9,012.8	9,000	3.0742	3.03401	4.6634	3.80685	9.77890	.9971704
10,000	9,984.3	2.6500^{+2}	2.61532^{-1}	4.1351^{-1}	3.37554^{-1}	9.77587	.9968612
10,016	10,000	2.6436	2.60903	4.1270	3.36896	9.77582	.9968562
11,000	10,981	2.2700	2.24030	3.6480	2.97792	9.77280	.9965481
11,019	11,000	2.2632	2.23358	3.6391	2.97069	9.77274	.9965421
12,000	11,977	1.9399	1.91455	3.1193	2.54637	9.76973	.9962352
12,023	12,000	1.9330	1.90774	3.1082	2.53731	9.76966	.9962281
13,000	12,973	1.6579	1.63626	2.1659	2.17624	9.76666	.9959224
13,027	13,000	1.6510	1.62943	2.6548	2.16716	9.76658	.9959140
14,000	13,969	1.4170	1.39849	2.2785	1.86001	9.76360	.9956098
14,031	14,000	1.4102	1.39172	2.2675	1.85100	9.76350	.9956001

METRIC TABLE II CONTINUED

ALTITUDE		PRESSURE		DENSITY		ACCELERATION OF GRAVITY	
Z,m	H,m'	P,mb	P/P_o	ρ, kg/m^3	ρ/ρ_o	g, m/sec^2	g/go
15,000	14,965	1.2112^{+2}	1.19533^{-1}	1.9475^{-1}	1.58980^{-1}	9.76053	.9952973
15,035	15,000	1.2044	1.18869	1.9367	1.58097	9.76042	.9952862
16,000	15,960	1.0353	1.02173	1.6647	1.35891	9.75747	.9949849
16,040	16,000	1.0287	1.01528	1.6542	1.35033	9.75735	.9949723
17,000	16,955	8.8496^{+1}	8.73388^{-2}	1.4230	1.16162	9.75441	.9946728
17,046	17,000	8.7866	8.67167	1.4129	1.15334	9.75427	.9946585
18,000	17,949	7.5652	7.46623	1.2165	9.93016^{-2}	9.75135	.9943607
18,051	18,000	7.5048	7.40662	1.2067	9.85088	9.75119	.9943448
19,000	18,943	6.4674	6.38285	1.0399	8.48925	9.74829	.9940448
19,057	19,000	6.4099	6.32611	1.0307	8.41379	9.74811	.9940311
20,000	19,937	5.5293^{+1}	5.45694^{-2}	8.8909^{-2}	7.25779^{-2}	9.74523	.9937371
20,063	20,000	5.4748	5.40323	8.8034	7.18634	9.74504	.9937174
21,000	20,931	4.7275	4.66564	7.6016	6.20534	9.74218	.9934255
21,070	21,000	4.6761	4.61498	7.5191	6.13797	9.74196	.9934038
22,000	21,924	4.0420	3.98918	6.4995	5.30565	9.73912	.9931140
22,076	22,000	3.9940	3.94173	6.4222	5.24255	9.73889	.9930902
23,000	22,917	3.4562	3.41101	5.5575	4.53667	9.73607	.9928027
23,084	23,000	3.4113	3.36670	5.4853	4.47774	9.73581	.9927767
24,000	23,910	2.9554	2.91677	4.7522	3.87934	9.73302	.9924916
24,091	24,000	2.9137	2.87555	4.6851	3.82451	9.73274	.9924633
25,000	24,902	2.5273^{+1}	2.49428^{-2}	4.0639^{-2}	3.31742^{-2}	9.72997	.9921805
25,099	25,000	2.4886	2.45606	4.0016	3.26658	9.72967	.9921498
26,000	25,894	2.1632	2.13493	3.4359	2.80476	9.72692	.9918697
26,107	26,000	2.1278	2.10001	3.3748	2.75490	9.72659	.9918365
27,000	26,886	1.8555	1.83126	2.9077	2.37361	9.72387	.9915589
27,115	27,000	1.8233	1.79943	2.8528	2.32877	9.72352	.9915232
28,000	27,877	1.5949	1.57407	2.4663	2.01332	9.72083	.9912484
28,124	28,000	1.5655	1.54504	2.4169	1.97296	9.72045	.9912099
29,000	28,868	1.3737	1.35573	2.0966	1.71147	9.71778	.9909379
29,133	29,000	1.3469	1.32930	2.0521	1.67520	9.71738	.9908967
30,000	29,859	1.1855^{+1}	1.17002^{-2}	1.7861^{-2}	1.45803^{-2}	9.71474	.9906276
30,142	30,000	1.1611	1.14592	1.7461	1.42540	9.71431	.9905835
31,000	30,850	1.0251	1.01167	1.5248	1.24472	9.71170	.9903175
31,152	31,000	1.0028	9.89735^{-3}	1.4889	1.21538	9.71124	.9902704
32,000	31,840	8.8801^{+0}	8.76402	1.3044	1.06478	9.70866	.9900075
32,162	32,000	8.6776	8.56423	1.2721	1.03840	9.70816	.9899573
33,000	32,830	7.7068	7.60604	1.1180	9.12666^{-3}	9.70562	.9896977
33,172	33,000	7.5224	7.42412	1.0890	8.88944	9.70510	.9896443
34,000	33,819	6.7006	6.61300	9.6019^{-3}	7.83821	9.70258	.9893879
34,183	34,000	6.5327	6.44726	9.3404	7.62473	9.70203	.9893314

METRIC TABLE II CONTINUED

ALTITUDE		PRESSURE		DENSITY		ACCELERATION OF GRAVITY	
Z,m	H,m'	P,mb	P/P_o	ρ,kg/m^3	ρ/ρ_o	g,m/sec^2	g/go
35,000	34,808	5.8359^{+0}	5.75960^{-3}	8.2619^{-3}	6.74437^{-3}	9.69955	.9890784
35,194	35,000	5.6829	5.60855	8.0265	6.55217	9.69896	.9890184
36,000	35,797	5.0914	5.02486	7.1221	5.81390	9.69651	.9887690
36,205	36,000	4.9519	4.88717	6.9101	5.64082	9.69589	.9887056
37,000	36,786	4.4493	4.39115	6.1507	5.02089	9.69348	.9884597
37,217	37,000	4.3221	4.26562	5.9597	4.86497	9.69282	.9883927
38,000	37,774	3.8944	3.84344	5.3209	4.34354	9.69045	.9881506
38,229	38,000	3.7785	3.72908	5.1489	4.20313	9.68975	.9880800
39,000	38,762	3.4142	3.36952	4.6112	3.76419	9.68742	.9878416
39,241	39,000	3.3084	3.26514	4.4560	3.63753	9.68669	.9877672
40,000	39,750	2.9977^{+0}	2.95851^{-3}	4.0027^{-3}	3.26751^{-3}	9.68439	.9875328
40,253	40,000	2.9013	2.86333	3.8629	3.15332	9.68362	.9874546
41,000	40,737	2.6361	2.60159	3.4803	2.84105	9.68136	.9872241
41,266	41,000	2.5481	2.51474	3.3541	2.73803	9.68056	.9871420
42,000	41,724	2.3215	2.29110	3.0310	2.47422	9.67834	.9869155
42,279	42,000	2.2411	2.21176	2.9169	2.38115	9.67749	.9868294
43,000	42,711	2.0474	2.02060	2.6438	2.15815	9.67531	.9866072
43,293	43,000	1.9739	1.94812	2.5408	2.07408	9.67443	.9865169
44,000	43,698	1.8082	1.78454	2.3096	1.88534	9.67229	.9862989
44,307	44,000	1.7411	1.71828	2.2165	1.80933	9.67136	.9862044
45,000	44,684	1.5991^{+0}	1.57820^{-3}	2.0206^{-3}	1.64946^{-3}	9.66927	.9859908
45,321	45,000	1.5378	1.51765	1.9364	1.58073	9.66830	.9858920
46,000	45,670	1.4161	1.39763	1.7704	1.44523	9.66625	.9856828
46,335	46,000	1.3600	1.34224	1.6942	1.38304	9.66523	.9855796
47,000	46,655	1.2558	1.23936	1.5535	1.26812	9.66323	.9853750
47,350	47,000	1.2044	1.18866	1.4845	1.21179	9.66217	.9852673
48,000	47,640	1.1147	1.10014	1.3739	1.12155	9.66021	.9850673
48,365	48,000	1.0673	1.05333	1.3155	1.07383	9.65911	.9849550
49,000	48,625	9.8961^{-1}	9.76671^{-4}	1.2197	9.95675^{-4}	9.65719	.9847598
49,381	49,000	9.4578	9.33411	1.1657	9.51574	9.65605	.9846428
50,000	49,610	8.7858^{-1}	8.67088^{-4}	1.0829^{-3}	8.83961^{-4}	9.65418	.9844524
50,396	50,000	8.3810	8.27142	1.0330	8.43237	9.65299	.9843306
51,000	50,594	7.8003	7.69829	9.6140^{-4}	7.84809	9.65117	.9841452
51,412	51,000	7.4269	7.32973	9.1537	7.47235	9.64992	.9840185
52,000	51,578	6.9256	6.83507	8.5360	6.96807	9.64815	.9838381
52,429	52,000	6.5813	6.49524	8.1116	6.62162	9.64686	.9837064
53,000	52,562	6.1493	6.06886	7.5791	6.18694	9.64515	.9835311
53,446	53,000	5.8320	5.75576	7.1881	5.86775	9.64380	.9833944
54,000	53,545	5.4588	5.38738	6.7790	5.53383	9.64214	.9832243
54,463	54,000	5.1637	5.09615	6.4534	5.26800	9.64074	.9830824

METRIC TABLE II CONTINUED

ALTITUDE		PRESSURE		DENSITY		ACCELERATION OF GRAVITY	
Z,m	H,m'	P,mb	P/P_o	ρ,kg/m^3	ρ/ρ_o	g,m/sec^2	g/go
55,000	54,528	4.8388^{-1}	4.77557^{-4}	6.0924^{-4}	4.97335^{-4}	9.63913	.9829176
55,480	55,000	4.5641	4.50445	5.7850	4.72241	9.63769	.9827705
56,000	55,511	4.2822	4.22624	5.4674	4.46310	9.63612	.9826111
56,498	56,000	4.0270	3.97438	5.1777	4.22666	9.63463	.9824586
57,000	56,493	3.7833	3.73384	4.8991	3.99926	9.63312	.9823047
57,516	57,000	3.5467	3.50036	4.6268	3.77692	9.63157	.9821467
58,000	57,476	3.3367	3.29306	4.3832	3.57809	9.63012	.9819985
58,534	58,000	3.1179	3.07713	4.1276	3.36945	9.62851	.9818350
59,000	58,457	2.9375	2.89912	3.9154	3.19620	9.62711	.9816924
59,553	59,000	2.7356	2.69987	3.6761	3.00083	9.62545	.9815232
60,000	59,439	2.5814^{-1}	2.54761^{-4}	3.4918^{-4}	2.85042^{-4}	9.62411	.9813864
60,572	60,000	2.3955	2.36417	3.2681	2.66784	9.62240	.9812116
61,000	60,420	2.2641	2.23453	3.1089	2.53783	9.62111	.9810806
61,591	61,000	2.0934	2.06598	2.9002	2.36750	9.61934	.9808999
62,000	61,401	1.9820	1.95606	2.7631	2.25558	9.61812	.9807749
62,611	62,000	1.8255	1.80159	2.5689	2.09705	9.61629	.9805884
63,000	62,382	1.7315	1.70885	2.4514	2.00114	9.61512	.9804694
63,631	63,000	1.5884	1.56764	2.2711	1.85394	9.61323	.9802768
64,000	63,362	1.5096	1.48982	2.1709	1.77218	9.61213	.9801640
64,651	64,000	1.3790	1.36099	2.0038	1.63573	9.61018	.9799653
65,000	64,342	1.3132^{-1}	1.29606^{-4}	1.9189^{-4}	1.56640^{-4}	9.60913	.9798588
65,672	65,000	1.1945	1.17885	1.7643	1.44026	9.60712	.9796539
66,000	65,322	1.1399	1.12503	1.6928	1.38185	9.60614	.9795537
66,692	66,000	1.0322	1.01866	1.5502	1.26546	9.60407	.9793425
67,000	66,301	9.8726^{-2}	9.74349^{-5}	1.4903	1.21658	9.60315	.9792488
67,714	67,000	8.8969	8.78052	1.3591	1.10944	9.60102	.9790312
68,000	67,280	8.5301	8.41856	1.3093	1.06883	9.60016	.9789439
68,735	68,000	7.6491	7.54912	1.1888	9.70447^{-5}	9.59796	.9787199
69,000	68,259	7.3523	7.25615	1.1479	9.37010	9.59717	.9786393
69,757	69,000	6.5590	6.47323	1.0374	8.46874	9.59491	.9784087
70,000	69,238	6.3212^{-2}	6.23854^{-5}	1.0040^{-4}	8.19618^{-5}	9.59419	.9783347
70,779	70,000	5.6088	5.53547	9.0313^{-5}	7.37244	9.59186	.9780975
71,000	70,216	5.4206	5.34974	8.7624	7.15289	9.59120	.9780304
71,802	71,000	4.7826	4.72010	7.8424	6.40188	9.58881	.9777864
72,000	71,194	4.6357	4.57513	7.6286	6.22739	9.58822	.9777261
72,825	72,000	4.0662	4.01299	6.7922	5.54461	9.58576	.9774753
73,000	72,171	3.9535	3.90181	6.6252	5.40830	9.58524	.9774220
73,848	73,000	3.4464	3.40132	5.8666	4.78903	9.58271	.9771642
74,000	73,148	3.3619	3.31794	5.7391	4.68489	9.58225	.9771181
74,872	74,000	2.9118	2.87373	5.0529	4.12479	9.57966	.9768532

METRIC TABLE II CONTINUED

ALTITUDE		PRESSURE		DENSITY		ACCELERATION OF GRAVITY	
Z,m	H,m'	P,mb	P/P_o	ρ,kg/m^3	ρ/ρ_o	g,m/sec^2	g/go
75,000	74,125	2.8503^{-2}	2.81299^{-5}	4.9582^{-5}	4.04747^{-5}	9.57928	.9768142
75,895	75,000	2.452	2.4200	4.339	3.5423	9.57661	.9765423
76,000	75,102	2.409	2.3775	4.263	3.4801	9.57630	.9765106
76,920	76,000	2.061	2.0344	3.648	2.9780	9.57356	.9762314
77,000	76,078	2.034	2.0069	3.599	2.9377	9.57332	.9762070
77,944	77,000	1.733	1.7103	3.067	2.5035	9.57051	.9759206
78,000	77,055	1.717	1.6942	3.038	2.4799	9.57035	.9759036
78,969	78,000	1.457	1.4378	2.578	2.1046	9.56746	.9756098
79,000	78,030	1.449	1.4303	2.565	2.0936	9.56737	.9756004
79,994	79,000	1.225	1.2087	2.167	1.7693	9.56442	.9752990
80,000	79,006	1.224^{-2}	1.2075^{-5}	2.165^{-5}	1.7676^{-5}	9.56440	.9752973
81,000	79,981	1.033	1.0195	1.828	1.4924	9.56143	.9749943
81,020	80,000	1.030	1.0162	1.822	1.4874	9.56137	.9749883
82,000	80,956	8.723^{-3}	8.6085^{-6}	1.544	1.2601	9.55846	.9746915
82,045	81,000	8.656	8.5425	1.532	1.2504	9.55832	.9746777
83,000	81,930	7.365	7.2690	1.303	1.0640	9.55549	.9743888
83,072	82,000	7.277	7.1815	1.288	1.0512	9.55528	.9743671
84,000	82,904	6.220	6.1383	1.101	8.9851^{-6}	9.55252	.9740862
84,098	83,000	6.117	6.0373	1.083	8.8373	9.55223	.9740566
85,000	83,878	5.252^{-3}	5.1837^{-6}	9.295^{-6}	7.5878^{-6}	9.54956	.9737838
85,125	84,000	5.143	5.0754	9.101	7.4293	9.54919	.9737461
86,000	84,852	4.436	4.3778	7.850	6.4081	9.54659	.9734816
86,152	85,000	4.323	4.2668	7.651	6.2456	9.54614	.9734356
87,000	85,825	3.746	3.6974	6.630	5.4121	9.54363	.9731795
87,179	86,000	3.635	3.5870	6.432	5.2506	9.54310	.9731253
88,000	86,798	3.164	3.1229	5.600	4.5712	9.54067	.9728774
88,207	87,000	3.055	3.0155	5.407	4.4140	9.54006	.9728149
89,000	87,771	2.673	2.6378	4.730	3.8611	9.53771	.9725756
89,235	88,000	2.569	2.5351	4.546	3.7108	9.53701	.9725046
90,000	88,744	2.258^{-3}	2.2282^{-6}	3.995^{-6}	3.2615^{-6}	9.53475	.9722739
90,264	89,000	2.159	2.1312	3.822	3.1196	9.53397	.9721944
91,000	89,716	1.907	1.8823	3.375	2.7552	9.53179	.9719724
91,293	90,000	1.815	1.7916	3.213	2.6225	9.53093	.9718842
92,000	90,688	1.612	1.5913	2.819	2.3012	9.52884	.9716709
92,322	91,000	1.528	1.5085	2.658	2.1695	9.52789	.9715740
93,000	91,659	1.367	1.3490	2.350	1.9181	9.52588	.9713697
93,351	92,000	1.291	1.2739	2.206	1.8006	9.52485	.9712639
94,000	92,630	1.162	1.1468	1.965	1.6037	9.52293	.9710685
94,381	93,000	1.093	1.0789	1.837	1.4992	9.52180	.9709539

METRIC TABLE II CONTINUED

ALTITUDE		PRESSURE		DENSITY		ACCELERATION OF GRAVITY	
Z,m	H,m'	P,mb	P/P_o	ρ,kg/m^3	ρ/ρ_o	g,m/sec^2	g/go
95,000	93,601	9.905^{-4}	9.7759^{-7}	1.647^{-6}	1.3449^{-6}	9.51998	.9707675
95,411	94,000	9.284	9.1622	1.534	1.2521	9.51876	.9706439
96,000	94,572	8.466	8.3552	1.386	1.1311	9.51703	.9704666
96,441	95,000	7.905	7.8021	1.285	1.0488	9.51573	.9703339
97,000	95,542	7.254	7.1590	1.169	9.5392^{-7}	9.51408	.9701659
97,472	96,000	6.749	6.6612	1.079	8.8106	9.51269	.9700240
98,000	96,512	6.231	6.1492	9.882^{-7}	8.0671	9.51113	.9698654
98,503	97,000	5.777	5.7015	9.092	7.4220	9.50965	.9697142
99,000	97,482	5.365	5.2944	8.379	6.8399	9.50818	.9695649
99,534	98,000	4.957	4.8920	7.680	6.2691	9.50661	.9694044
100,000	98,451	4.629^{-4}	4.5689^{-7}	7.123^{-7}	5.8142^{-7}	9.50524	.9692646
100,566	99,000	4.263	4.2073	6.504	5.3091	9.50357	.9690946
101,000	99,420	4.004	3.9516	6.069	4.9545	9.50230	.9689644
101,598	100,000	3.675	3.6268	5.522	4.5074	9.50053	.9687849
102,000	100,389	3.471	3.4253	5.184	4.2321	9.49935	.9686644
102,631	101,000	3.175	3.1333	4.699	3.8363	9.49750	.9684753
103,000	101,358	3.015	2.9753	4.439	3.6234	9.49641	.9683645
103,663	102,000	2.749	2.7129	4.009	3.2728	9.49446	.9681657
104,000	102,326	2.624	2.5896	3.809	3.1092	9.49347	.9680648
104,696	103,000	2.385	2.3538	3.428	2.7986	9.49143	.9678561
105,000	103,294	2.288^{-4}	2.2585^{-7}	3.276^{-7}	2.6739^{-7}	9.49053	.9677652
105,730	104,000	2.073	2.0463	2.938	2.3984	9.48839	.9675466
106,000	104,261	2.000	1.9735	2.823	2.3044	9.48760	.9674657
106,764	105,000	1.806	1.7826	2.523	2.0600	9.48536	.9672372
107,000	105,229	1.751	1.7277	2.438	1.9901	9.48466	.9671664
107,798	106,000	1.576	1.5559	2.172	1.7731	9.48232	.9669278
108,000	106,196	1.535	1.5153	2.110	1.7222	9.48173	.9668672
108,832	107,000	1.378	1.3605	1.873	1.5292	9.47929	.9666184
109,000	107,162	1.349	1.3314	1.829	1.4932	9.47880	.9665682
109,867	108,000	1.208	1.1918	1.619	1.3216	9.47626	.9663091
110,000	108,129	1.187^{-4}	1.1718^{-7}	1.589^{-7}	1.2972^{-7}	9.47586	.9662692
110,902	109,000	1.060	1.0459	1.402	1.1444	9.47322	.9659999
111,000	109,095	1.047	1.0331	1.383	1.1289	9.47293	.9659705
111,937	110,000	9.316^{-5}	9.1941^{-8}	1.216	9.9280^{-8}	9.47019	.9656906
112,000	110,061	9.244	9.1229	1.206	9.8432	9.47001	.9656718
112,973	111,000	8.203	8.0961	1.057	8.6292	9.46716	.9653815
113,000	111,026	8.176	8.0692	1.053	8.5975	9.46708	.9653733
114,000	111,992	7.243	7.1484	9.215^{-8}	7.5224	9.46415	.9650750
114,009	112,000	7.235	7.1408	9.204	7.5137	9.46413	.9650724

METRIC TABLE II CONTINUED

ALTITUDE		PRESSURE		DENSITY		ACCELERATION OF GRAVITY	
Z,m	H,m'	P,mb	P/P_o	ρ,kg/m^3	ρ/ρ_o	g,m/sec^2	g/go
115,000	112,957	6.426^{-5}	6.3422^{-8}	8.076^{-8}	6.5928^{-8}	9.46123	.9647768
115,045	113,000	6.392	6.3083	8.029	6.5540	9.46110	.9647633
116,000	113,921	5.710	5.6354	7.090	5.7875	9.45830	.9644787
116,082	114,000	5.655	5.5815	7.015	5.7265	9.45807	.9644543
117,000	114,885	5.081	5.0145	6.234	5.0887	9.45538	.9641807
117,119	115,000	5.011	4.9459	6.140	5.0120	9.45503	.9641454
118,000	115,850	4.528	4.4684	5.490	4.4812	9.45246	.9638829
118,156	116,000	4.447	4.3893	5.383	4.3938	9.45201	.9638365
119,000	116,813	4.040	3.9873	4.842	3.9524	9.44954	.9635853
119,194	117,000	3.952	3.9008	4.726	3.8579	9.44898	.9635276
120,000	117,777	3.610^{-5}	3.5628^{-8}	4.277^{-8}	3.4911^{-8}	9.44663	.9632877
120,232	118,000	3.518	3.4716	4.156	3.3927	9.44595	.9632188
121,000	118,740	3.230	3.1876	3.783	3.0881	9.44371	.9629904
121,270	119,000	3.135	3.0939	3.660	2.9881	9.44292	.9629100
122,000	119,703	2.894	2.8557	3.351	2.7355	9.44079	.9626931
122,309	120,000	2.798	2.7610	3.229	2.6357	9.43989	.9626013
123,000	120,665	2.595	2.5615	2.973	2.4265	9.43788	.9623960
123,348	121,000	2.500	2.4671	2.852	2.3282	9.43687	.9622927
124,000	121,627	2.331	2.3005	2.640	2.1554	9.43497	.9620990
124,387	122,000	2.237	2.2074	2.523	2.0595	9.43384	.9619840
125,000	122,589	2.096^{-5}	2.0685^{-8}	2.348^{-8}	1.9171^{-8}	9.43206	.9618022
125,427	123,000	2.004	1.9775	2.235	1.8243	9.43081	.9616755
126,000	123,551	1.887	1.8622	2.092	1.7073	9.42915	.9615055
126,467	124,000	1.797	1.7737	1.982	1.6181	9.42779	.9613670
127,000	124,512	1.701	1.6783	1.865	1.5225	9.42624	.9612089
127,507	125,000	1.614	1.5928	1.761	1.4372	9.42476	.9610585
128,000	125,473	1.534	1.5143	1.665	1.3593	9.42333	.9609125
128,548	126,000	1.451	1.4320	1.566	1.2781	9.42174	.9607501
129 000	126,434	1.386	1.3681	1.476	1.2049	9.42043	.9606162
129,589	127,000	1.307	1.2903	1.368	1.1170	9.41872	.9604417
130,000	127,395	1.256^{-5}	1.2394^{-8}	1.299^{-8}	1.0604^{-8}	9.41752	.9603200
130,630	128,000	1.182	1.1662	1.201	9.8014^{-9}	9.41569	.9601334
131,000	128,355	1.141	1.1259	1.147	9.3657	9.41462	.9600240
131,672	129,000	1.071	1.0571	1.058	8.6327	9.41267	.9598251
132,000	129,315	1.039	1.0255	1.017	8.3007	9.41172	.9597281
132,714	130,000	9.736^{-6}	9.6084^{-9}	9.347^{-9}	7.6303	9.40965	.9595169
135,000	132,193	7.967	7.8632	7.214	5.8886	9.40302	.9588413
137,929	135,000	6.264	6.1817	5.285	4.3146	9.39454	.9579766

METRIC TABLE II CONTINUED

ALTITUDE		PRESSURE		DENSITY		ACCELERATION OF GRAVITY	
Z,m	H,m'	P,mb	P/P_o	ρ,kg/m^3	ρ/ρ_o	g,m/sec^2	g/go
140,000	136,983	5.336^{-6}	5.2662^{-9}	4.296^{-9}	3.5071^{-9}	9.38855	.9573660
143,153	140,000	4.238	4.1831	3.190	2.6042	9.37945	.9564375
145,000	141,766	3.729	3.6807	2.704	2.2072	9.37412	.9558941
148,385	145,000	2.985	2.9464	2.028	1.6555	9.36437	.9548996
150,000	146,542	2.698	2.6628	1.779	1.4525	9.35972	.9544256
153,625	150,000	2.173	2.1442	1.345	1.0977	9.34930	.9533630
155,000	151,311	2.008	1.9820	1.215	9.9162^{-10}	9.34535	.9529605
158,874	155,000	1.624	1.6032	9.234^{-10}	7.5380	9.33424	.9518276
160,000	156,072	1.531^{-6}	1.5110^{-9}	8.554^{-10}	6.9824^{-10}	9.33101	.9514987
164,131	160,000	1.243	1.2264	6.531	5.3312	9.31920	.9502935
165,000	160,826	1.191	1.1756	6.183	5.0476	9.31671	.9500403
169,397	165,000	9.692^{-7}	9.5657^{-10}	4.737	3.8668	9.30416	.9487606
170,000	165,572	9.431	9.3080	4.573	3.7326	9.30244	.9485852
174,671	170,000	7.689	7.5881	3.511	2.8663	9.28914	.9472289
175,000	170,311	7.582	7.4832	3.449	2.8152	9.28821	.9471335
179,954	175,000	6.189	6.1085	2.653	2.1655	9.27413	.9456985
180,000	175,043	6.178^{-7}	6.0974^{-10}	2.647^{-10}	2.1609^{-10}	9.27400	.9456852
185,000	179,768	5.082	5.0159	2.107	1.7196	9.25983	.9442401
185,245	180,000	5.035	4.9689	2.083	1.7008	9.25913	.9441693
190,000	184,486	4.208	4.1533	1.689	1.3790	9.24569	.9427984
190,545	185,000	4.124	4.0704	1.650	1.3469	9.24415	.9426413
195,000	189,196	3.506	3.4602	1.364	1.1138	9.23159	.9413599
195,854	190,000	3.400	3.3559	1.316	1.0747	9.22918	.9411146
200,000	193,899	2.938^{-7}	2.8995^{-10}	1.110^{-10}	9.0572^{-11}	9.21751	.9399247
201,171	195,000	2.821	2.7840	1.058	8.6369	9.21422	.9395891
205,000	198,595	2.475	2.4427	9.079^{-11}	7.4117	9.20347	.9384929
206,497	200,000	2.354	2.3229	8.560	6.9880	9.19927	.9380648
210,000	203,284	2.096	2.0685	7.474	6.1014	9.18946	.9370643
211,831	205,000	1.974	1.9486	6.970	5.6899	9.18433	.9365418
215,000	207,966	1.783	1.7600	6.188	5.0511	9.17548	.9356389
217,175	210,000	1.665	1.6430	5.709	4.6605	9.16941	.9350200
220,000	212,641	1.524^{-7}	1.5044^{-10}	5.150^{-11}	4.2039^{-11}	9.16154	.9342168
222,526	215,000	1.410	1.3920	4.703	3.8389	9.15450	.9334995
225,000	217,308	1.308	1.2914	4.308	3.5164	9.14762	.9327979
227,887	220,000	1.200	1.1846	3.894	3.1789	9.13960	.9319802
230,000	221,969	1.128	1.1131	3.620	2.9554	9.13374	.9313823
233,256	225,000	1.026	1.0126	3.241	2.6457	9.12472	.9304621
235,000	226,622	9.759^{-8}	9.6315^{-11}	3.057	2.4953	9.11989	.9299699
238,634	230,000	8.805	8.6900	2.710	2.2124	9.10984	.9289453

METRIC TABLE II CONTINUED

ALTITUDE		PRESSURE		DENSITY		ACCELERATION OF GRAVITY	
Z,m	H,m'	P,mb	P/P_o	ρ,kg/m^3	ρ/ρ_o	g,m/sec^2	g/go
240,000	231,268	8.475^{-8}	8.3646^{-11}	2.592^{-11}	2.1158^{-11}	9.10607	.9285607
244,021	235,000	7.586	7.4869	2.277	1.8585	9.09498	.9274297
245,000	235,908	7.387	7.2900	2.207	1.8014	9.09228	.9271547
249,417	240,000	6.560	6.4741	1.921	1.5679	9.08013	.9259154
250,000	240,540	6.459	6.3746	1.886	1.5398	9.07852	.9257519
254,821	245,000	5.692	5.6179	1.627	1.3282	9.06529	.9244022
255,000	245,165	5.666	5.5920	1.618	1.3210	9.06480	.9243522
260,000	249,784	4.986^{-8}	4.9204^{-11}	1.393^{-11}	1.1374^{-11}	9.05110	.9229558
260,235	250,000	4.956	4.8913	1.384	1.1295	9.05046	.9228904
265,000	254,395	4.400	4.3421	1.204	9.8261^{-12}	9.03744	.9215625
265,657	255,000	4.329	4.2722	1.181	9.6414	9.03565	.9213797
270,000	258,999	3.893	3.8423	1.043	8.5166	9.02381	.9201724
271,088	260,000	3.792	3.7428	1.012	8.2591	9.02085	.9198703
275,000	263,597	3.454	3.4092	9.071^{-12}	7.4048	9.01021	.9187854
276,528	265,000	3.332	3.2886	8.697	7.0992	9.00606	.9183622
280,000	268,187	3.073^{-8}	3.0327^{-11}	7.910^{-12}	6.4574^{-12}	8.99664	.9174015
281,977	270,000	2.936	2.8975	7.499	6.1220	8.99128	.9168552
285,000	272,771	2.740	2.7044	6.918	5.6473	8.98309	.9160207
287,435	275,000	2.594	2.5598	6.487	5.2958	8.97651	.9153496
290,000	277,347	2.449	2.4173	6.067	4.9525	8.96958	.9146431
292,902	280,000	2.297	2.2672	5.629	4.5948	8.96176	.9138451
295,000	281,917	2.194	2.1655	5.334	4.3546	8.95611	.9132686
298,377	285,000	2.040	2.0130	4.898	3.9980	8.94702	.9123419
300,000	286,480	1.970^{-8}	1.9442^{-11}	4.703^{-12}	3.8388^{-12}	8.94266	.9118972
303,862	290,000	1.815	1.7914	4.273	3.4883	8.93229	.9108399
305,000	291,036	1.772	1.7492	4.156	3.3923	8.92924	.9105288
309,356	295,000	1.619	1.5979	3.738	3.0517	8.91757	.9093391
310,000	295,585	1.598	1.5769	3.681	3.0048	8.91585	.9091636
314,859	300,000	1.447	1.4284	3.279	2.6765	8.90287	.9078396
320,000	304,663	1.306^{-8}	1.2890^{-11}	2.908^{-12}	2.3736^{-12}	8.88916	.9064422
325,893	310,000	1.164	1.1485	2.541	2.0739	8.87349	.9048444
330,000	313,714	1.075	1.0613	2.316	1.8909	8.86259	.9037331
336,963	320,000	9.431^{-9}	9.3073^{-12}	1.987	1.6217	8.84417	.9018540
340,000	322,738	8.914	8.7978	1.860	1.5183	8.83615	.9010361
348,069	330,000	7.698	7.5971	1.567	1.2788	8.81489	.8988686

METRIC TABLE II CONTINUED

ALTITUDE		PRESSURE		DENSITY		ACCELERATION OF GRAVITY	
Z,m	H,m'	P,mb	P/P_o	ρ,kg/m^3	ρ/ρ_o	g,m/sec^2	g/go
350,000	331,735	7.437^{-9}	7.3393^{-12}	1.505^{-12}	1.2282^{-12}	8.80982	.8983512
359,213	340,000	6.326	6.2432	1.245	1.0165	8.78566	.8958882
360,000	340,705	6.241	6.1589	1.226	1.0005	8.78360	.8956782
370,000	349,648	5.266	5.1973	1.005	8.2026^{-13}	8.75751	.8930172
370,394	350,000	5.232	5.1631	9.971^{-13}	8.1396	8.75648	.8929127
380,000	358,565	4.467	4.4088	8.289	6.7665	8.73153	.8903680
381,612	360,000	4.352	4.2954	8.040	6.5634	8.72735	.8899421
390,000	367,456	3.808	3.7584	6.877	5.6141	8.70566	.8877305
392,867	370,000	3.641	3.5934	6.526	5.3270	8.69827	.8869765
400,000	376,320	3.262^{-9}	3.2190^{-12}	5.737^{-13}	4.6836^{-13}	8.67991	.8851048
404,160	380,000	3.062	3.0220	5.329	4.3501	8.66923	.8840159
410,000	385,158	2.806	2.7691	4.811	3.9273	8.65428	.8824907
415,491	390,000	2.586	2.5517	4.373	3.5696	8.64025	.8810602
420,000	393,970	2.424	2.3922	4.054	3.3096	8.62876	.8798882
426,860	400,000	2.197	2.1687	3.615	2.9508	8.61131	.8781094
430,000	402,756	2.102	2.0747	3.432	2.7848	8.60335	.8772971
438,267	410,000	1.874	1.8496	3.000	2.4495	8.58242	.8751636
440,000	411,516	1.830	1.8062	2.918	2.3824	8.57805	.8747175
449,713	420,000	1.605	1.5841	2.503	2.0434	8.55358	.8722228
450,000	420,250	1.599^{-9}	1.5780^{-12}	2.492^{-13}	2.0343^{-13}	8.55286	.8721492
460,000	428,959	1.402	1.3834	2.136	1.7440	8.52779	.8695923
461,197	430,000	1.380	1.3621	2.098	1.7126	8.52479	.8692869
470,000	437,642	1.233	1.2168	1.839	1.5008	8.50282	.8670466
472,721	440,000	1.191	1.1756	1.766	1.4417	8.49605	.8663559
480,000	446,300	1.088	1.0735	1.588	1.2963	8.47797	.8645120
484,283	450,000	1.032	1.0184	1.493	1.2187	8.46735	.8634299
490,000	454,932	9.625^{-10}	9.4990^{-13}	1.376	1.1234	8.45322	.8619885
495,884	460,000	8.969	8.8512	1.267	1.0344	8.43871	.8605088
500,000	463,540	8.541^{-10}	8.4293^{-13}	1.197^{-13}	9.7692^{-14}	8.42858	.8594761
507,525	470,000	7.821	7.7184	1.080	8.8124	8.41011	.8575927
510,000	472,122	7.600	7.5003	1.044	8.5219	8.40405	.8569746
519,205	480,000	6.841	6.7515	9.231^{-14}	7.5353	8.38156	.8546816
520,000	480,679	6.780	6.6912	9.135	7.4567	8.37963	.8544840
530,000	489,212	6.064	5.9842	8.016	6.5435	8.35531	.8520043
530,925	490,000	6.002	5.9234	7.921	6.4658	8.35306	.8517754
540,000	497,719	5.436	5.3647	7.054	5.7583	8.33110	.8495354
542,686	500,000	5.281	5.2117	6.819	5.5666	8.32461	.8488741

METRIC TABLE III

Velocity of Sound, Particle Speed, Molecular-Scale Temperature Gradient, and Scale Height as Functions of Geometric and Geopotential Altitude

ALTITUDE		MOL-SCALE TEMP. GRAD.	SCALE HEIGHT		PARTICLE SPEED →	RATIO	← SOUND SPEED
Z, m	H, m'	L_M, °C/m'	H_s, km	H_s/H_{so}	$\bar{V}$, m/sec	$\bar{V}/\bar{V}_o$ C_s/C_{so}	C_s, m/sec
-5,000	-5,003.9		9.3717	1.11112	484.15	1.05493	358.98
-4,996.1	-5,000		9.3709	1.11104	484.13	1.05489	358.97
-4,000	-4,002.5		9.1843	1.08891	479.21	1.04417	355.32
-3,997.5	-4,000		9.1839	1.08886	479.20	1.04414	355.31
-3,000	-3,001.4		8.9969	1.06670	474.22	1.03330	351.62
-2,998.6	-3,000		8.9967	1.06666	474.22	1.03328	351.62
-2,000	-2,000.6		8.8095	1.04447	469.19	1.02232	347.89
-1,999.4	-2,000		8.8094	1.04446	469.18	1.02231	347.88
-1,000	-1,000.2		8.6220	1.02224	464.09	1.01122	344.11
- 999.8	-1,000		8.6220	1.02224	464.09	1.01122	344.11
0	0		8.4344	1.00000	458.94	1.00000	340.29
1,000	999.8		8.2468	.977754	453.74	.988659	336.43
1,000.2	1,000		8.2468	.977751	453.74	.988657	336.43
2,000	1,999.4		8.0591	.955501	448.47	.977190	332.53
2,000.6	2,000		8.0590	.955487	448.47	.977183	332.53
3,000	2,998.6		7.8713	.933241	443.15	.965589	328.58
3,001.4	3,000	-0.0065	7.8711	.933210	443.11	.965572	328.58
4,000	3,997.5		7.6835	.910975	437.76	.953850	324.59
4,002.5	4,000		7.6831	.910918	437.75	.953820	324.58
5,000	4,996.1		7.4957	.888700	432.31	.941968	320.54
5,003.9	5,000		7.4949	.888613	432.29	.941921	320.53
6,000	5,994.3		7.3077	.866419	426.79	.929939	316.45
6,005.7	6,000		7.3067	.866293	426.76	.929870	316.43
7,000	6,992.3		7.1198	.844131	421.20	.917756	312.30
7,007.7	7,000		7.1183	.843959	421.15	.917661	312.27
8,000	7,989.9		6.9317	.821836	415.53	.905412	308.10
8,010.1	8,000		6.9298	.821611	415.47	.905287	308.06
9,000	8,987.3		6.7436	.799534	409.79	.892902	303.85
9,012.8	9,000		6.7412	.799249	409.72	.892742	303.79
10,000	9,984.3		6.5554	.777225	403.97	.880219	299.53
10,016	10,000		6.5525	.776873	403.88	.880018	299.46
11,000	10,981		6.3672	.754908	398.07	.867354	295.15
11,019	11,000		6.3636	.754483	397.95	.867107	295.07
12,000	11,977		6.3656	.754715	397.95	.867107	295.07
12,023	12,000		6.3656	.754720	397.95	.867107	295.07
13,000	12,973		6.3676	.754952	397.95	.867107	295.07
13,027	13,000	0.0000	6.3676	.754959	397.95	.867107	295.07
14,000	13,969		6.3696	.755189	397.95	.867107	295.07
14,031	14,000		6.3696	.755197	397.95	.867107	295.07

METRIC TABLE III CONTINUED

ALTITUDE		MOL-SCALE TEMP.GRAD.	SCALE HEIGHT		PARTICLE SPEED	→ RATIO ←	SOUND SPEED
Z,m	H,m'	L_M, °C/m'	H_s, km	H_s/H_{so}	$\overline{V}$,m/sec	$\overline{V}/\overline{V}_o$ C_s/C_{so}	C_s,m/sec
15,000	14,965		6.3716	.755426	397.95	.867107	295.07
15,035	15,000		6.3717	.755435	397.95	.867107	295.07
16,000	15,960		6.3736	.755664	397.95	.867107	295.07
16,040	16,000		6.3737	.755673	397.95	.867107	295.07
17,000	16,955		6.3756	.755901	397.95	.867107	295.07
17,046	17,000		6.3757	.755915	397.95	.867107	295.07
18,000	17,949		6.3776	.756140	397.95	.867107	295.07
18,051	18,000		6.3777	.756152	397.95	.867107	295.07
19,000	18,943		6.3796	.756377	397.95	.867107	295.07
19,057	19,000		6.3797	.756389	397.95	.867107	295.07
20,000	19,937	0.0000	6.3816	.756615	397.95	.867107	295.07
20,063	20,000		6.3817	.756626	397.95	.867107	295.07
21,000	20,931		6.3836	.756852	397.95	.867107	295.07
21,070	21,000		6.3837	.756864	397.95	.867107	295.07
22,000	21,924		6.3856	.757089	397.95	.867107	295.07
22,076	22,000		6.3857	.757101	397.95	.867107	295.07
23,000	22,917		6.3876	.757326	397.95	.867107	295.07
23,084	23,000		6.3878	.757350	397.95	.867107	295.07
24,000	23,910		6.3896	.757563	397.95	.867107	295.07
24,091	24,000		6.3898	.757587	397.95	.867107	295.07
25,000	24,902		6.3916	.757800	397.95	.867107	295.07
25,099	25,000		6.3918	.757824	397.95	.867107	295.07
26,000	25,894		6.4728	.767427	400.41	.872458	296.89
26,107	26,000		6.4823	.768554	400.70	.873089	297.11
27,000	26,886		6.5626	.778074	403.11	.878355	298.90
27,115	27,000		6.5730	.779307	403.42	.879031	299.13
28,000	27,877		6.6525	.788733	405.80	.884206	300.89
28,124	28,000		6.6636	.790049	406.13	.884933	301.14
29,000	28,868		6.7424	.799392	408.58	.890026	302.87
29,133	29,000	+0.0030	6.7543	.800803	408.82	.890796	303.13
30,000	29,859		6.8323	.810050	411.12	.895802	304.83
30,142	30,000		6.8451	.811568	411.50	.896621	305.11
31,000	30,850		6.9223	.820721	413.75	.901539	306.79
31,152	31,000		6.9360	.822345	414.15	.902407	307.08
32,000	31,840		7.0123	.831392	416.37	.907238	308.13
32,162	32,000		7.0269	.833123	416.79	.908158	309.04
33,000	32,830		7.1023	.842062	418.97	.912900	310.65
33,172	33,000		7.1178	.843900	419.41	.913871	310.98
34,000	33,819		7.1923	.852733	421.55	.918525	312.57
34,183	34,000		7.2088	.854689	422.02	.919550	312.92

METRIC TABLE III CONTINUED

ALTITUDE		MOL-SCALE TEMP. GRAD.	SCALE HEIGHT		PARTICLE SPEED → RATIO ← SOUND SPEED		
Z,m	H,m'	L_M, °C/m'	H_s, km	H_s/H_{so}	$\bar{V}$,m/sec	$\bar{V}/\bar{V}_o$ C_s/C_{so}	C_s,m/sec
35,000	34,808		7.2824	.863415	424.11	.924114	314.47
35,194	35,000		7.2999	.865490	424.61	.925193	314.84
36,000	35,797		7.3725	.874098	426.66	.929668	316.36
36,205	36,000		7.3910	.876291	427.18	.930803	316.74
37,000	36,786		7.4627	.884792	429.20	.935187	318.24
37,217	37,000		7.4822	.887104	429.74	.936378	318.64
38,000	37,774		7.5528	.895474	431.71	.940672	320.10
38,229	38,000		7.5734	.897917	432.29	.941921	320.53
39,000	38,762		7.6430	.906169	434.22	.946124	321.96
39,241	39,000		7.6647	.908741	434.82	.947433	322.40
40,000	39,750		7.7332	.916863	436.70	.951543	323.80
40,253	40,000		7.7561	.919578	437.33	.952910	324.27
41,000	40,737		7.8235	.927569	439.17	.956929	325.64
41,266	41,000	+0.0030	7.8475	.930414	439.83	.958357	326.12
42,000	41,724		7.9138	.938275	441.63	.962283	327.46
42,279	42,000		7.9389	.941251	442.32	.963773	327.96
43,000	42,711		8.0040	.948969	444.08	.967606	329.27
43,293	43,000		8.0305	.952111	444.79	.969159	329.80
44,000	43,698		8.0943	.959676	446.50	.972899	331.07
44,307	44,000		8.1220	.962960	447.25	.974516	331.62
45,000	44,684		8.1847	.970394	448.92	.978161	332.86
45,321	45,000		8.2137	.973832	449.69	.979843	333.43
46,000	45,670		8.2751	.981111	451.32	.983393	334.64
46,335	46,000		8.3054	.984704	452.12	.985141	335.24
47,000	46,655		8.3655	.991829	453.71	.988596	336.41
47,350	47,000		8.3971	.995576	454.54	.990411	337.03
48,000	47,640		8.3988	.995778	454.54	.990411	337.03
48,365	48,000		8.3998	.995896	454.54	.990411	337.03
49,000	48,625		8.4015	.996098	454.54	.990411	337.03
49,381	49,000		8.4025	.996216	454.54	.990411	337.03
50,000	49,610		8.4041	.996406	454.54	.990411	337.03
50,396	50,000	0.0000	8.4051	.996525	454.54	.990411	337.03
51,000	50,594		8.4067	.996714	454.54	.990411	337.03
51,412	51,000		8.4078	.996845	454.54	.990411	337.03
52,000	51,578		8.4093	.997023	454.54	.990411	337.03
52,429	52,000		8.4105	.997165	454.54	.990411	337.03
53,000	52,562		8.4120	.997343	454.54	.990411	337.03
53,446	53,000		8.4131	.997473	454.54	.990411	337.03
54,000	53,545		8.3513	.990146	452.83	.986679	335.76
54,463	54,000	-0.0039	8.2997	.984026	451.39	.983554	334.70

ALTITUDE		MOL-SCALE TEMP. GRAD.	SCALE HEIGHT		PARTICLE SPEED →	RATIO	← SOUND SPEED
Z, m	H, m'	L_M, °C/m'	H_S, km	H_S/H_{SO}	$\bar{V}$, m/sec	$\bar{V}/\bar{V}_o$ C_S/C_{SO}	C_S, m/sec
55,000	54,528		8.2397	.976918	449.72	.979913	333.46
55,480	55,000		8.1862	.970567	448.23	.976650	332.35
56,000	55,511		8.1281	.963687	446.60	.973103	331.14
56,498	56,000		8.0726	.957100	445.03	.969696	329.98
57,000	56,493		8.0165	.950451	443.45	.966247	328.81
57,516	57,000		7.9589	.943623	441.82	.962692	327.60
58,000	57,476		7.9048	.937211	440.28	.959344	326.46
58,534	58,000		7.8452	.930139	438.58	.955637	325.20
59,000	58,457		7.7931	.923967	437.09	.952393	324.09
59,553	59,000		7.7314	.916645	435.32	.948530	322.78
60,000	59,439		7.6814	.910719	433.88	.945393	321.71
60,572	60,000		7.6175	.903143	432.03	.941368	320.34
61,000	60,420		7.5696	.897467	430.65	.938343	319.31
61,591	61,000		7.5035	.889632	428.72	.934152	317.88
62,000	61,401		7.4578	.884210	427.39	.931242	316.89
62,611	62,000		7.3895	.876113	425.38	.926880	315.41
63,000	62,382		7.3459	.870949	424.10	.924088	314.46
63,631	63,000		7.2754	.862585	422.02	.919550	312.92
64,000	63,362		7.2341	.857685	420.80	.916881	312.01
64,651	64,000	-0.0039	7.1612	.849048	418.63	.912161	310.40
65,000	64,342		7.1221	.844415	417.46	.909620	309.54
65,672	65,000		7.0470	.835503	415.21	.904712	307.87
66,000	65,322		7.0102	.831142	414.10	.902302	307.05
66,692	66,000		6.9327	.821949	411.76	.897201	305.31
67,000	66,301		6.8982	.817865	410.72	.894926	304.54
67,714	67,000		6.8183	.808386	408.29	.889627	302.73
68,000	67,280		6.7862	.804583	407.31	.887492	302.01
68,735	68,000		6.7108	.795644	404.78	.881987	300.13
69,000	68,259		6.6741	.791297	403.87	.879997	299.46
69,757	69,000		6.5893	.781235	401.24	.874281	297.51
70,000	69,238		6.5620	.778007	400.40	.872440	296.88
70,779	70,000		6.4746	.767646	397.68	.866506	294.87
71,000	70,216		6.4499	.764713	396.90	.864820	294.29
71,802	71,000		6.3600	.754049	394.08	.858661	292.20
72,000	71,194		6.3377	.751415	393.37	.857134	291.68
72,825	72,000		6.2452	.740443	390.44	.850744	289.50
73,000	72,171		6.2255	.738113	389.82	.849381	289.04
73,848	73,000		6.1304	.726828	386.77	.842752	286.78
74,000	73,148		6.1133	.724806	386.23	.841559	286.38
74,872	74,000		6.0155	.713205	383.07	.834683	284.04

ALTITUDE		MOL-SCALE TEMP.GRAD.	SCALE HEIGHT		PARTICLE SPEED →	RATIO	← SOUND SPEED
Z,m	H,m'	L_M, °C/m'	H_s, km	H_s/H_{so}	$\overline{V}$,m/sec	$\overline{V}/\overline{V}_o$ C_s/C_{so}	C_s,m/sec
75,000	74,125	-0.0039	6.0010	.71150	382.60	.833666	283.69
75,895	75,000		5.900	.69957	379.3	.82654	281.26
76,000	75,102		5.901	.69960	379.3	.82654	281.26
76,920	76,000		5.902	.69980	379.3	.82654	281.26
77,000	76,078		5.903	.69981	379.3	.82654	281.26
77,944	77,000		5.904	.70002	379.3	.82654	281.26
78,000	77,055		5.904	.70003	379.3	.82654	281.26
78,969	78,000		5.906	.70024	379.3	.82654	281.26
79,000	78,030		5.906	.70025	379.3	.82654	281.26
79,994	79,000		5.908	.70046	379.3	.82654	281.26
80,000	79,006		5.908	.70047	379.3	.82654	281.26
81,000	79,981		5.910	.70068	379.3	.82654	281.26
81,020	80,000		5.910	.70069	379.3	.82654	281.26
82,000	80,956		5.912	.70090	379.3	.82654	281.26
82,045	81,000		5.912	.70091	379.3	.82654	281.26
83,000	81,930		5.914	.70112	379.3	.82654	281.26
83,072	82,000		5.914	.70113	379.3	.82654	281.26
84,000	82,904	0.0000	5.915	.70134	379.3	.82654	281.26
84,098	83,000		5.916	.70136	379.3	.82654	281.26
85,000	83,878		5.917	.70155	379.3	.82654	281.26
85,125	84,000		5.917	.70158	379.3	.82654	281.26
86,000	84,852		5.919	.70177	379.3	.82654	281.26
86,152	85,000		5.919	.70181	379.3	.82654	281.26
87,000	85,825		5.921	.70199	379.3	.82654	281.26
87,179	86,000		5.921	.70203	379.3	.82654	281.26
88,000	86,798		5.923	.70221	379.3	.82654	281.26
88,207	87,000		5.923	.70225	379.3	.82654	281.26
89,000	87,771		5.925	.70243	379.3	.82654	281.26
89,235	88,000		5.925	.70248	379.3	.82654	281.26
90,000	88,744		5.926	.70264	379.3	.82654	281.26
90,264	89,000		5.927	.70270	379.3	.82654	281.26
91,000	89,716		5.928	.70286	379.3	.82654	281.26
91,293	90,000		5.929	.70293	379.3	.82654	281.26
92,000	90,688		5.984	.70944	381.6	.83157	
92,322	91,000		6.036	.71565	382.7	.83385	
93,000	91,659		6.107	.72404	384.9	.83863	
93,351	92,000	+0.0035	6.144	.72839	386.0	.84110	
94,000	92,630		6.211	.73641	388.1	.84564	
94,381	93,000		6.251	.74113	389.3	.84829	

METRIC TABLE III CONTINUED

ALTITUDE		MOL-SCALE TEMP.GRAD	SCALE HEIGHT		PARTICLE SPEED	
Z,m	H,m'	L_M,°C/m'	H_s,km	H_s/H_{so}	$\bar{V}$,m/sec	$\bar{V}/\bar{V}_o$
95,000	93,601		6.316	.74879	391.3	.85259
95,411	94,000		6.359	.75388	392.6	.85542
96,000	94,572		6.420	.76117	394.4	.85947
96,441	95,000		6.466	.76664	395.8	.86249
97,000	95,542		6.524	.77355	397.6	.86630
97,472	96,000		6.574	.77940	399.1	.86950
98,000	96,512		6.629	.78594	400.7	.87307
98,503	97,000		6.682	.79218	402.2	.87646
99,000	97,482		6.733	.79833	403.8	.87979
99,534	98,000		6.789	.80496	405.4	.88336
100,000	98,451		6.838	.81073	406.8	.88646
100,566	99,000		6.897	.81775	408.6	.89021
101,000	99,420		6.943	.82313	409.9	.89307
101,598	100,000		7.005	.83055	411.7	.89701
102,000	100,389		7.047	.83553	412.9	.89964
102,631	101,000		7.113	.84335	414.8	.90375
103,000	101,358		7.152	.84794	415.9	.90615
103,663	102,000		7.221	.85617	417.8	.91045
104,000	102,326		7.257	.86035	418.8	.91262
104,696	103,000	+0.0035	7.329	.86899	420.9	.91709
105,000	103,294		7.361	.87276	421.8	.91904
105,730	104,000		7.438	.88182	423.9	.92369
106,000	104,261		7.466	.88518	424.7	.92541
106,764	105,000		7.546	.89466	426.9	.93024
107,000	105,229		7.571	.89760	427.6	.93174
107,798	106,000		7.654	.90751	429.9	.93675
108,000	106,196		7.676	.91003	430.5	.93802
108,832	107,000		7.763	.92037	432.9	.94321
109,000	107,162		7.780	.92246	433.4	.94426
109,867	108,000		7.871	.93323	435.8	.94963
110,000	108,129		7.885	.93489	436.2	.95045
110,902	109,000		7.980	.94610	438.7	.95600
111,000	109,095		7.990	.94733	439.0	.95660
111,937	110,000		8.088	.95898	441.7	.96233
112,000	110,061		8.095	.95977	441.8	.96272
112,973	111,000		8.197	.97187	444.5	.96862
113,000	111,026		8.200	.97221	444.6	.96879
114,000	111,992		8.305	.98466	447.4	.97482
114,009	112,000		8.306	.98477	447.4	.97487

ALTITUDE		MOL-SCALE TEMP.GRAD.	SCALE HEIGHT		PARTICLE SPEED	
Z,m	H,m'	L_M, °C/m'	H_s,km	H_s/H_{so}	$\bar{V}$,m/sec	$\bar{V}/\bar{V}_o$
115,000	112,957		8.410	.99712	450.1	.98081
115,045	113,000		8.415	.99768	450.3	.98108
116,000	113,921		8.515	1.0096	452.9	.98677
116,082	114,000		8.524	1.0106	453.1	.98725
117,000	114,885		8.620	1.0220	455.6	.99268
117,119	115,000		8.633	1.0235	455.9	.99338
118,000	115,850		8.725	1.0345	458.3	.99856
118,156	116,000		8.742	1.0364	458.7	.99948
119,000	116,813		8.831	1.0470	461.0	1.0044
119,194	117,000		8.851	1.0494	461.5	1.0055
120,000	117,777		8.936	1.0594	463.6	1.0102
120,232	118,000		8.960	1.0623	464.2	1.0116
121,000	118,740		9.041	1.0719	466.3	1.0160
121,270	119,000		9.069	1.0753	467.0	1.0175
122,000	119,703	+0.0035	9.146	1.0844	468.9	1.0217
122,309	120,000		9.179	1.0882	469.7	1.0235
123,000	120,665		9.251	1.0969	471.5	1.0274
123,348	121,000		9.288	1.1012	472.4	1.0294
124,000	121,627		9.357	1.1094	474.1	1.0331
124,387	122,000		9.398	1.1142	475.1	1.0353
125,000	122,589		9.462	1.1218	476.7	1.0387
125,427	123,000		9.507	1.1272	477.8	1.0411
126,000	123,551		9.567	1.1343	479.3	1.0444
126,467	124,000		9.617	1.1402	480.5	1.0470
127,000	124,512		9.673	1.1468	481.9	1.0499
127,507	125,000		9.726	1.1532	483.1	1.0527
128,000	125,473		9.778	1.1593	484.4	1.0555
128,548	126,000		9.836	1.1662	485.8	1.0585
129,000	126,434		9.970	1.1820	489.0	1.0656
129,589	127,000		10.14	1.2027	493.3	1.0748
130,000	127,395	+0.0100	10.27	1.2171	496.2	1.0811
130,630	128,000		10.45	1.2392	500.6	1.0908
131,000	128,355		10.56	1.2522	503.2	1.0964
131,672	129,000		10.76	1.2758	507.9	1.1066
132,000	129,315		10.86	1.2873	510.1	1.1115
132,774	130,000		11.07	1.3124	515.0	1.1222
135,000	132,193		11.75	1.3926	530.3	1.1556
137,929	135,000		12.61	1.4956	549.3	1.1970

METRIC TABLE III CONTINUED

ALTITUDE		MOL-SCALE TEMP. GRAD.	SCALE HEIGHT		PARTICLE SPEED	
Z, m	H, m'	L_M, °C/m'	H_s, km	H_s/H_{so}	$\overline{V}$, m/sec	$\overline{V}/\overline{V}_o$
140,000	136,983		13.23	1.5684	562.4	1.2254
143,153	140,000		14.16	1.6794	581.7	1.2674
145,000	141,766		14.71	1.7445	592.6	1.2913
148,385	145,000		15.72	1.8638	608.8	1.3266
150,000	146,542		16.20	1.9208	621.4	1.3540
153,625	150,000		17.28	2.0488	641.4	1.3976
155,000	151,311		17.69	2.0974	648.8	1.4138
158,874	155,000	+0.0100	18.85	2.2344	669.3	1.4584
160,000	156,072		19.18	2.2743	675.1	1.4711
164,131	160,000		20.42	2.4206	696.1	1.5167
165,000	160,826		20.68	2.4514	700.4	1.5261
169,397	165,000		21.99	2.6074	721.8	1.5728
170,000	165,572		22.17	2.6288	724.7	1.5791
174,671	170,000		23.57	2.7948	746.7	1.6271
175,000	170,311		23.67	2.8065	748.3	1.6304
179,954	175,000		25.16	2.9828	770.8	1.6795
180,000	175,043		25.17	2.9838	770.9	1.6798
185,000	179,768		26.05	3.0891	781.1	1.7020
185,245	180,000		26.10	3.0943	784.4	1.7092
190,000	184,486		26.94	3.1945	796.5	1.7355
190,545	185,000		27.04	3.2060	797.8	1.7384
195,000	189,196		27.83	3.3001	808.9	1.7626
195,854	190,000		27.99	3.3182	811.0	1.7671
200,000	193,899		28.73	3.4059	821.1	1.7892
201,171	195,000		28.94	3.4307	824.0	1.7954
205,000	198,595		29.62	3.5118	833.1	1.8154
206,497	200,000	+0.0058	29.89	3.5435	836.7	1.8232
210,000	203,284		30.51	3.6179	845.0	1.8412
211,831	205,000		30.84	3.6567	849.3	1.8506
215,000	207,966		31.41	3.7241	856.7	1.8667
217,175	210,000		31.80	3.7703	861.7	1.8776
220,000	212,641		32.31	3.8305	868.2	1.8917
222,526	215,000		32.76	3.8843	873.9	1.9042
225,000	217,308		33.21	3.9370	879.5	1.9164
227,887	220,000		33.73	3.9986	886.0	1.9304
230,000	221,969		34.11	4.0437	890.7	1.9407
233,256	225,000		34.69	4.1133	897.8	1.9563
235,000	226,622		35.01	4.1506	901.7	1.9647
238,634	230,000		35.66	4.2283	909.6	1.9819

METRIC TABLE III CONTINUED

ALTITUDE		MOL-SCALE TEMP. GRAD.	SCALE HEIGHT		PARTICLE SPEED	
Z, m	H, m'	L_M, °C/m'	H_s, km	H_s/H_{so}	$\bar{V}$, m/sec	$\bar{V}/\bar{V}_o$
240,000	231,268		35.91	4.2576	912.5	1.9883
244,021	235,000		36.64	4.3438	921.2	2.0071
245,000	235,908		36.81	4.3647	923.2	2.0117
249,417	240,000		37.61	4.4595	932.6	2.0320
250,000	240,540		37.72	4.4721	933.8	2.0347
254,821	245,000		38.59	4.5757	943.9	2.0566
255,000	245,165		38.63	4.5796	944.3	2.0575
260,000	249,784		39.53	4.6872	954.6	2.0799
260,235	250,000		39.58	4.6923	955.0	2.0810
265,000	254,395		40.44	4.7950	964.8	2.1021
265,657	255,000		40.56	4.8092	966.1	2.1050
270,000	258,999		41.35	4.9030	974.8	2.1240
271,088	260,000		41.55	4.9265	977.0	2.1288
275,000	263,597		42.27	5.0111	984.8	2.1457
276,528	265,000		42.54	5.0442	987.8	2.1523
280,000	268,187	+0.0058	43.18	5.1194	994.6	2.1671
281,977	270,000		43.54	5.1622	998.4	2.1755
285,000	272,771		44.09	5.2278	1,004	2.1883
287,435	275,000		44.54	5.2806	1,009	2.1986
290,000	277,347		45.01	5.3364	1,014	2.2093
292,902	280,000		45.54	5.3995	1,017	2.2168
295,000	281,917		45.93	5.4451	1,023	2.2300
298,377	285,000		46.55	5.5187	1,030	2.2439
300,000	286,480		46.84	5.5540	1,033	2.2505
303,862	290,000		47.56	5.6383	1,040	2.2662
305,000	291,036		47.76	5.6631	1,042	2.2708
309,356	295,000		48.57	5.7582	1,050	2.2883
310,000	295,585		48.69	5.7723	1,051	2.2908
314,859	300,000		49.58	5.8786	1,060	2.3102
320,000	304,663		50.53	5.9912	1,070	2.3304
325,893	310,000		51.62	6.1205	1,080	2.3533
330,000	313,714		52.38	6.2108	1,087	2.3691
336,963	320,000		53.68	6.3640	1,099	2.3957
340,000	322,738		54.24	6.4309	1,105	2.4072
348,069	330,000		55.74	6.6090	1,119	2.4373

METRIC TABLE III CONTINUED

ALTITUDE		MOL-SCALE TEMP. GRAD.	SCALE HEIGHT		PARTICLE SPEED	
Z,m	H,m'	L_M, °C/m'	H_s,km	H_s/H_{so}	$\bar{V}$,m/sec	$\bar{V}/\bar{V}_o$
350,000	331,735		56.10	6.6517	1,122	2.4445
359,213	340,000		57.82	6.8557	1,137	2.4783
360,000	340,705		57.97	6.8731	1,139	2.4812
370,000	349,648		59.84	7.0952	1,155	2.5172
370,394	350,000		59.92	7.1040	1,156	2.5186
380,000	358,565		61.72	7.3179	1,171	2.5526
381,612	360,000		62.03	7.3538	1,174	2.5582
390,000	367,456		63.61	7.5412	1,187	2.5874
392,867	370,000		64.15	7.6053	1,192	2.5973
400,000	376,320		65.49	7.7652	1,201	2.6178
404,160	380,000		66.28	7.8585	1,210	2.6357
410,000	385,158		67.39	7.9897	1,219	2.6553
415,491	390,000		68.43	8.1133	1,227	2.6736
420,000	393,970		69.29	8.2149	1,234	2.6885
426,860	400,000		70.59	8.3698	1,244	2.7110
430,000	402,756		71.19	8.4408	1,249	2.7212
438,267	410,000	+0.0058	72.77	8.6280	1,261	2.7479
440,000	411,516		73.10	8.6672	1,264	2.7534
449,713	420,000		74.96	8.8878	1,278	2.7843
450,000	420,250		75.02	8.8943	1,278	2.7852
460,000	428,959		76.94	9.1221	1,293	2.8165
461,197	430,000		77.17	9.1494	1,294	2.8202
470,000	437,642		78.87	9.3504	1,307	2.8473
472,721	440,000		79.39	9.4127	1,311	2.8556
480,000	446,300		80.80	9.5794	1,322	2.8812
484,283	450,000		81.63	9.6777	1,327	2.8907
490,000	454,932		82.73	9.8090	1,335	2.9078
495,884	460,000		83.88	9.9444	1,343	2.9253
500,000	463,540		84.68	10.0393	1,348	2.9374
507,525	470,000		86.14	10.2129	1,358	2.9595
510,000	472,122		86.62	10.2701	1,362	2.9667
519,205	480,000		88.42	10.4832	1,374	2.9933
520,000	480,679		88.58	10.5016	1,375	2.9956
530,000	489,212		90.53	10.7338	1,388	3.0241
530,925	490,000		90.71	10.7553	1,389	3.0267
540,000	497,719		92.50	10.9665	1,401	3.0523
542,686	500,000		93.02	11.0292	1,404	3.0598

METRIC TABLE IV

VISCOSITY, KINEMATIC VISCOSITY, AND SPECIFIC WEIGHT
AS FUNCTIONS OF GEOMETRIC AND GEOPOTENTIAL ALTITUDE

ALTITUDE		VISCOSITY		KINEMATIC VISCOSITY		SPECIFIC WEIGHT	
Z,m	H,m'	μ, $\frac{kg}{m\ sec}$	μ/μ_o	η, $\frac{m^2}{sec}$	η/η_o	ω, $\frac{kg}{m^2 sec^2}$	ω/ω_o
-5,000	-5,003.9	1.9423^{-5}	1.08542	1.0058^{-5}	6.88539^{-1}	1.8968^{+1}	1.57892
-4,996.1	-5,000	1.9422	1.08536	1.0060	6.88717	1.8961	1.57839
-4,000	-4,002.5	1.9123	1.06868	1.0805	7.39715	1.7378	1.44654
-3,997.5	-4,000	1.9123	1.06864	1.0808	7.39864	1.7373	1.44619
-3,000	-3,001.4	1.8821	1.05177	1.1625	7.95852	1.5891	1.32282
-2,998.6	-3,000	1.8820	1.05175	1.1627	7.95935	1.5889	1.32265
-2,000	-2,000.6	1.8515	1.03469	1.2526	8.57478	1.4505	1.20743
-1,999.4	-2,000	1.8515	1.03468	1.2526	8.57517	1.4504	1.20735
-1,000	-1,000.2	1.8206	1.01744	1.3516	9.25274	1.3214	1.09994
999.8	-1,000	1.8206	1.01743	1.3516	9.25289	1.3214	1.09993
0	0	1.7894^{-5}	1.00000	1.4607^{-5}	1.00000^{+0}	1.2013^{+1}	1.00000
1,000	999.8	1.7579	.982380	1.5813	1.08254	1.0898	9.07189^{-1}
1,000.2	1,000	1.7579	.982377	1.5813	1.08255	1.0898	9.07179
2,000	1,999.4	1.7260	.964571	1.7148	1.17391	9.8648^{+0}	8.21154
2,000.6	2,000	1.7260	.964560	1.7149	1.17397	9.8642	8.21105
3,000	2,998.6	1.6938	.946567	1.8629	1.27528	8.9083	7.41543
3,001.4	3,000	1.6938	.946542	1.8631	1.27543	8.9071	7.41437
4,000	3,997.5	1.6612	.928364	2.0275	1.38801	8.0249	6.68006
4,002.5	4,000	1.6612	.928318	2.0279	1.38830	8.0228	6.67830
5,000	4,996.1	1.6285^{-5}	.909955	2.2111^{-5}	1.51366^{+0}	7.2106^{+0}	6.00216^{-1}
5,003.9	5,000	1.6282	.909882	2.2118	1.51418	7.2075	5.99961
6,000	5,994.3	1.5950	.891335	2.4162	1.65411	6.4613	5.37843
6,005.7	6,000	1.5948	.891229	2.4175	1.65496	6.4572	5.37503
7,000	6,992.3	1.5613	.872499	2.6461	1.81150	5.7734	4.80584
7,007.7	7,000	1.5610	.872352	2.6480	1.81281	5.7683	4.80157
8,000	7,989.9	1.5272	.853439	2.9045	1.98841	5.1432	4.28128
8,010.1	8,000	1.5268	.853246	2.9073	1.99031	5.1372	4.27623
9,000	8,987.3	1.4927	.834151	3.1958	2.18782	4.5674	3.80193
9,012.8	9,000	1.4922	.833903	3.1998	2.19053	4.5603	3.79608
10,000	9,984.3	1.4577^{-5}	.814627	3.5252^{-5}	2.41332^{+0}	4.0424^{+0}	3.36494^{-1}
10,016	10,000	1.4572	.814317	3.5308	2.41711	4.0344	3.35831
11,000	10,981	1.4223	.794861	3.8990	2.66918	3.5651	2.96764
11,019	11,000	1.4217	.794482	3.9066	2.67440	3.5564	2.96042
12,000	11,977	1.4217	.794482	4.5576	3.12005	3.0475	2.53678
12,023	12,000	1.4217	.794482	4.5739	3.13119	3.0366	2.52774
13,000	12,973	1.4217	.794482	5.3327	3.65070	2.6037	2.16737
13,027	13,000	1.4217	.794482	5.3551	3.66601	2.5928	2.15830
14,000	13,969	1.4217	.794482	6.2394	4.27139	2.2247	1.85184
14,031	14,000	1.4217	.794482	6.2697	4.29217	2.2139	1.84286

METRIC TABLE IV CONTINUED

ALTITUDE		VISCOSITY		KINEMATIC VISCOSITY		SPECIFIC WEIGHT	
Z,m	H,m'	μ, $\frac{kg}{m\ sec}$	μ/μ_o	η, $\frac{m^2}{sec}$	η/η_o	ω, $\frac{kg}{m^2 sec^2}$	ω/ω_o
15,000	14,965	1.4217^{-5}	.794482	7.2999^{-5}	4.99737^{+0}	1.9009^{+0}	1.58232^{-1}
15,035	15,000	1.4217	.794482	7.3406	5.02527	1.8903	1.57352
16,000	15,960	1.4217	.794482	8.5401	5.84644	1.6243	1.35209
16,040	16,000	1.4217	.794482	8.5944	5.88359	1.6140	1.34354
17,000	16,955	1.4217	.794482	9.9907	6.83946	1.3881	1.15543
17,046	17,000	1.4217	.794482	1.0062^{-4}	6.88852	1.3781	1.14718
18,000	17,949	1.4217	.794482	1.1687	8.00069	1.1862	9.87416^{-2}
18,051	18,000	1.4217	.794482	1.1781	8.06509	1.1767	9.79517
19,000	18,943	1.4217	.794482	1.3671	9.35868	1.0138	8.43869
19,057	19,000	1.4217	.794482	1.3793	9.44261	1.0047	8.36357
20,000	19,937	1.4217^{-5}	.794482	1.5990^{-4}	1.09466^{+1}	8.6644^{-1}	7.21234^{-2}
20,063	20,000	1.4217	.794482	1.6149	1.10554	8.5789	7.14119
21,000	20,931	1.4217	.794482	1.8702	1.28032	7.4056	6.16454
21,070	21,000	1.4217	.794482	1.8907	1.29437	7.3251	6.09748
22,000	21,924	1.4217	.794482	2.1874	1.49743	6.3300	5.26912
22,076	22,000	1.4217	.794482	2.2137	1.51545	6.2545	5.20633
23,000	22,917	1.4217	.794482	2.5581	1.75124	5.4108	4.50402
23,084	23,000	1.4217	.794482	2.5918	1.77429	5.3404	4.44540
24,000	23,910	1.4217	.794482	2.9916	2.04798	4.6254	3.85021
24,091	24,000	1.4217	.794482	3.0345	2.07734	4.5599	3.79569
25,000	24,902	1.4217^{-5}	.794482	3.4983^{-4}	2.39488^{+1}	3.9541^{-1}	3.29148^{-2}
25,099	25,000	1.4217	.794482	3.5527	2.43215	3.8934	3.24094
26,000	25,894	1.4364	.802698	4.1805	2.86191	3.3420	2.78196
26,107	26,000	1.4381	.803668	4.2613	2.91724	3.2825	2.73241
27,000	26,886	1.4526	.811760	4.9956	3.41994	2.8274	2.35357
27,115	27,000	1.4544	.812801	5.0984	3.49026	2.7739	2.30903
28,000	27,877	1.4687	.820768	5.9550	4.07667	2.3975	1.99570
28,124	28,000	1.4707	.821880	6.0850	4.16571	2.3493	1.95562
29,000	28,868	1.4847	.829720	7.0817	4.84801	2.0374	1.69596
29,133	29,000	1.4868	.830906	7.2453	4.96004	1.9941	1.65995
30,000	29,859	1.5006^{-5}	.838619	8.4018^{-4}	5.75171^{+1}	1.7352^{-1}	1.44436^{-2}
30,142	30,000	1.5029	.839880	8.6070	5.89224	1.6962	1.41198
31,000	30,850	1.5165	.847464	9.9454	6.80848	1.4808	1.23267
31,152	31,000	1.5189	.848803	1.0202^{-3}	6.98383	1.4459	1.20355
32,000	31,840	1.5322	.856257	1.1747	8.04162	1.2664	1.05414
32,162	32,000	1.5347	.857676	1.2065	8.25957	1.2349	1.02797
33,000	32,830	1.5479	.864998	1.3844	9.47770	1.0851	9.03263^{-3}
33.172	33,000	1.5505	.866498	1.4239	9.74750	1.0569	8.79738
34,000	33,819	1.5634	.873688	1.6282	1.11465^{+2}	9.3163^{-2}	7.75503
34,183	34,000	1.5662	.875271	1.6768	1.14794	9.0621	7.54338

METRIC TABLE IV CONTINUED

ALTITUDE		VISCOSITY		KINEMATIC VISCOSITY		SPECIFIC WEIGHT	
Z,m	H,m'	μ, $\frac{kg}{m\ sec}$	μ/μ_o	η, $\frac{m^2}{sec}$	η/η_o	ω, $\frac{kg}{m^2 sec^2}$	ω/ω_o
35,000	34,808	1.5789^{-5}	.882327	1.9110^{-3}	1.30824^{+2}	8.0137^{-2}	6.67071^{-3}
35,194	35,000	1.5818	.883996	1.9708	1.34916	7.7849	6.48022
36,000	35,797	1.5942	.890917	2.2384	1.53239	6.9060	5.81390
36,205	36,000	1.5974	.892672	2.3117	1.58252	6.6999	5.57711
37,000	36,786	1.6095	.899457	2.6168	1.79143	5.9621	4.96294
37,217	37,000	1.6128	.901300	2.7062	1.85263	5.7766	4.80850
38,000	37,774	1.6247	.907948	3.0535	2.09034	5.1562	4.29207
38,229	38,000	1.6282	.909882	3.1622	2.16477	4.9892	4.15303
39,000	38,762	1.6398	.916392	3.5562	2.43450	4.4670	3.71842
39,241	39,000	1.6434	.918417	3.6881	2.52483	4.3164	3.59303
40,000	39,750	1.6548^{-5}	.924788	4.1343^{-3}	2.83025^{+2}	3.8764^{-2}	3.22677^{-3}
40,253	40,000	1.6586	.926907	4.2938	2.93946	3.7407	3.11376
41,000	40,737	1.6698	.933137	4.7978	3.28448	3.3694	2.80475
41,266	41,000	1.6737	.935352	4.9901	3.41615	3.2470	2.70282
42,000	41,724	1.6846	.941440	5.5581	3.80500	2.9335	2.44185
42,279	42,000	1.6888	.943751	5.7895	3.96342	2.8229	2.34979
43,000	42,711	1.6994	.949698	6.4280	4.40051	2.5579	2.12925
43,293	43,000	1.7037	.952107	6.7055	4.59050	2.4581	2.04611
44,000	43,698	1.7141	.957910	7.4218	5.08083	2.2339	1.85951
44,307	44,000	1.7186	.960420	7.7538	5.30816	2.1436	1.78437
45,000	44,684	1.7287^{-5}	.966078	8.5554^{-3}	5.85692^{+2}	1.9538^{-2}	1.62635^{-3}
45,321	45,000	1.7334	.968689	8.9516	6.12811	1.8722	1.55843
46,000	45,670	1.7433	.974202	9.8466	6.74082	1.7113	1.42454
46,335	46,000	1.7481	.976916	1.0318^{-2}	7.06356	1.6375	1.36310
47,000	46,655	1.7577	.982282	1.1315	7.74600	1.5011	1.24957
47,350	47,000	1.7628	.985101	1.1875	8.12930	1.4343	1.19394
48,000	47,640	1.7628	.985101	1.2830	8.78339	1.3272	1.10480
48,365	48,000	1.7628	.985101	1.3400	9.17372	1.2706	1.05767
49,000	48,625	1.7628	.985101	1.4452	9.89380	1.1779	9.80501^{-4}
49,381	49,000	1.7628	.985101	1.5122	1.03523^{+3}	1.1256	9.36960
50,000	49,610	1.7628^{-5}	.985101	1.6279^{-2}	1.11442^{+3}	1.0454^{-2}	8.70218^{-4}
50,396	50,000	1.7628	.985101	1.7065	1.16824	9.9713^{-3}	8.30024
51,000	50,594	1.7628	.985101	1.8335	1.25521	9.2786	7.72366
51,412	51,000	1.7628	.985101	1.9257	1.31833	8.8333	7.35293
52,000	51,578	1.7628	.985101	2.0651	1.41374	8.2356	6.85545
52,429	52,000	1.7628	.985101	2.1731	1.48770	7.8251	6.51373
53,000	52,562	1.7628	.985101	2.3258	1.59223	7.3101	6.08505
53,446	53,000	1.7628	.985101	2.4524	1.67884	6.9320	5.77031
54,000	53,545	1.7524	.979305	2.5850	1.76967	6.5364	5.44100
54,463	54,000	1.7437	.974452	2.7020	1.84976	6.2215	5.17887

METRIC TABLE IV CONTINUED

ALTITUDE		VISCOSITY		KINEMATIC VISCOSITY		SPECIFIC WEIGHT	
Z,m	H,m'	$\mu, \frac{kg}{m\ sec}$	μ/μ_o	$\eta, \frac{m^2}{sec}$	η/η_o	$\omega, \frac{kg}{m^2 sec^2}$	ω/ω_o
55,000	54,528	1.7336^{-5}	.968799	2.8455^{-2}	1.94798^{+3}	5.8726^{-3}	4.88839^{-4}
55,480	55,000	1.7245	.963733	2.9810	2.04076	5.5754	4.64105
56,000	55,511	1.7147	.958227	3.1362	2.14700	5.2684	4.38549
56,498	56,000	1.7052	.952941	3.2934	2.25459	4.9885	4.15252
57,000	56,493	1.6956	.947588	3.4611	2.36941	4.7194	3.92849
57,516	57,000	1.6858	.942075	3.6435	2.49429	4.4563	3.70949
58,000	57,476	1.6765	.936882	3.8248	2.61839	4.2211	3.51368
58,534	58,000	1.6662	.931135	4.0367	2.76346	3.9743	3.30824
59,000	58.457	1.6572	.926105	4.2325	2.89752	3.7693	3.13769
59,553	59,000	1.6465	.920119	4.4790	3.06622	3.5384	2.94538
60,000	59,439	1.6378^{-5}	.915259	4.6904^{-2}	3.21097^{+3}	3.3606^{-3}	2.79736^{-4}
60,572	60,000	1.6266	.909026	4.9772	3.40734	3.1447	2.61772
61,000	60,420	1.6183	.904341	5.2053	3.56345	2.9911	2.48982
61,591	61,000	1.6066	.89754	5.5397	3.79241	2.7898	2.32281
62,000	61,401	1.5986	.893351	5.7855	3.96064	2.6576	2.21222
62,611	62,000	1.5865	.886603	6.1758	4.22786	2.4703	2.05634
63,000	62,382	1.5788	.882287	6.4403	4.40893	2.3571	1.96206
63,631	63,000	1.5662	.875271	6.8964	4.72115	2.1833	1.81737
64,000	63,362	1.5589	.871148	7.1805	4.91569	2.0867	1.73703
64,651	64,000	1.5458	.863857	7.7144	5.28116	1.9257	1.60296
65,000	64,342	1.5388^{-5}	.859933	8.0193^{-2}	5.48985^{+3}	1.8439^{-3}	1.53485^{-4}
65,672	65,000	1.5252	.852358	8.6448	5.91811	1.6950	1.41096
66,000	65,322	1.5186	.848640	8.9709	6.14136	1.6261	1.35360
66,692	66,000	1.5045	.840775	9.7052	6.64401	1.4888	1.23932
67,000	66,301	1.4982	.837269	1.0053^{-1}	6.88216	1.4312	1.19134
67,714	67,000	1.4836	.829105	1.0916	7.47317	1.3049	1.08618
68,000	67,280	1.4777	.825818	1.1286	7.72637	1.2570	1.04632
68,735	68,000	1.4626	.817347	1.2303	8.42238	1.1410	9.49796^{-5}
69,000	68,259	1.4571	.814286	1.2694	8.69026	1.1016	9.16995
69,757	69,000	1.4414	.805499	1.3894	9.51143	9.9541^{-4}	8.28589
70,000	69,238	1.4363^{-5}	.802671	1.4305^{-1}	9.79324^{+3}	9.6330^{-4}	8.01867^{-5}
70,779	70,000	1.4200	.793560	1.5723	1.07639^{+4}	8.6627	7.21097
71,000	70,216	1.4154	.790972	1.6153	1.10581	8.4042	6.99574
71,802	71,000	1.3985	.781528	1.7832	1.22078	7.5200	6.25967
72,000	71,194	1.3943	.779188	1.8277	1.25123	7.3145	6.08868
72,825	72,000	1.3768	.769402	2.0270	1.38766	6.5109	5.41972
73,000	72,171	1.3731	.767317	2.0725	1.41878	6.3504	5.28619
73,848	73,000	1.3549	.757180	2.3095	1.58107	5.6218	4.67967
74,000	73,148	1.3517	.755358	2.3552	1.61233	5.4993	4.57769
74,872	74,000	1.3329	.744861	2.6378	1.80582	4.8405	4.02931

METRIC TABLE IV CONTINUED

ALTITUDE		VISCOSITY		KINEMATIC VISCOSITY		SPECIFIC WEIGHT	
Z,m	H,m'	$\mu, \frac{kg}{m\ sec}$	μ/μ_o	$\eta, \frac{m^2}{sec}$	η/η_o	$\omega, \frac{kg}{m^2 sec^2}$	ω,ω_o
75,000	74,125	1.3301^{-5}	.743309	2.6826^{-1}	1.83648^{+4}	4.7496^{-4}	3.95363^{-5}
75,895	75,000	1.311	.73244	3.020	2.0677	4.156	3.4592
76,000	75,102	1.311	.73244	3.074	2.1047	4.083	3.3984
76,920	76,000	1.311	.73244	3.593	2.4595	3.493	2.9072
77,000	76,078	1.311	.73244	3.642	2.4933	3.445	2.8678
77,944	77,000	1.311	.73244	4.274	2.9257	2.935	2.4432
78,000	77,055	1.311	.73244	4.314	2.9535	2.907	2.4201
78,969	78,000	1.311	.73244	5.084	3.4801	2.467	2.0533
79,000	78,030	1.311	.73244	5.110	3.4985	2.454	2.0425
79,994	79,000	1.311	.73244	6.047	4.1397	2.073	1.7256
80,000	79,006	1.311^{-5}	.73244	6.053^{-1}	4.1438^{+4}	2.071^{-4}	1.7239^{-5}
81,000	79,981	1.311	.73244	7.169	4.9079	1.748	1.4551
81,020	80,000	1.311	.73244	7.193	4.9242	1.742	1.4502
82,000	80,956	1.311	.73244	8.491	5.8126	1.475	1.2282
82,045	81,000	1.311	.73244	8.556	5.8575	1.464	1.2187
83,000	81,930	1.311	.73244	1.006^{+0}	6.8837	1.245	1.0367
83,072	82,000	1.311	.73244	1.018	6.9676	1.230	1.0243
84,000	82,904	1.311	.73244	1.191	8.1518	1.051	8.7523^{-6}
84,098	83,000	1.311	.73244	1.211	8.2881	1.034	8.6080
85,000	83,878	1.311^{-5}	.73244	1.410^{+0}	9.6529^{+4}	8.876^{-5}	7.3889^{-6}
85,125	84,000	1.311	.73244	1.440	9.8588	8.691	7.2343
86,000	84,852	1.311	.73244	1.670	1.1430^{+5}	7.494	6.2382
86,152	85,000	1.311	.73244	1.713	1.1727	7.304	6.0797
87,000	85,825	1.311	.73244	1.977	1.3533	6.327	5.2669
87,179	86,000	1.311	.73244	2.038	1.3950	6.138	5.1095
88,000	86,798	1.311	.73244	2.341	1.6023	5.343	4.4772
88,207	87,000	1.311	.73244	2.424	1.6594	5.159	4.2940
89,000	87,771	1.311	.73244	2.771	1.8970	4.511	3.7552
89,235	88,000	1.311	.73244	2.883	1.9738	4.335	3.6088
90,000	88,744	1.311^{-5}	.73244	3.280^{+0}	2.2457^{+5}	3.810^{-5}	3.1711^{-6}
90,264	89,000	1.311	.73244	3.430	2.3479	3.643	3.0329
91,000	89,716	1.311	.73244	3.883	2.6584	3.217	2.6779
91,293	90,000	1.311	.73244	4.080	2.7929	3.062	2.5488
92,000	90,688					2.686	2.2360
92,322	91,000					2.532	2.1078
93,000	91,659					2.238	1.8632
93,351	92,000					2.101	1.7489
94,000	92,630					1.871	1.5573
94,381	93,000					1.749	1.4557

METRIC TABLE IV CONTINUED

ALTITUDE		SPECIFIC WEIGHT	
Z,m	H,m'	$\omega, \frac{kg}{m^2 sec^2}$	ω/ω_o
95,000	93,601	1.568^{-5}	1.3056^{-6}
95,411	94,000	1.460	1.2153
96,000	94,572	1.319	1.0977
96,441	95,000	1.223	1.0177
97,000	95,542	1.112	9.2546^{-7}
97,472	96,000	1.027	8.5465
98,000	96,512	9.399^{-6}	7.8240
98,503	97,000	8.646	7.1972
99,000	97,482	7.967	6.6317
99,534	98,000	7.301	6.0773
100,000	98,451	6.770^{-6}	5.6355^{-7}
100,566	99,000	6.181	5.1450
101,000	99,420	5.767	4.8007
101,598	100,000	5.246	4.3667
102,000	100,389	4.925	4.0995
102,631	101,000	4.463	3.7154
103,000	101,358	4.215	3.5088
103,663	102,000	3.807	3.1686
104,000	102,326	3.616	3.0099
104,696	103,000	3.254	2.7086
105,000	103,294	3.109^{-6}	2.5877^{-7}
105,730	104,000	2.788	2.3206
106,000	104,261	2.678	2.2294
106,764	105,000	2.394	1.9925
107,000	105,229	2.312	1.9248
107,798	106,000	2.060	1.7145
108,000	106,196	2.000	1.6651
108,832	107,000	1.776	1.4782
109,000	107,162	1.734	1.4435
109,867	108,000	1.534	1.2771
110,000	108,129	1.506^{-6}	1.2534^{-7}
110,902	109,000	1.328	1.1055
111,000	109,095	1.310	1.0905
111,937	110,000	1.152	9.5874^{-8}
112,000	110,061	1.142	9.5053
112,973	111,000	1.001	8.3305
113,000	111,026	9.971^{-7}	8.2998
114,000	111,992	8.721	7.2597
114,009	112,000	8.711	7.2513
115,000	112,957	7.641^{-7}	6.3606^{-8}
115,045	113,000	7.596	6.3231
116,000	113,921	6.706	5.5819
116,082	114,000	6.635	5.5230
117,000	114,885	5.894	4.9064
117,119	115,000	5.805	4.8323
118,000	115,850	5.189	4.3194
118,156	116,000	5.088	4.2349
119,000	116,813	4.575	3.8085
119,194	117,000	4.466	3.7172
120,000	117,777	4.040^{-7}	3.3629^{-8}
120,232	118,000	3.926	3.2679
121,000	118,740	3.573	2.9738
121,270	119,000	3.457	2.8773
122,000	119,703	3.164	2.6335
122,309	120,000	3.048	2.5371
123,000	120,665	2.805	2.3353
123,348	121,000	2.692	2.2404
124,000	121,627	2.491	2.0737
124,387	122,000	2.380	1.9812
125,000	122,589	2.215^{-7}	1.8439^{-8}
125,427	123,000	2.108	1.7544
126,000	123,551	1.972	1.6416
126,467	124,000	1.869	1.5556
127,000	124,512	1.758	1.4634
127,507	125,000	1.659	1.3812
128,000	125,473	1.569	1.3062
128,548	126,000	1.475	1.2280
129,000	126,434	1.391	1.1575
129,589	127,000	1.289	1.0728
130,000	127,395	1.223^{-7}	1.0183^{-8}
130,630	128,000	1.131	9.4107^{-9}
131,000	128,355	1.080	8.9913
131,672	129,000	9.954^{-8}	8.2859
132,000	129,315	9.570	7.9664
132,714	130,000	8.795	7.3214
135,000	132,193	6.783	5.6462
137,929	135,000	4.965	4.1333

METRIC TABLE IV CONTINUED

ALTITUDE		SPECIFIC WEIGHT	
Z,m	H,m'	$\omega, \frac{kg}{m^2 sec^2}$	ω/ω_0
140,000	136,983	4.034^{-8}	3.3576^{-9}
143,153	140,000	2.992	2.4908
145,000	141,766	2.535	2.1098
148,385	145,000	1.899	1.5808
150,000	146,542	1.665	1.3863
153,625	150,000	1.257	1.0465
155,000	151,311	1.135	9.4498^{-10}
158,874	155,000	8.619^{-9}	7.1749
160,000	156,072	7.981^{-9}	6.6437^{-10}
164,131	160,000	6.086	5.0662
165,000	160,826	5.761	4.7954
169,397	165,000	4.407	3.6687
170,000	165,572	4.254	3.5407
174,671	170,000	3.262	2.7150
175,000	170,311	3.203	2.6664
179,954	175,000	2.460	2.0479
180,000	175,043	2.455^{-9}	2.0435^{-10}
185,000	179,768	1.951	1.6237
185,245	180,000	1.929	1.6058
190,000	184,486	1.562	1.3001
190,545	185,000	1.525	1.2696
195,000	189,196	1.260	1.0485
195,854	190,000	1.215	1.0114
200,000	193,899	1.023^{-9}	8.5131^{-11}
201,171	195,000	9.749^{-10}	8.1151
205,000	198,595	8.356	6.9558
206,497	200,000	7.875	6.5552

ALTITUDE		SPECIFIC WEIGHT	
Z,m	H,m'	$\omega, \frac{kg}{m^2 sec^2}$	ω/ω_0
210,000	203,284	6.869^{-10}	5.7174^{-11}
211,831	205,000	6.402	5.3288
215,000	207,966	5.678	4.7260
217,175	210,000	5.235	4.3577
220,000	212,641	4.718^{-10}	3.9274^{-11}
222,526	215,000	4.305	3.5836
225,000	217,308	3.941	3.2801
227,887	220,000	3.559	2.9627
230,000	221,969	3.307	2.7526
233,256	225,000	2.957	2.4617
235,000	226,622	2.788	2.3206
238,634	230,000	2.469	2.0552
240,000	231,268	2.360^{-10}	1.9647^{-11}
244,021	235,000	2.071	1.7236
245,000	235,908	2.006	1.6702
249,417	240,000	1.744	1.4517
250,000	240,540	1.713	1.4255
254,821	245,000	1.475	1.2278
255,000	245,165	1.467	1.2211
260,000	249,784	1.261^{-10}	1.0498^{-11}
260,235	250,000	1.252	1.0424
265,000	254,395	1.088	9.0554^{-12}
265,657	255,000	1.067	8.8834
270,000	258,999	9.415^{-11}	7.8367
271,088	260,000	9.127	7.5973
275,000	263,597	8.173	6.8034
276,528	265,000	7.832	6.5196

METRIC TABLE IV CONTINUED

ALTITUDE		SPECIFIC WEIGHT	
Z,m	H,m'	$\omega, \frac{kg}{m^2 sec^2}$	ω/ω_o
280,000	268,187	7.117^{-11}	5.9240^{-12}
281,977	270,000	6.743	5.6130
285,000	272,771	6.215	5.1730
287,435	275,000	5.823	4.8475
290,000	277,347	5.442	4.5298
292,902	280,000	5.044	4.1989
295,000	281,917	4.778	3.9769
298,377	285,000	4.382	3.6475
300,000	286,480	4.205^{-11}	3.5006^{-12}
303,862	290,000	3.817	3.1773
305,000	291,036	3.711	3.0888
309,356	295,000	3.334	2.7750
310,000	295,585	3.282	2.7319
314,859	300,000	2.919	2.4298
320,000	304,663	2.585	2.1515
325,893	310,000	2.254	1.8766
330,000	313,714	2.053	1.7081
336,963	320,000	1.757	1.4625
340,000	322,738	1.644	1.3680
348,069	330,000	1.381	1.1495
350,000	331,735	1.326^{-11}	1.1034^{-12}
359,213	340,000	1.094	9.1067^{-13}
360,000	340,705	1.077	8.9613
370,000	349,648	8.800^{-12}	7.3251
370,394	350,000	8.731	7.2680
380,000	358,565	7.238	6.0247
381,612	360,000	7.017	5.8410
390,000	367,456	5.987	4.9838
392,867	370,000	5.676	4.7249

ALTITUDE		SPECIFIC WEIGHT	
Z,m	H,m'	$\omega, \frac{kg}{m^2 sec^2}$	ω/ω_o
400,000	376,320	4.980^{-12}	4.1455^{-13}
404,160	380,000	4.620	3.8456
410,000	385,158	4.164	3.4658
415,491	390,000	3.778	3.1450
420,000	393,970	3.498	2.9121
426,860	400,000	3.113	2.5911
430,000	402,756	2.935	2.4431
438,267	410,000	2.575	2.1437
440,000	411,516	2.504	2.0839
449,713	420,000	2.141	1.7823
450,000	420,250	2.131^{-12}	1.7742^{-13}
460,000	428,959	1.822	1.5166
461,197	430,000	1.789	1.4887
470,000	437,642	1.563	1.3013
472,721	440,000	1.501	1.2490
480,000	446,300	1.346	1.1207
484,283	450,000	1.264	1.0523
490,000	454,932	1.163	9.6836^{-14}
495,884	460,000	1.069	8.9011
500,000	463,540	1.009^{-12}	8.3964^{-14}
507,525	470,000	9.080^{-13}	7.5575
510,000	472,122	8.773	7.3031
519,205	480,000	7.737	6.4403
520,000	480,679	7.654	6.3716
530,000	489,212	6.698	5.5751
530,925	490,000	6.616	5.5074
540,000	497,719	5.877	4.8919
542,686	500,000	5.677	4.7253

METRIC TABLE V

MEAN FREE PATH, COLLISION FREQUENCY AND NUMBER DENSITY AS FUNCTIONS OF GEOMETRIC AND GEOPOTENTIAL ALTITUDE

ALTITUDE		MEAN FREE PATH		COLLISION FREQUENCY		NUMBER DENSITY	
Z,m	H,m'	L,m	L/L_o	ν, sec^{-1}	ν/ν_o	n, m^{-3}	n/n_o
-5,000	-5,003.9	4.2068^{-8}	6.34342^{-1}	1.1509^{10}	1.66303	4.0161^{25}	1.57644
-4,996.1	-5,000	4.2082	6.34554	1.1505	1.66241	4.0147	1.57591
-4,000	-4,002.5	4.5903	6.92177	1.0440	1.50853	3.6805	1.44472
-3,997.5	-4,000	4.5914	6.92343	1.0437	1.50812	3.6796	1.44437
-3,000	-3,001.4	5.0181	7.56678	9.4503^{9}	1.36557	3.3668	1.32157
-2,998.6	-3,000	5.0187	7.56774	9.4490	1.36538	3.3663	1.32140
-2,000	-2,000.6	5.4959	8.28729	8.5370	1.23359	3.0740	1.20667
-1,999.4	-2,000	5.4962	8.28776	8.5364	1.23352	3.0739	1.20660
-1,000	-1,000.2	6.0310	9.09419	7.6951	1.11194	2.8013	1.09960
- 999.8	-1,000	6.0311	9.09435	7.6949	1.11192	2.8012	1.09958
0	0	6.6317^{-8}	1.00000^{0}	6.9204^{9}	1.00000	2.5476^{25}	1.00000
1,000	999.8	7.3079	1.10196	6.2089	8.97183^{-1}	2.3118	9.07475^{-1}
1,000.2	1,000	7.3080	1.10197	6.2088	8.97171	2.3118	9.07464
2,000	1,999.4	8.0710	1.21703	5.5566	8.02929	2.0933	8.21671
2,000.6	2,000	8.0715	1.21695	5.5565	8.02910	2.0931	8.21622
3,000	2,998.6	8.9347	1.34727	4.9599	7.16701	1.8909	7.42243
3,001.4	3,000	8.9360	1.34746	4.9591	7.16587	1.8906	7.42137
4,000	3,997.5	9.9151	1.49511	4.4151	6.37980	1.7039	6.68847
4,002.5	4,000	9.9178	1.49550	4.4138	6.37792	1.7035	6.68671
5,000	4,996.1	1.1032^{-7}	1.66345^{0}	3.9189^{9}	5.66275^{-1}	1.5315^{25}	6.01161^{-1}
5,003.9	5,000	1.1036	1.66415	3.9170	5.66006	1.5308	6.00906
6,000	5,994.3	1.2307	1.85577	3.4679	5.01106	1.3728	5.38859
6,005.7	6,000	1.2315	1.85694	3.4654	5.00753	1.3719	5.38519
7,000	6,992.3	1.3769	2.07623	3.0590	4.42031	1.2270	4.81643
7,007.7	7,000	1.3781	2.07807	3.0560	4.41594	1.2259	4.81216
8,000	7,989.9	1.5451	2.32988	2.6893	3.88608	1.0934	4.29206
8,010.7	8,000	1.5469	2.33263	2.6858	3.88097	1.0921	4.28701
9,000	8,987.3	1.7394	2.62282	2.3560	3.40437	9.7130^{24}	3.81270
9,012.8	9,000	1.7421	2.62684	2.3519	3.39853	9.6981	3.80685
10,000	9,984.3	1.9646^{-7}	2.96249^{0}	2.0562^{9}	2.97121^{-1}	8.5994^{24}	3.37554^{-1}
10,016	10,000	1.9685	2.96827	2.0517	2.96474	8.5826	3.36896
11,000	10,981	2.2269	3.35805	1.7875	2.58291	7.5864	2.97792
11,019	11,000	2.2324	3.36622	1.7826	2.57590	7.5680	2.97069
12,000	11,977	2.6044	3.92715	1.5280	2.20798	6.4870	2.54637
12,023	12,000	2.6137	3.94118	1.5226	2.20012	6.4639	2.53731
13,000	12,973	3.0473	4.59508	1.3059	1.88703	5.5441	2.17624
13,027	13,000	3.0601	4.61434	1.3005	1.87916	5.5210	2.16716
14,000	13,979	3.5653	5.37614	1.1162	1.61288	4.7385	1.86001
14,031	14,000	3.5828	5.40247	1.1107	1.60502	4.7155	1.85100

ALTITUDE		MEAN FREE PATH		COLLISION FREQUENCY		NUMBER DENSITY	
Z,m	H,m'	L,m	L/L_o	ν, sec^{-1}	ν/ν_o	n, m^{-3}	n/n_o
15,000	14,965	4.1714^{-7}	6.29011^{0}	9.5399^{8}	1.37852^{-1}	4.0501^{24}	1.58980^{-1}
15,035	15,000	4.1947	6.32521	9.4870	1.37087	4.0276	1.58097
16,000	15,960	4.8802	7.35882	8.1545	1.17832	3.4619	1.35891
16,040	16,000	4.9112	7.40557	8.1030	1.17088	3.4400	1.35033
17,000	16,955	5.7091	8.60870	6.9705	1.00724	2.9593	1.16162
17,046	17,000	5.7500	8.67046	6.9209	1.00007	2.9382	1.15334
18,000	17,949	6.6784	1.00703^{1}	5.9588	8.61051^{-2}	2.5298	9.93016^{-2}
18,051	18,000	6.7321	1.01514	5.9112	8.54176	2.5096	9.85088
19,000	18,943	7.8119	1.17796	5.0942	7.36108	2.1627	8.48925
19,057	19,000	7.8820	1.18852	5.0489	7.29565	2.1435	8.41379
20,000	19,937	9.1374^{-7}	1.37783^{1}	4.3552^{8}	6.29328^{-2}	1.8490^{24}	7.25779^{-2}
20,063	20,000	9.2282	1.39153	4.3123	6.23133	1.8308	7.18634
21,000	20,931	1.0687^{-6}	1.61151	3.7237	5.38070	1.5808	6.20534
21,070	21,000	1.0804	1.62920	3.6832	5.32227	1.5637	6.13797
22,000	21,924	1.2499	1.88478	3.1838	4.60056	1.3516	5.30565
22,076	22,000	1.2650	1.90747	3.1459	4.54585	1.3356	5.24255
23,000	22,917	1.4618	2.20426	2.7223	3.93378	1.1557	4.53667
23,084	23,000	1.4810	2.23327	2.6870	3.88268	1.1407	4.47774
24,000	23,910	1.7095	2.57776	2.3279	3.36380	9.8828^{23}	3.87934
24,091	24,000	1.7340	2.61471	2.2950	3.31626	9.7431	3.82451
25,000	24,902	1.9991^{-6}	3.01439^{1}	1.9907^{8}	2.87655^{-2}	8.4513^{23}	3.31742^{-2}
25,099	25,000	2.0302	3.06131	1.9602	2.83247	8.3218	3.26658
26,000	25,894	2.3645	3.56537	1.6934	2.44703	7.1453	2.80476
26,107	26,000	2.4073	3.62990	1.6645	2.40527	7.0182	2.75490
27,000	26,886	2.7939	4.21299	1.4428	2.08487	6.0469	2.37361
27,115	27,000	2.8477	4.29412	1.4166	2.04706	5.9327	2.32877
28,000	27,877	3.2939	4.96693	1.2320	1.78019	5.1290	2.01332
28,124	28,000	3.3613	5.06852	1.2083	1.74594	5.0262	1.97296
29,000	28,868	3.8749	5.84294	1.0542	1.52325	4.3601	1.71147
29,133	29,000	3.9588	5.96943	1.0327	1.49226	4.2679	1.67520
30,000	29,859	4.5484^{-6}	6.85855^{1}	9.0388^{7}	1.30611^{-2}	3.7144^{23}	1.45803^{-2}
30,142	30,000	4.6525	7.01556	8.8446	1.27804	3.6313	1.42540
31,000	30,850	5.3279	8.03395	7.7658	1.12216	3.1710	1.24472
31,152	31,000	5.4565	8.22785	7.5901	1.09677	3.0963	1.21538
32,000	31,840	6.2282	9.39160	6.6852	9.66010^{-3}	2.7126	1.06478
32,162	32,000	6.3865	9.63017	6.5262	9.43053	2.6454	1.03840
33,000	32,830	7.2663	1.09569^{2}	5.7659	8.33173	2.3251	9.12666^{-3}
33,172	33,000	7.4602	1.12493	5.6220	8.12381	2.2646	8.88944
34,000	33,819	8.4608	1.27580	4.9824	7.19959	1.9968	7.83821
34,183	34,000	8.6976	1.31152	4.8521	7.01132	1.9424	7.62473

ALTITUDE		MEAN FREE PATH		COLLISION FREQUENCY		NUMBER DENSITY	
Z,m	H,m'	L,m	L/L_o	ν, sec^{-1}	ν/ν_o	n, m^{-3}	n/n_o
35,000	34,808	9.8330^{-6}	1.48272^{2}	4.3132^{7}	6.23256^{-3}	1.7182^{23}	6.74437^{-3}
35,194	35,000	1.0121^{-5}	1.52621	4.1952	6.06203	1.6692	6.55217
36,000	35,797	1.1407	1.72002	3.7405	5.40500	1.4811	5.81390
36,205	36,000	1.1757	1.77279	3.6336	5.25049	1.4370	5.64082
37,000	36,786	1.3208	1.99168	3.2495	4.69547	1.2791	5.02089
37,217	37,000	1.3632	2.05551	3.1526	4.55545	1.2394	4.86497
38,000	37,774	1.5268	2.30227	2.8276	4.08585	1.1065	4.34354
38,229	38,000	1.5778	2.37918	2.7398	3.95902	1.0708	4.20313
39,000	38,762	1.7618	2.65661	2.4646	3.56139	9.5895^{22}	3.76419
39,241	39,000	1.8231	2.74911	2.3850	3.44631	9.2668	3.63753
40,000	39,750	2.0296^{-5}	3.06044^{2}	2.1517^{7}	3.10917^{-3}	8.3241^{22}	3.26751^{-3}
40,253	40,000	2.1031	3.17126	2.0795	3.00483	8.0332	3.15332
41,000	40,737	2.3342	3.51982	1.8814	2.71869	7.2377	2.84105
41,266	41,000	2.4221	3.65228	1.8159	2.62401	6.9753	2.73803
42,000	41,724	2.6803	4.04168	1.6477	2.38090	6.3032	2.47422
42,279	42,000	2.7851	4.19964	1.5882	2.29489	6.0661	2.38115
43,000	42,711	3.0729	4.63359	1.4451	2.08824	5.4980	2.15815
43,293	43,000	3.1974	4.82141	1.3911	2.01011	5.2838	2.07408
44,000	43,698	3.5175	5.30407	1.2694	1.83425	4.8030	1.88534
44,307	44,000	3.6653	5.52691	1.2202	1.76322	4.6094	1.80933
45,000	44,684	4.0205^{-5}	6.06258^{2}	1.1166^{7}	1.61344^{-3}	4.2021^{22}	1.64946^{-3}
45,321	45,000	4.1954	6.32619	1.0719	1.54887	4.0270	1.58073
46,000	45,670	4.5887	6.91932	9.8355^{6}	1.42123	3.6818	1.44523
46,335	46,000	4.7950	7.23046	9.4290	1.36249	3.5234	1.38304
47,000	46,655	5.2296	7.88572	8.6758	1.25365	3.2306	1.26812
47,350	47,000	5.4727	8.25225	8.3057	1.20017	3.0871	1.21179
48,000	47,640	5.9130	8.91623	7.6872	1.11080	2.8572	1.12155
48,365	48,000	6.1758	9.31247	7.3601	1.06353	2.7356	1.07383
49,000	48,625	6.6605	1.00434^{3}	6.8244	9.86127^{-4}	2.5365	9.95675
49,381	49,000	6.9692	1.05089	6.5221	9.42449	2.4242	9.51574
50,000	49,610	7.5023^{-5}	1.13127^{3}	6.0587^{6}	8.75484^{-4}	2.2519^{22}	8.83961^{-4}
50,396	50,000	7.8646	1.18591	5.7796	8.35151	2.1482	8.43237
51,000	50,594	8.4501	1.27420	5.3791	7.77283	1.9993	7.84809
51,412	51,000	8.8750	1.33827	5.1216	7.40070	1.9036	7.47235
52,000	51,578	9.5173	1.43512	4.7759	6.90125	1.7752	6.96807
52,429	52,000	1.0015^{-4}	1.51020	4.5385	6.55813	1.6869	6.62162
53,000	52,562	1.0719	1.61631	4.2406	6.12761	1.5761	6.18694
53,446	53,000	1.1302	1.70423	4.0218	5.81148	1.4948	5.86775
54,000	53,545	1.1984	1.80707	3.7786	5.46012	1.4098	5.53383
54,463	54,000	1.2589	1.89826	3.5857	5.18136	1.3420	5.26800

ALTITUDE		MEAN FREE PATH		COLLISION FREQUENCY		NUMBER DENSITY	
Z,m	H,m'	L,m	L/L_o	ν, sec^{-1}	ν/ν_o	n, m^{-3}	n/n_o
55,000	54,528	1.3335^{-4}	2.01072^{3}	3.3726^{6}	4.87346^{-4}	1.2670^{22}	4.97335^{-4}
55,480	55,000	1.4043	2.11756	3.1918	4.61214	1.2031	4.72241
56,000	55,511	1.4859	2.24060	3.0056	4.34305	1.1370	4.46310
56,498	56,000	1.5690	2.36593	2.8364	4.09858	1.0768	4.22666
57,000	56,493	1.6582	2.50046	2.6742	3.86427	1.0188	3.99926
57,516	57,000	1.7559	2.64766	2.5163	3.63601	9.6219^{21}	3.77692
58,000	57,476	1.8534	2.79479	2.3755	3.43261	9.1154	3.57809
58,534	58,000	1.9682	2.96784	2.2284	3.21998	8.5839	3.36945
59,000	58,457	2.0749	3.12871	2.1066	3.04404	8.1425	3.19620
59,553	59,000	2.2100	3.33242	1.9698	2.84637	7.6448	3.00083
60,000	59,439	2.3266^{-4}	3.50826^{3}	1.8649^{6}	2.69476^{-4}	7.2616^{21}	2.85042^{-4}
60,572	60,000	2.4858	3.74835	1.7380	2.51142	6.7965	2.66784
61,000	60,420	2.6131	3.94038	1.6480	2.38135	6.4652	2.53783
61,591	61,000	2.8011	4.22386	1.5305	2.21160	6.0313	2.36750
62,000	61,401	2.9401	4.43346	1.4536	2.10049	5.7462	2.25558
62,611	62,000	3.1624	4.76860	1.3451	1.94371	5.3423	2.09705
63,000	62,382	3.3140	4.99716	1.2797	1.84923	5.0980	2.00114
63,631	63,000	3.5771	5.39393	1.1798	1.70479	4.7230	1.85394
64,000	63,362	3.7421	5.64277	1.1245	1.62488	4.5147	1.77218
64,651	64,000	4.0543	6.11347	1.0326	1.49205	4.1671	1.63573
65,000	64,342	4.2337^{-4}	6.38404^{3}	9.8604^{5}	1.42483^{-4}	3.9905^{21}	1.56640^{-4}
65,672	65,000	4.6045	6.94321	9.0174	1.30302	3.6691	1.44026
66,000	65,322	4.7992	7.23670	8.6286	1.24684	3.5203	1.38185
66,692	66,000	5.2406	7.90225	7.8572	1.13537	3.2238	1.26546
67,000	66,301	5.4511	8.21977	7.5346	1.08875	3.0993	1.21658
67,714	67,000	5.9775	9.01354	6.8304	9.86989^{-5}	2.8264	1.10944
68,000	67,280	6.2046	9.35602	6.5646	9.48579	2.7229	1.06883
68,735	68,000	6.8337	1.03045^{4}	5.9233	8.55922	2.4723	9.70447^{-5}
69,000	68,259	7.0775	1.06722	5.7063	8.24566	2.3871	9.37010
69,757	69,000	7.8308	1.18081	5.1239	7.40406	2.1575	8.46874
70,000	69,238	8.0912^{-4}	1.22008^{4}	4.9486^{5}	7.15067^{-5}	2.0880^{21}	8.19618^{-5}
70,779	70,000	8.9953	1.35640	4.4209	6.38827	1.8782	7.37244
71,000	70,216	9.2714	1.39804	4.2809	6.18596	1.8222	7.15289
71,802	71,000	1.0359^{-3}	1.56204	3.8042	5.49705	1.6309	6.40188
72,000	71,194	1.0649	1.60581	3.6939	5.33770	1.5865	6.22739
72,825	72,000	1.1961	1.80355	3.2644	4.71704	1.4125	5.54461
73,000	72,171	1.2262	1.84901	3.1790	4.59371	1.3778	5.40830
73,848	73,000	1.3848	2.08810	2.7931	4.03597	1.2200	4.78903
74,000	73,148	1.4156	2.13452	2.7284	3.94261	1.1935	4.68489
74,872	74,000	1.6078	2.42437	2.3826	3.44289	1.0508	4.12479

METRIC TABLE V CONTINUED

ALTITUDE		MEAN FREE PATH		COLLISION FREQUENCY		NUMBER DENSITY	
Z,m	H,m'	L,m	L/L_o	ν, sec^{-1}	ν/ν_o	n, m^{-3}	n/n_o
75,000	74,125	1.6385^{-3}	2.47068^{4}	2.3351^{5}	3.37424^{-5}	1.0311^{21}	4.04747^{-5}
75,895	75,000	1.872	2.8230	2.026	2.9279	9.024^{20}	3.5423
76,000	75,102	1.906	2.8735	1.991	2.8764	8.866	3.4801
76,920	76,000	2.227	3.3580	1.703	2.4614	7.587	2.9780
77,000	76,078	2.257	3.4040	1.680	2.4281	7.484	2.9377
77,944	77,000	2.649	3.9944	1.432	2.0692	6.378	2.5035
78,000	77,055	2.674	4.0324	1.419	2.0498	6.318	2.4799
78,969	78,000	3.151	4.7514	1.204	1.7396	5.362	2.1046
79,000	78,030	3.168	4.7764	1.198	1.7304	5.334	2.0936
79,994	79,000	3.748	5.6519	1.012	1.4624	4.507	1.7693
80,000	79,006	3.752^{-3}	5.6575^{4}	1.011^{5}	1.4610^{-5}	4.503^{20}	1.7676^{-5}
81,000	79,981	4.444	6.7007	8.536^{4}	1.2335	3.802	1.4924
81,020	80,000	4.459	6.7230	8.508	1.2294	3.789	1.4874
82,000	80,956	5.263	7.9359	7.208	1.0415	3.210	1.2601
82,045	81,000	5.304	7.9972	7.152	1.0335	3.186	1.2504
83,000	81,930	6.233	9.3983	6.086	8.7945^{-6}	2.711	1.0640
83,072	82,000	6.309	9.5128	6.013	8.6887	2.678	1.0512
84,000	82,904	7.381	1.1130^{5}	5.139	7.4265	2.289	8.9851^{-6}
84,098	83,000	7.504	1.1316	5.055	7.3044	2.251	8.8373
85,000	83,878	8.740^{-3}	1.3179^{5}	4.340^{4}	6.2716^{-6}	1.933^{20}	7.5878^{-6}
85,125	84,000	8.926	1.3460	4.250	6.1406	1.893	7.4293
86,000	84,852	1.035^{-2}	1.5605	3.665	5.2965	1.632	6.4081
86,152	85,000	1.062	1.6011	3.572	5.1623	1.591	6.2456
87,000	85,825	1.225	1.8477	3.096	4.4733	1.379	5.4121
87,179	86,000	1.263	1.9046	3.003	4.3398	1.338	5.2506
88,000	86,798	1.451	2.1876	2.615	3.7783	1.165	4.5712
88,207	87,000	1.502	2.2655	2.525	3.6484	1.124	4.4140
89,000	87,771	1.718	2.5899	2.209	3.1914	9.836^{19}	3.8611
89,235	88,000	1.787	2.6949	2.123	3.0671	9.453	3.7108
90,000	88,744	2.033^{-2}	3.0660^{5}	1.866^{4}	2.6958^{-6}	8.309^{19}	3.2615^{-6}
90,264	89,000	2.126	3.2056	1.784	2.5784	7.947	3.1196
91,000	89,716	2.407	3.6295	1.576	2.2773	7.019	2.7552
91,293	90,000	2.529	3.8131	1.500	2.1676	6.681	2.6225
92,000	90,688	2.849	4.2956	1.340	1.9359	5.931	2.3280
92,322	91,000	3.007	4.5340	1.273	1.8391	5.619	2.2055
93,000	91,659	3.369	5.0795	1.143	1.6510	5.015	1.9687
93,351	92,000	3.572	5.3859	1.080	1.5617	4.730	1.8567
94,000	92,630	3.978	5.9991	9.755^{3}	1.4096	4.247	1.6669
94,381	93,000	4.237	6.3891	9.188	1.3277	3.987	1.5652

METRIC TABLE V CONTINUED

ALTITUDE		MEAN FREE PATH		COLLISION FREQUENCY		NUMBER DENSITY	
Z,m	H,m'	L,m	L/L_o	ν, sec^{-1}	ν/ν_o	n,m^{-3}	n/n_o
95,000	93,601	4.692^{-2}	7.0748^{5}	8.340^{+3}	1.2051^{-6}	3.601^{19}	1.4135^{-6}
95,411	94,000	5.018	7.5674	7.823	1.1304	3.366	1.3215
96,000	94,572	5.524	8.3298	7.141	1.0318	3.058	1.2005
96,441	95,000	5.934	8.9474	6.671	9.6396^{-7}	2.847	1.1176
97,000	95,542	6.493	9.7905	6.123	8.8483	2.602	1.0214
97,472	96,000	7.003	1.0560^{6}	5.698	8.2341	2.413	9.4699^{-7}
98,000	96,512	7.617	1.1486	5.260	7.6010	2.218	8.7060
98,503	97,000	8.249	1.2439	4.876	7.0461	2.048	8.0393
99,000	97,482	8.920	1.3451	4.526	6.5407	1.894	7.4344
99,534	98,000	9.698	1.4624	4.180	6.0404	1.742	6.8380
100,000	98,451	1.043^{-1}	1.5722^{6}	4.902^{+3}	5.6382^{-7}	1.620^{19}	6.3605^{-7}
100,566	99,000	1.138	1.7160	3.590	5.1878	1.485	5.8276
101,000	99,420	1.216	1.8342	3.370	4.8690	1.389	5.4520
101,598	100,000	1.333	2.0095	3.089	4.4637	1.268	4.9763
102,000	100,389	1.416	2.1357	2.915	4.2123	1.193	4.6822
102,631	101,000	1.558	2.3487	2.663	3.8479	1.085	4.2577
103,000	101,358	1.646	2.4822	2.526	3.6506	1.026	4.0286
103,663	102,000	1.817	2.7397	2.300	3.3231	9.299^{18}	3.6500
104,000	102,326	1.910	2.8795	2.193	3.1693	8.847	3.4728
104,696	103,000	2.115	3.1897	1.990	2.8752	7.987	3.1351
105,000	103,294	2.211^{-1}	3.3342^{6}	1.908^{+3}	2.7564^{-7}	7.641^{18}	2.9992^{-7}
105,730	104,000	2.458	3.7065	1.725	2.4921	6.873	2.6979
106,000	104,261	2.556	3.8537	1.662	2.4014	6.611	2.5949
106,764	105,000	2.851	4.2990	1.497	2.1639	5.926	2.3261
107,000	105,229	2.949	4.4461	1.450	2.0956	5.730	2.2492
107,798	106,000	3.300	4.9768	1.303	1.8822	5.119	2.0093
108,000	106,196	3.396	5.1205	1.268	1.8319	4.975	1.9529
108,832	107,000	3.814	5.7512	1.135	1.6400	4.430	1.7388
109,000	107,162	3.904	5.8869	1.110	1.6040	4.327	1.6987
109,867	108,000	4.400	6.6341	9.906^{+2}	1.4314	3.840	1.5074
110,000	108,129	4.481^{-1}	6.7565^{6}	9.735^{+2}	1.4067^{-7}	3.771^{18}	1.4801^{-7}
110,902	109,000	5.066	7.6391	8.661	1.2514	3.335	1.3090
111,000	109,095	5.134	7.7416	8.551	1.2357	3.291	1.2917
111,937	110,000	5.824	8.7814	7.584	1.0959	2.901	1.1388
112,000	110,061	5.873	8.8556	7.523	1.0871	2.877	1.1292
112,973	111,000	6.683	1.0077^{7}	6.652	9.6120^{-8}	2.528	9.9234^{-8}
113,000	111,026	6.707	1.0114	6.629	9.5790	2.519	9.8876
114,000	111,992	7.648	1.1532	5.850	8.4531	2.209	8.6715
114,009	112,000	7.656	1.1545	5.844	8.4439	2.207	8.6616

METRIC TABLE V CONTINUED

ALTITUDE		MEAN FREE PATH		COLLISION FREQUENCY		NUMBER DENSITY	
Z,m	H,m'	L,m	L/L_o	ν, sec^{-1}	ν/ν_o	n,m^{-3}	n/n_o
115,000	112,957	8.707^{-1}	1.3129^{7}	5.170^{+2}	7.4705^{-8}	1.940^{18}	7.6167^{-8}
115,045	113,000	8.758	1.3206	5.141	7.4293	1.929	7.5725
116,000	113,921	9.898	1.4925	4.576	6.6117	1.707	6.7003
116,082	114,000	1.000^{0}	1.5081	4.530	6.5463	1.689	6.6308
117,000	114,885	1.123	1.6941	4.055	5.8598	1.504	5.9030
117,119	115,000	1.140	1.7196	3.998	5.7769	1.481	5.8154
118,000	115,850	1.273	1.9201	3.599	5.2007	1.327	5.2081
118,156	116,000	1.298	1.9577	3.533	5.1053	1.301	5.1080
119,000	116,813	1.441	2.1731	3.199	4.6220	1.172	4.6017
119,194	117,000	1.476	2.2255	3.127	4.5182	1.145	4.4933
120,000	117,777	1.629^{0}	2.4561^{7}	2.846^{+2}	4.1131^{-8}	1.037^{18}	4.0715^{-8}
120,232	118,000	1.675	2.5263	2.771	4.0041	1.008	3.9584
121,000	118,740	1.838	2.7721	2.536	3.6651	9.190^{17}	3.6074
121,270	119,000	1.899	2.8636	2.459	3.5534	8.896	3.4921
122,000	119,703	2.072	3.1245	2.263	3.2700	8.153	3.2005
122,309	120,000	2.150	3.2413	2.185	3.1577	7.860	3.0852
123,000	120,665	2.332	3.5171	2.022	2.9212	7.243	2.8432
123,348	121,000	2.430	3.6639	1.944	2.8096	6.953	2.7293
124,000	121,627	2.622	3.9539	1.808	2.6129	6.443	2.5291
124,387	122,000	2.743	4.1359	1.732	2.5032	6.160	2.4178
125,000	122,589	2.944^{0}	4.4394^{7}	1.619^{+2}	2.3398^{-8}	5.738^{17}	2.2525^{-8}
125,427	123,000	3.092	4.6627	1.545	2.2329	5.464	2.1447
126,000	123,551	3.302	4.9784	1.452	2.0978	5.117	2.0087
126,467	124,000	3.481	5.2497	1.380	1.9943	4.853	1.9049
127,000	124,512	3.698	5.5759	1.303	1.8830	4.569	1.7934
127,507	125,000	3.915	5.9032	1.234	1.7834	4.316	1.6940
128,000	125,473	4.137	6.2376	1.171	1.6921	4.084	1.6032
128,548	126,000	4.397	6.6298	1.105	1.5966	3.843	1.5083
129,000	126,434	4.662	7.0291	1.049	1.5160	3.624	1.4227
129,589	127,000	5.025	7.5770	9.816^{+1}	1.4185	3.362	1.3198
130,000	127,395	5.291^{0}	7.9784^{7}	9.378^{+1}	1.3551^{-8}	3.193^{17}	1.2534^{-8}
130,630	128,000	5.720	8.6255	8.752	1.2646	2.953	1.1593
131,000	128,355	5.984	9.0233	8.409	1.2151	2.823	1.1082
131,672	129,000	6.488	9.7827	7.828	1.1312	2.604	1.0222
132,000	129,315	6.745	1.0171^{8}	7.563	1.0929	2.505	9.8323^{-9}
132,714	130,000	7.332	1.1056	7.024	1.0149	2.304	9.0446
135,000	132,193	9.481	1.4296	5.594	8.0832^{-9}	1.782	6.9951
137,929	135,000	1.291^{+1}	1.9462	4.256	6.1502	1.309	5.1381

METRIC TABLE V CONTINUED

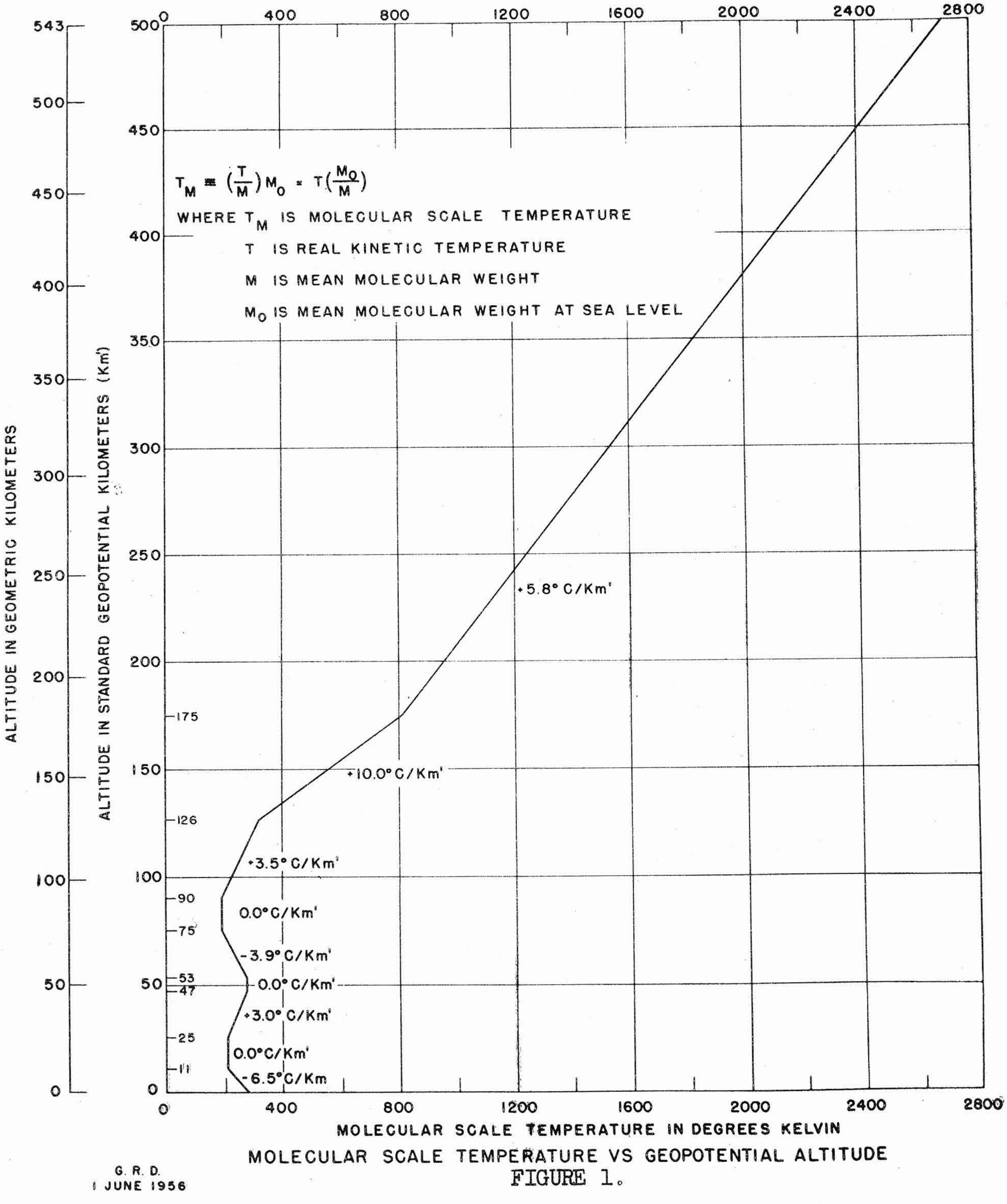
ALTITUDE		MEAN FREE PATH		COLLISION FREQUENCY		NUMBER DENSITY	
Z,m	H,m'	L,m	L/L_o	ν ,sec^{-1}	ν/ν_o	n,m^{-3}	n/n_o
140,000	136,983	1.585^{+1}	2.390^{+8}	3.548^{+1}	5.126^{-9}	1.066^{17}	4.1834^{-9}
143,153	140,000	2.130	3.212	2.731	3.946	7.932^{16}	3.1135
145,000	141,766	2.510	3.785	2.361	3.412	6.731	2.6421
148,385	145,000	3.340	5.036	1.823	2.634	5.059	1.9858
150,000	146,542	3.803	5.734	1.634	2.361	4.443	1.7439
153,625	150,000	5.022	7.573	1.277	1.846	3.364	1.3205
155,000	151,311	5.555	8.377	1.168	1.688	3.041	1.1937
158,874	155,000	7.295	1.100^{+9}	9.175^{0}	1.326	2.316	9.0908^{-10}
160,000	156,072	7.872^{+1}	1.187^{+9}	8.577^{0}	1.239^{-9}	2.146^{16}	8.4250^{-10}
164,131	160,000	1.029^{+2}	1.552	6.763	9.773^{-10}	1.642	6.4436
165,000	160,826	1.087	1.639	6.451	9.322	1.555	6.1029
169,397	165,000	1.416	2.136	5.097	7.365	1.193	4.6827
170,000	165,572	1.467	2.212	4.941	7.140	1.152	4.5212
174,671	170,000	1.907	2.876	3.915	5.658	8.858^{15}	3.4772
175,000	170,311	1.942	2.928	3.854	5.569	8.701	3.4155
179,954	175,000	2.521	3.801	3.058	4.419	6.703	2.6311
180,000	175,043	2.525^{+2}	3.808^{+9}	3.053^{0}	4.411^{-10}	6.690^{15}	2.6259^{-10}
185,000	179,768	3.119	4.703	2.505	3.619	5.417	2.1264
185,245	180,000	3.151	4.751	2.490	3.598	5.362	2.1049
190,000	184,486	3.826	5.769	2.082	3.008	4.416	1.7334
190,545	185,000	3.910	5.896	2.040	2.948	4.320	1.6959
195,000	189,196	4.664	7.033	1.734	2.506	3.622	1.4218
195,854	190,000	4.822	7.271	1.682	2.430	3.504	1.3754
200,000	193,899	5.653^{+2}	8.524^{+9}	1.453^{0}	2.099^{-10}	2.989^{15}	1.1731^{-10}
201,171	195,000	5.909	8.909	1.395	2.015	2.859	1.1224
205,000	198,595	6.814	1.027^{+10}	1.223	1.767	2.480	9.7332^{-11}
206,497	200,000	7.198	1.085	1.162	1.680	2.347	9.2134
210,000	203,284	8.169	1.232	1.034	1.495	2.068	8.1177
211,831	205,000	8.720	1.316	9.740^{-1}	1.407	1.937	7.6051
215,000	207,966	9.747	1.470	8.789	1.270	1.733	6.8040
217,175	210,000	1.051^{+3}	1.585	8.199	1.185	1.608	6.3103
220,000	212,641	1.157^{+3}	1.745^{+10}	7.501^{-1}	1.084^{-10}	1.460^{15}	5.7296^{-11}
222,526	215,000	1.260	1.900	6.934	1.002	1.341	5.2621
225,000	217,308	1.368	2.063	6.427	9.288^{-11}	1.235	4.8465
227,887	220,000	1.504	2.268	5.890	8.511	1.123	4.4086
230,000	221,969	1.611	2.429	5.529	7.990	1.049	4.1169
233,256	225,000	1.788	2.695	5.023	7.258	9.452^{14}	3.7100
235,000	226,622	1.889	2.848	4.774	6.899	8.945	3.5114
238,634	230,000	2.115	3.189	4.300	6.214	7.988	3.1354

METRIC TABLE V CONTINUED

ALTITUDE		MEAN FREE PATH		COLLISION FREQUENCY		NUMBER DENSITY	
Z,m	H,m'	L,m	L/L_o	ν, sec^{-1}	ν/ν_o	n, m^{-3}	n/n_o
240,000	231,268	2.206^{+3}	3.3263^{+10}	4.137^{-1}	5.9776^{-11}	7.659^{14}	3.006^{-11}
244,021	235,000	2.493	3.7586	3.696	5.3400	6.778	2.661
245,000	235,908	2.567	3.8708	3.597	5.1971	6.581	2.583
249,417	240,000	2.926	4.4126	3.187	4.6051	5.773	2.266
250,000	240,540	2.977	4.4886	3.137	4.5330	5.676	2.228
254,821	245,000	3.423	5.1612	2.758	3.9848	4.936	1.938
255,000	245,165	3.440	5.1877	2.745	3.9660	4.911	1.928
260,000	249,784	3.963^{+3}	5.9765^{+10}	2.408^{-1}	3.4802^{-11}	4.263^{14}	1.673^{-11}
260,235	250,000	3.990	6.0158	2.394	3.4591	4.235	1.662
265,000	254,395	4.552	5.8641	2.119	3.0625	3.711	1.457
265,657	255,000	4.635	6.9885	2.084	3.0121	3.645	1.431
270,000	258,999	5.213	7.8603	1.870	2.7022	3.241	1.272
271,088	260,000	5.367	8.0925	1.820	2.6306	3.148	1.236
275,000	263,597	5.952	8.9757	1.654	2.3906	2.838	1.114
276,528	265,000	6.195	9.3421	1.594	2.3039	2.727	1.070
280,000	268,187	6.779^{+3}	1.0222^{+11}	1.467^{-1}	2.1202^{-11}	2.492^{14}	9.783^{-12}
281,977	270,000	7.131	1.0753	1.400	2.0232	2.369	9.300
285,000	272,771	7.700	1.1610	1.304	1.8848	2.194	8.613
287,435	275,000	8.185	1.2342	1.233	1.7814	2.064	8.103
290,000	277,347	8.724	1.3154	1.162	1.6795	1.937	7.602
292,902	280,000	9.369	1.4127	1.086	1.5692	1.803	7.079
295,000	281,917	9.860	1.4868	1.038	1.4999	1.713	6.726
298,377	285,000	1.070^{+4}	1.6129	9.628	1.3912	1.580	6.200
300,000	286,480	1.112^{+4}	1.6766^{+11}	9.289^{-2}	1.3423^{-11}	1.520^{14}	5.965^{-12}
303,862	290,000	1.218	1.8368	8.538	1.2338	1.387	5.444
305,000	291,036	1.251	1.8863	8.331	1.2038	1.351	5.301
309,356	295,000	1.384	2.0867	7.589	1.0966	1.221	4.792
310,000	295,585	1.404	2.1177	7.486	1.0817	1.203	4.722
314,859	300,000	1.568	2.3651	6.760	9.7678^{-12}	1.077	4.228
320,000	304,663	1.759^{+4}	2.6527^{+11}	6.079^{-2}	8.7848^{-12}	9.603^{13}	3.770^{-12}
325,893	310,000	2.002	3.0182	5.396	7.7972	8.441	3.313
330,000	313,714	2.187	3.2971	4.973	7.1856	7.727	3.033
336,963	320,000	2.533	3.8194	4.341	6.2724	6.670	2.618
340,000	322,738	2.698	4.0682	4.095	5.9171	6.262	2.458
348,069	330,000	3.180	4.7959	3.517	5.0822	5.312	2.085

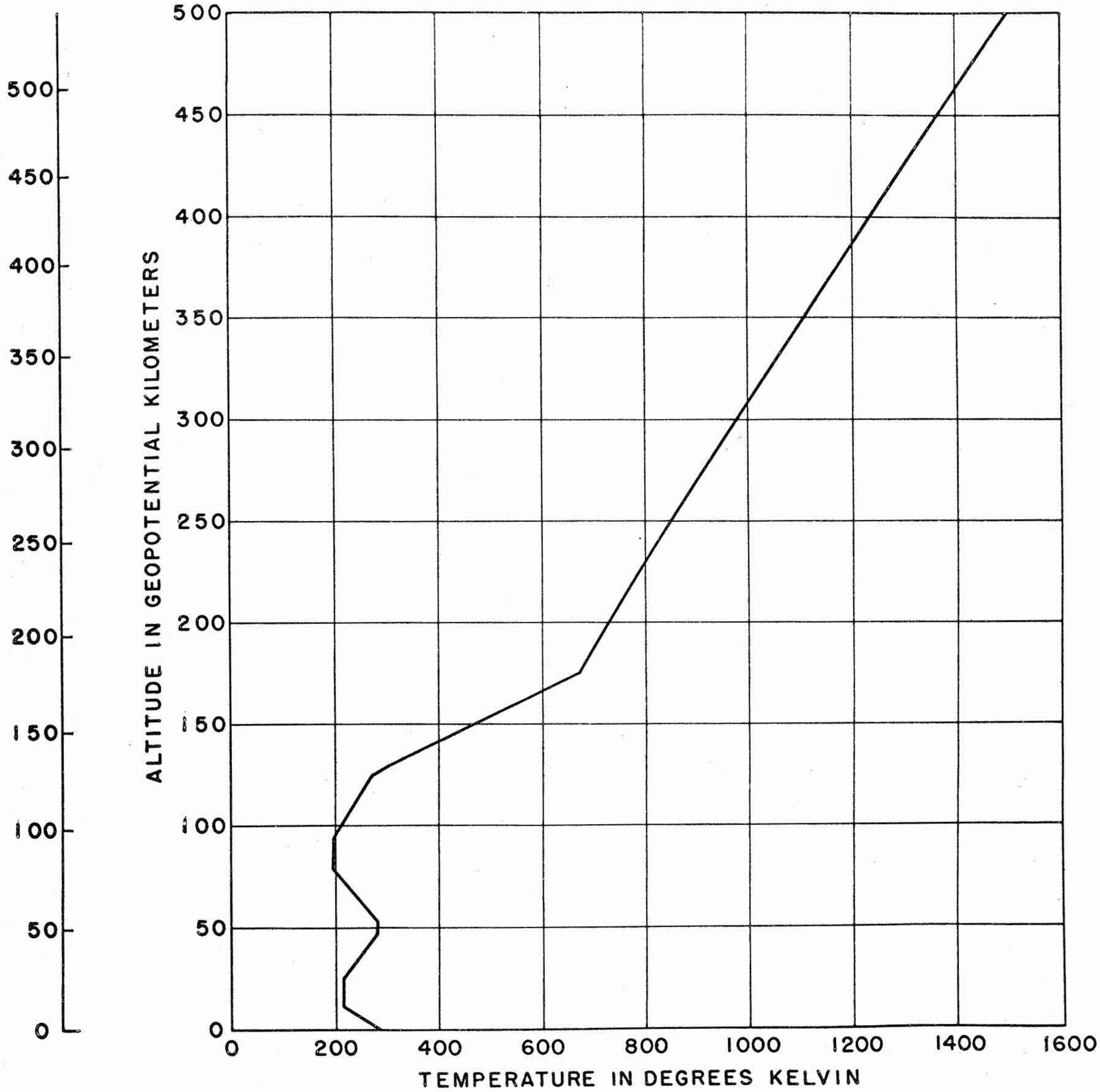
METRIC TABLE V CONTINUED

ALTITUDE		MEAN FREE PATH		COLLISION FREQUENCY		NUMBER DENSITY	
Z,m	H,m'	L,m	L/L_o	ν, sec^{-1}	ν/ν_o	n, m^{-3}	n/n_o
350,000	331,735	3.306^{+4}	4.9853^{+11}	3.393^{-2}	4.9035^{-12}	5.110^{13}	2.0059^{-12}
359,213	340,000	3.964	5.9781	2.869	4.1456	4.261	1.6728
360,000	340,705	4.026	6.0701	2.829	4.0875	4.197	1.6474
370,000	349,648	4.872	7.3463	2.371	3.4264	3.468	1.3612
370,394	350,000	4.908	7.4009	2.355	3.4030	3.442	1.3512
380,000	358,565	5.863	8.8404	1.998	2.8874	2.882	1.1312
381,612	360,000	6.037	9.1035	1.945	2.8101	2.798	1.0985
390,000	367,456	7.017	1.0581^{+12}	1.692	2.4453	2.408	9.4507^{-13}
392,867	370,000	7.381	1.1130	1.615	2.3335	2.289	8.9846
400,000	376,320	8.356^{+4}	1.2600^{+12}	1.438^{-2}	2.0776^{-12}	2.022^{13}	7.9363^{-13}
404,160	380,000	8.973	1.3531	1.348	1.9480	1.883	7.3906
410,000	385,158	9.903	1.4933	1.231	1.7782	1.706	6.6967
415,491	390,000	1.085^{+5}	1.6360	1.131	1.6342	1.557	6.1125
420,000	393,970	1.168	1.7616	1.056	1.5262	1.446	5.6767
426,860	400,000	1.305	1.9680	9.533^{-3}	1.3775	1.294	5.0812
430,000	402,756	1.372	2.0691	9.102	1.3152	1.231	4.8331
438,267	410,000	1.562	2.3561	8.071	1.1663	1.081	4.2444
440,000	411,516	1.605	2.4202	7.873	1.1377	1.053	4.1319
449,713	420,000	1.862	2.8076	6.863	9.9168^{-13}	9.074^{12}	3.5617
450,000	420,250	1.870^{+5}	2.8198^{+12}	6.835^{-3}	9.8771^{-13}	9.034^{12}	3.5463^{-13}
460,000	428,959	2.171	3.2731	5.955	8.6049	7.783	3.0552
461,197	430,000	2.209	3.3312	5.859	8.4660	7.648	3.0019
470,000	437,642	2.510	3.7856	5.205	7.5215	6.730	2.6416
472,721	440,000	2.610	3.9360	5.021	7.2551	6.472	2.5406
480,000	446,300	2.894	4.3634	4.570	6.6032	5.838	2.2918
484,283	450,000	3.072	4.6323	4.318	6.2402	5.499	2.1587
490,000	454,932	3.324	5.0129	4.014	5.8006	5.082	1.9948
495,884	460,000	3.602	5.4314	3.727	5.3859	4.690	1.8412
500,000	463,540	3.807^{+5}	5.7411^{+12}	3.541^{-3}	5.1165^{-13}	4.437^{12}	1.7418^{-13}
507,525	470,000	4.208	6.3453	3.228	4.6640	4.015	1.5760
510,000	472,122	4.347	6.5553	3.132	4.5256	3.886	1.5255
519,205	480,000	4.899	7.3876	2.804	4.0518	3.448	1.3536
520,000	480,679	4.949	7.4633	2.778	4.0137	3.413	1.3399
530,000	489,212	5.620	8.4737	2.470	3.5688	3.006	1.1801
530,925	490,000	5.685	8.5728	2.443	3.5306	2.972	1.1665
540,000	497,719	6.365	9.5976	2.201	3.1803	2.654	1.0419
542,686	500,000	6.577	9.9168	2.135	3.0855	2.569	1.0084



MOLECULAR SCALE TEMPERATURE VS GEOPOTENTIAL ALTITUDE

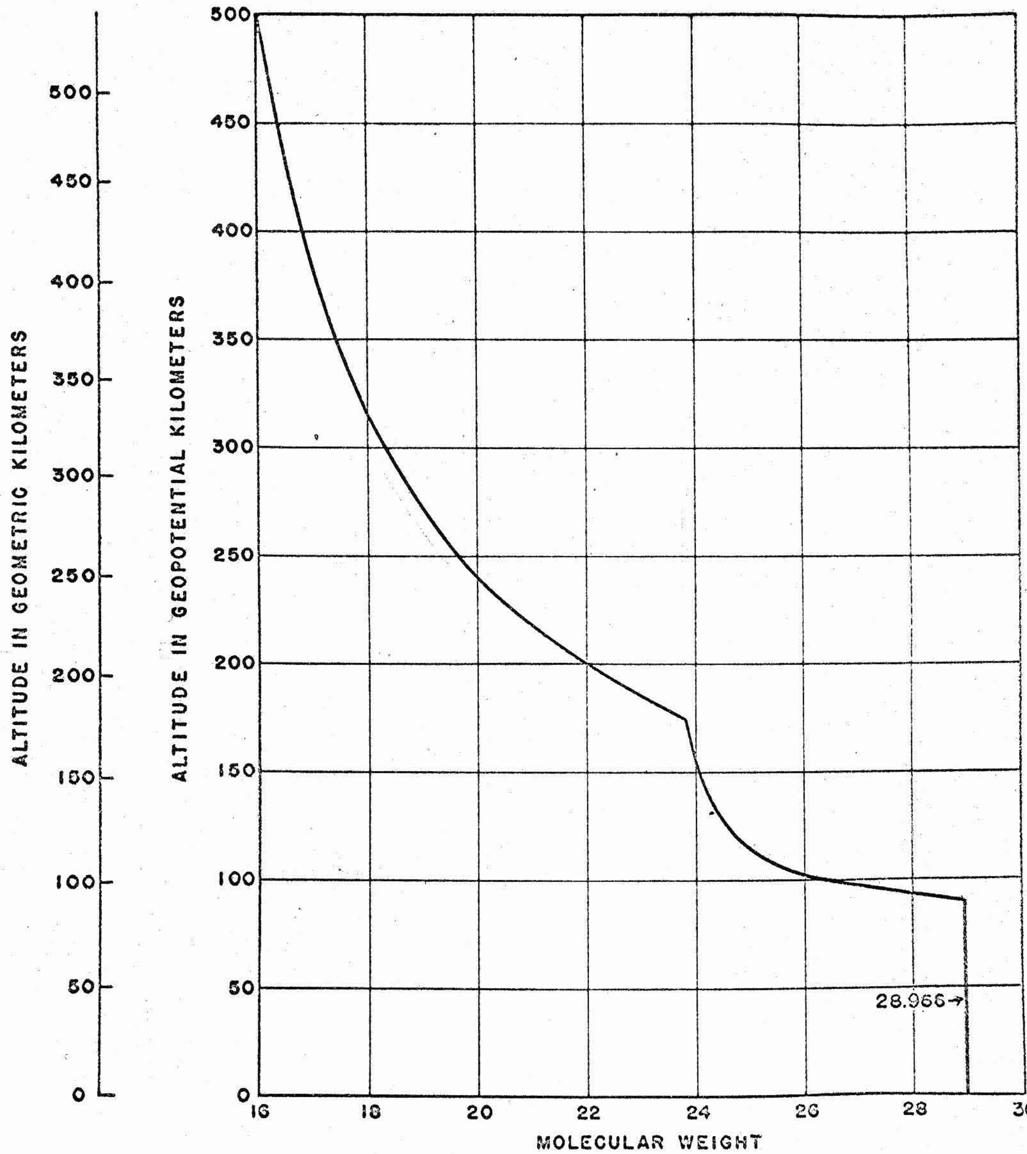
FIGURE 1.



REAL KINETIC TEMPERATURE VS GEOPOTENTIAL ALTITUDE

FIGURE 2.

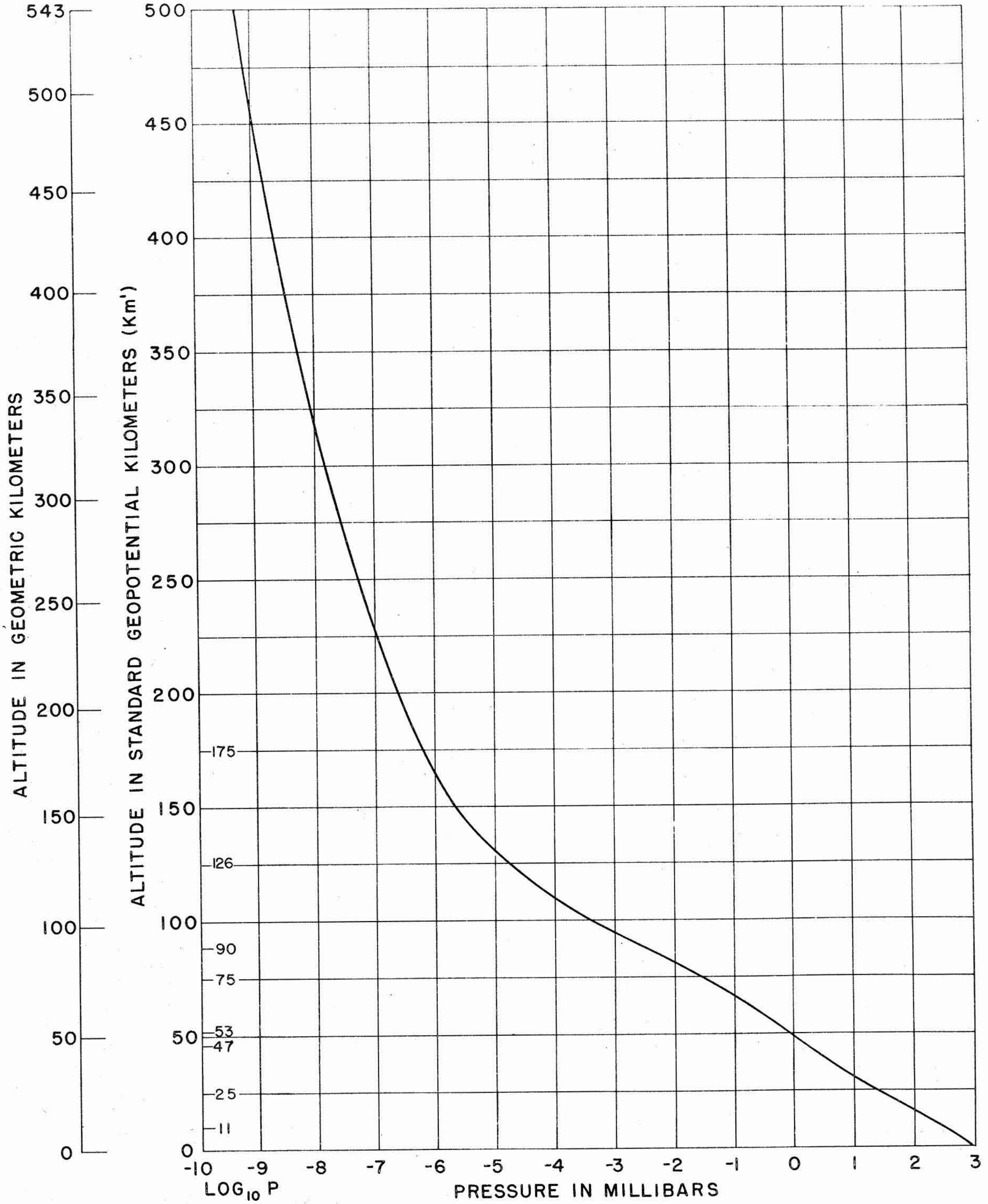
GRD
MARCH 1957



MOLECULAR WEIGHT VS GEOPOTENTIAL ALTITUDE

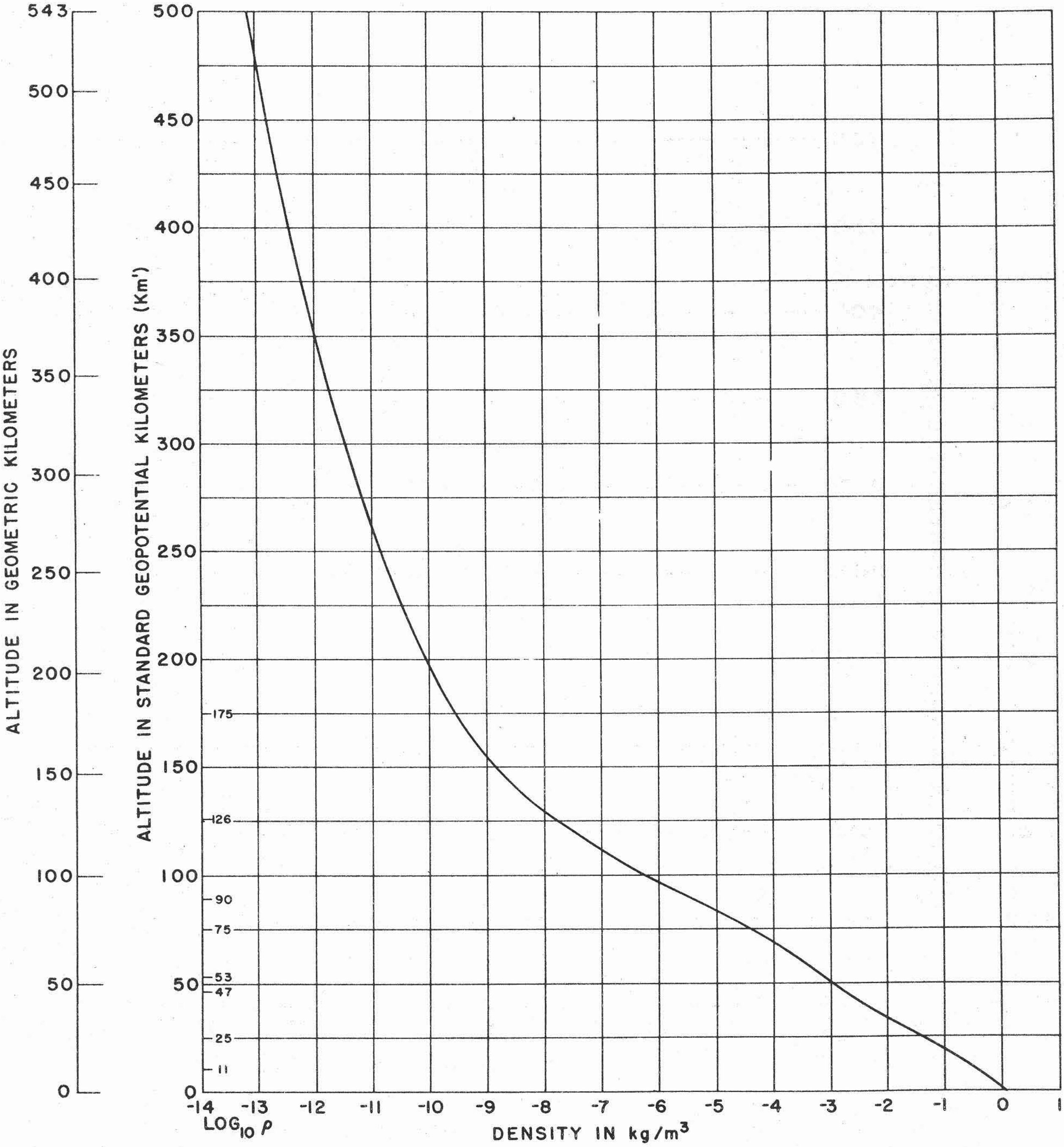
FIGURE 3

GRD
FEB. 1957



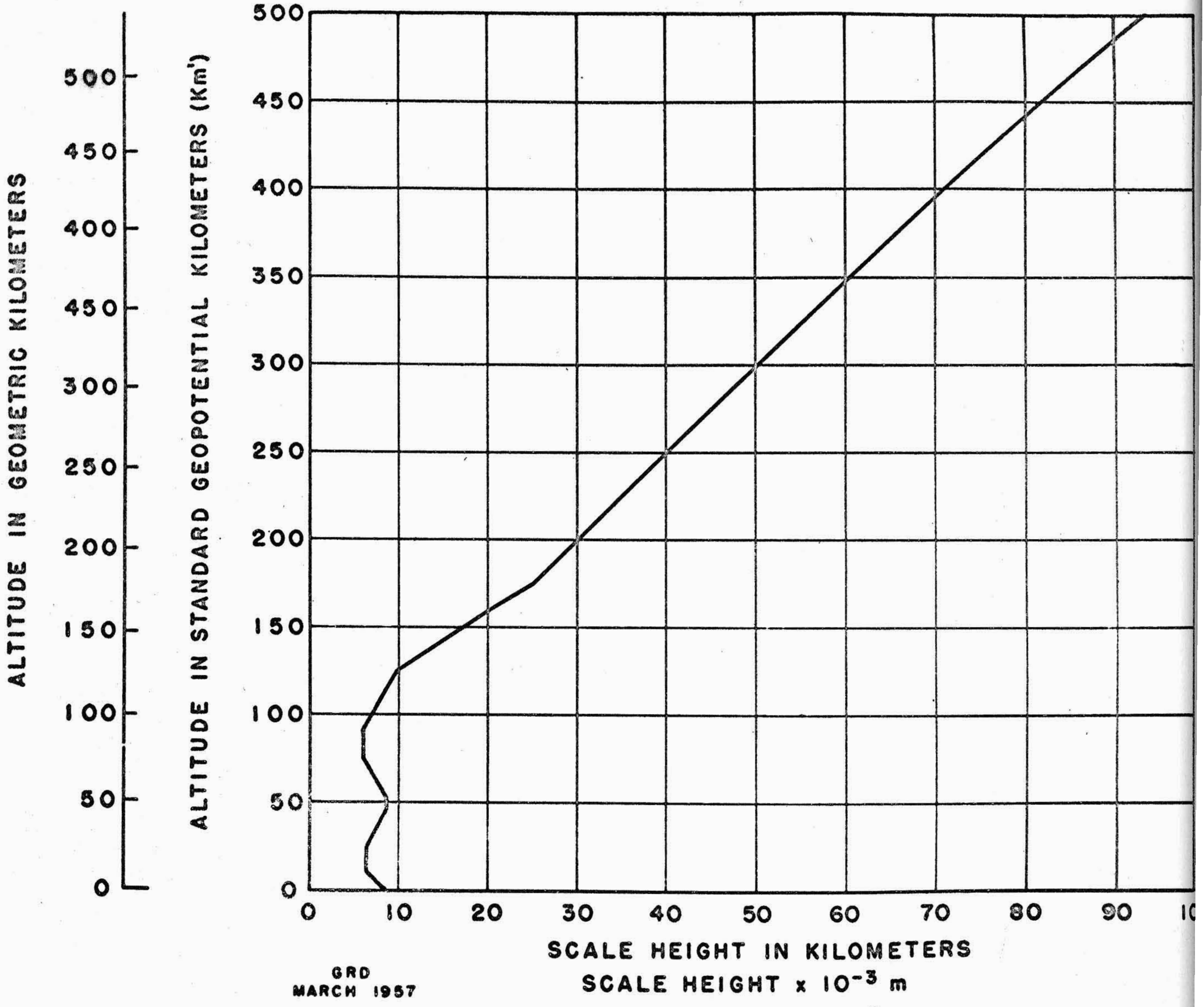
PRESSURE VS. GEOPOTENTIAL ALTITUDE

FIGURE 4.



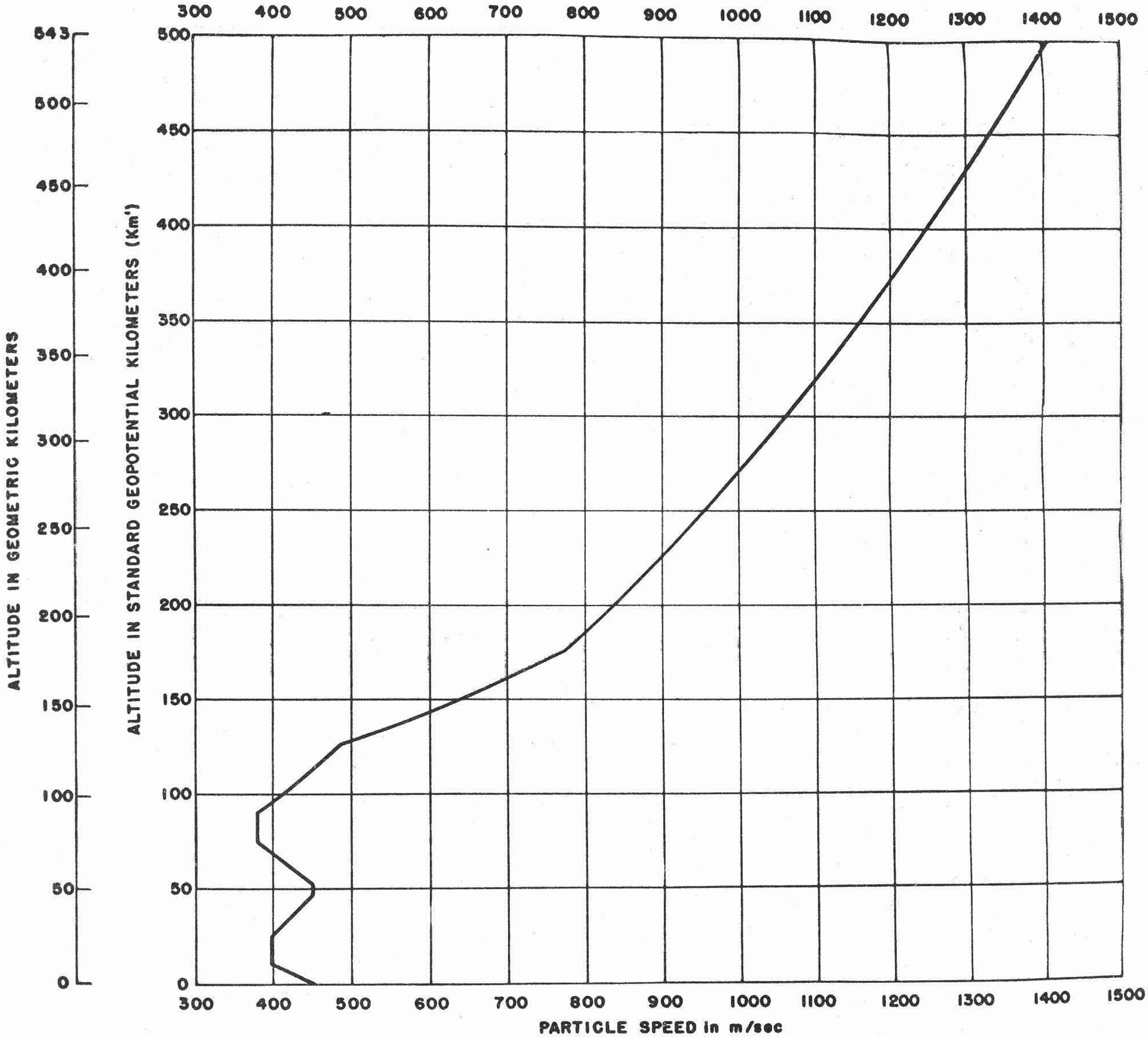
DENSITY VS. GEOPOTENTIAL ALTITUDE

FIGURE 5.



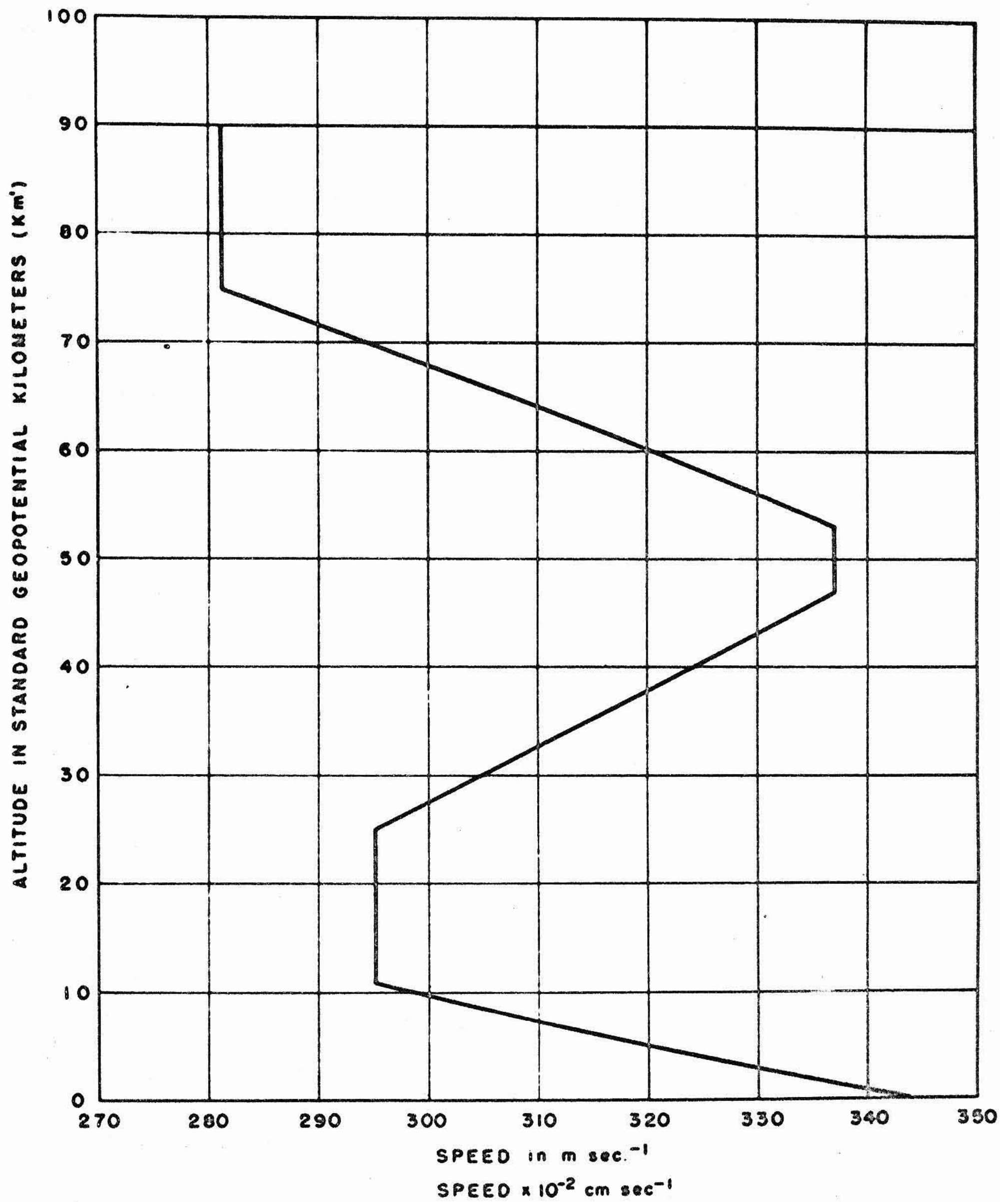
SCALE HEIGHT VS GEOPOTENTIAL ALTITUDE

FIGURE 6.



PARTICLE SPEED VS GEOPOTENTIAL ALTITUDE

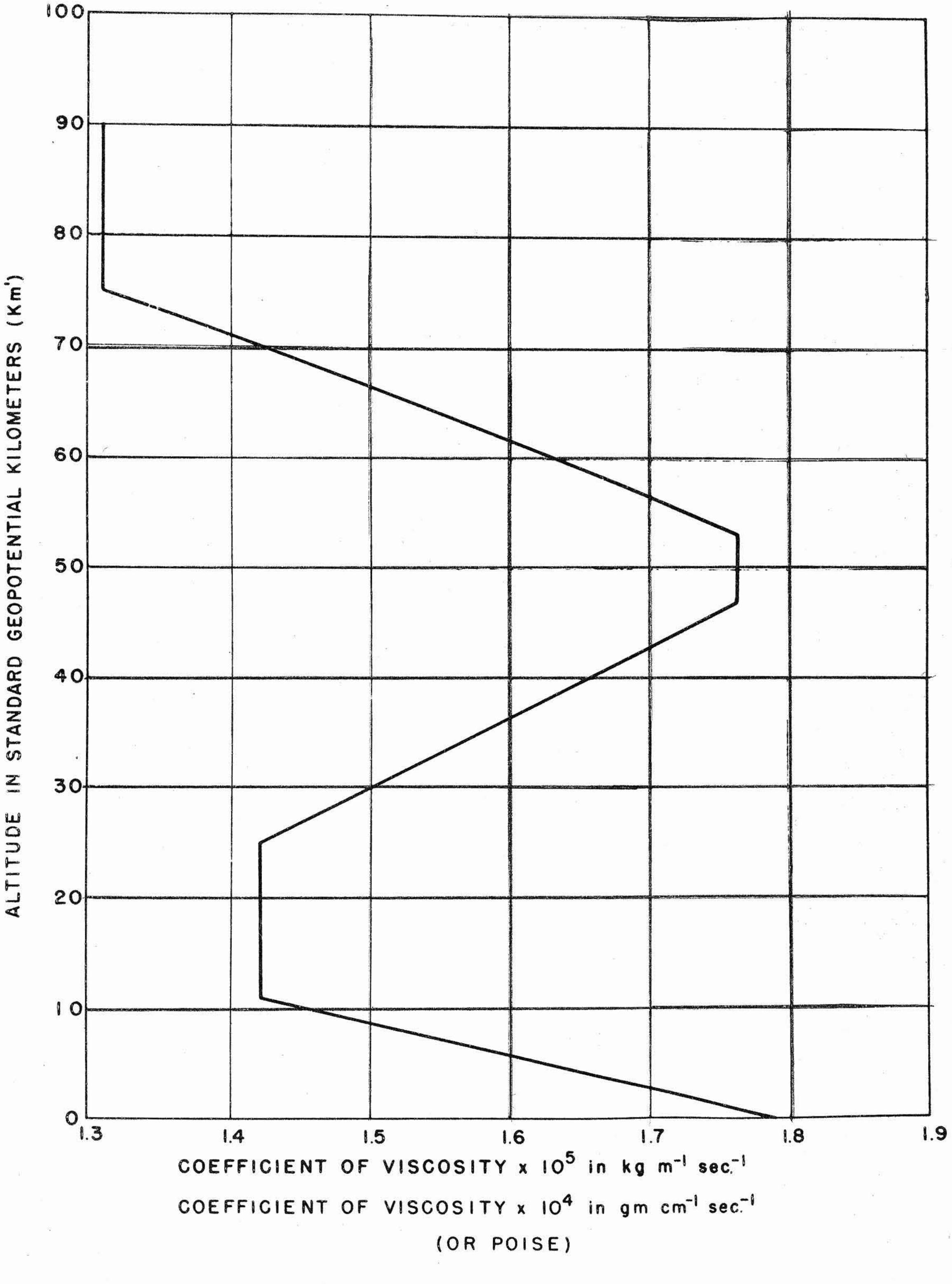
GRD
SEPT 1956



SOUND SPEED VS GEOPOTENTIAL ALTITUDE

GRD
MARCH 1957

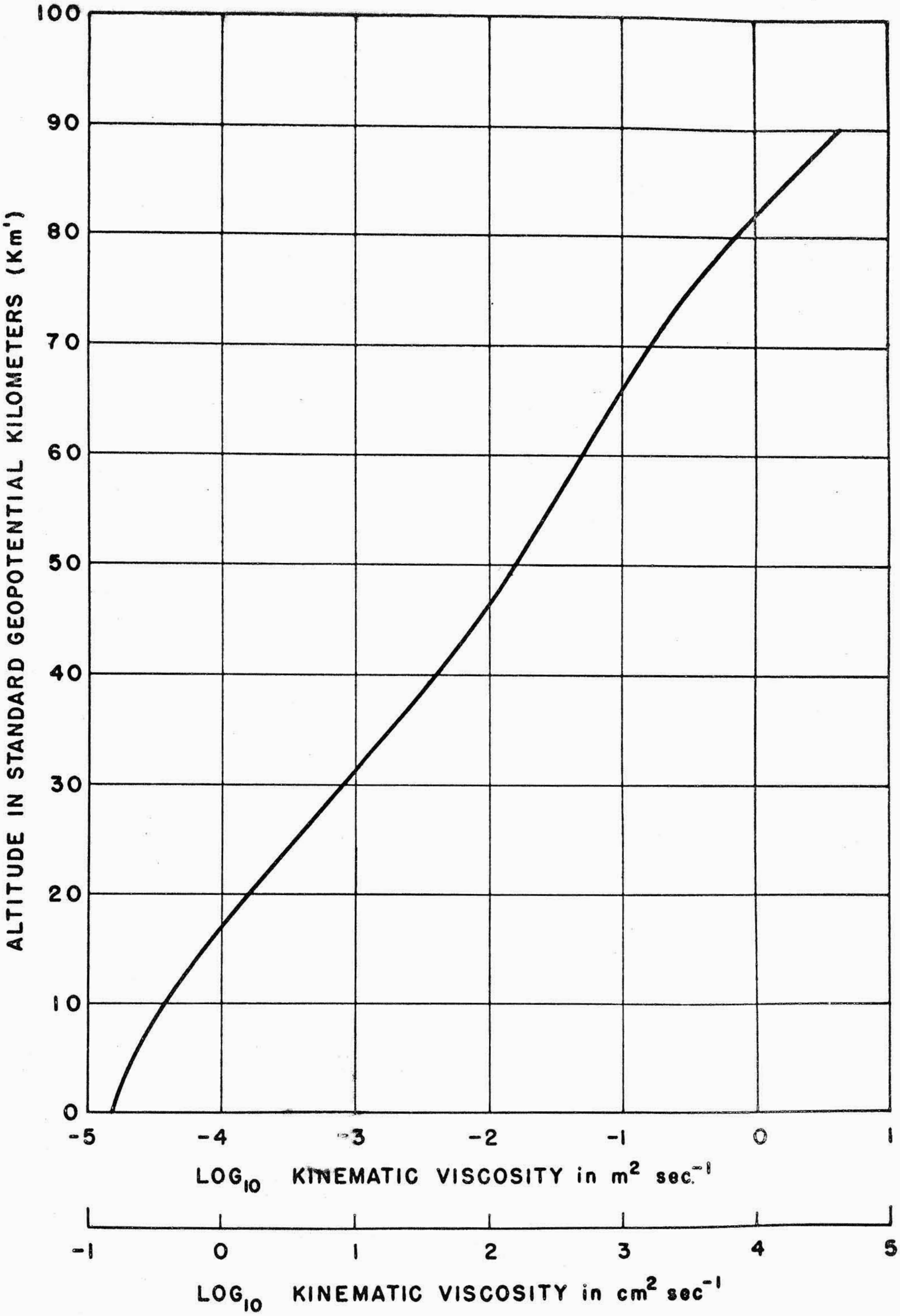
Figure 8



COEFFICIENT OF VISCOSITY VS GEOPOTENTIAL ALTITUDE

GRD
MARCH 1957

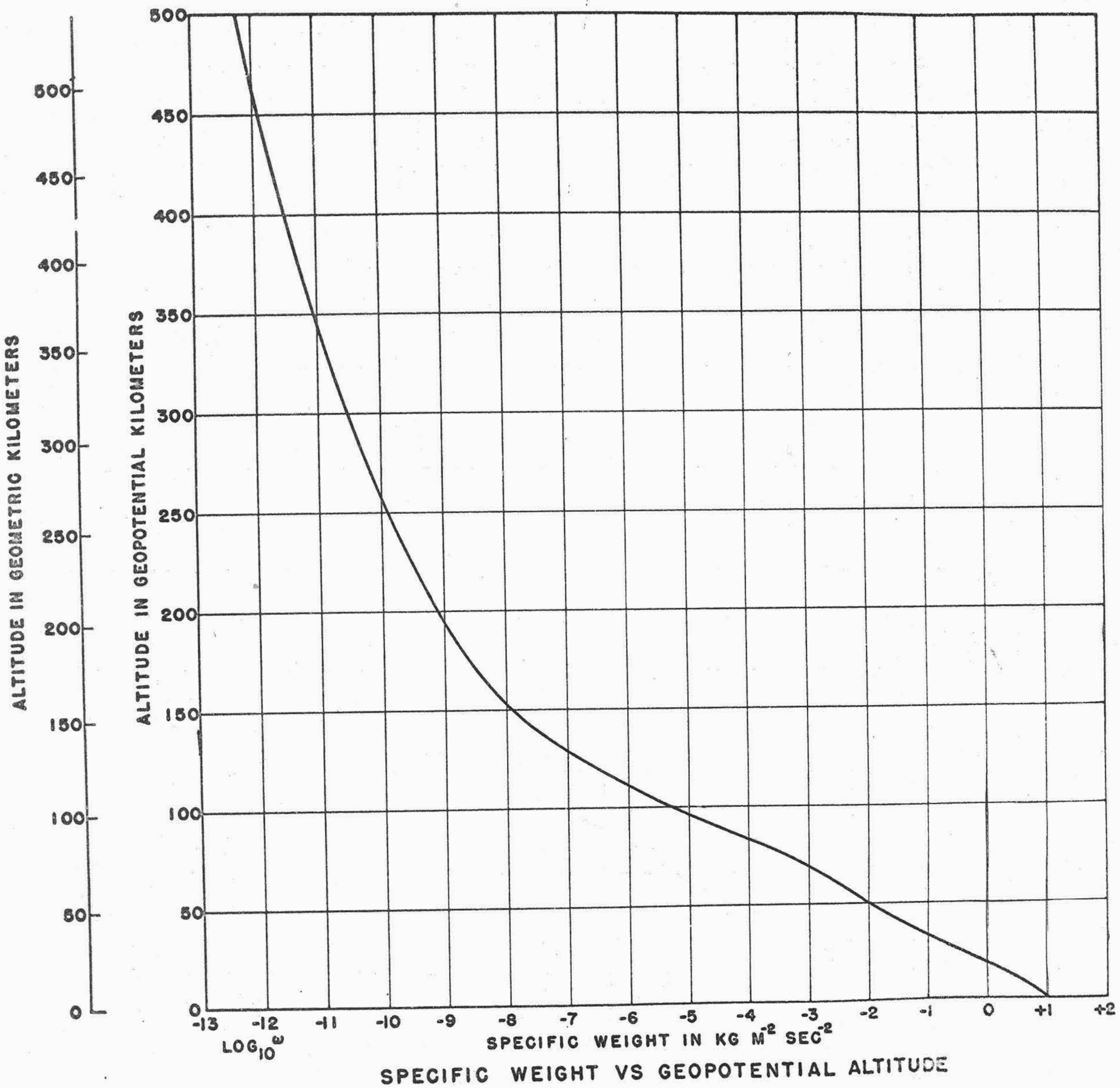
Figure 9



KINEMATIC VISCOSITY VS GEOPOTENTIAL ALTITUDE

GRD
MARCH 1957

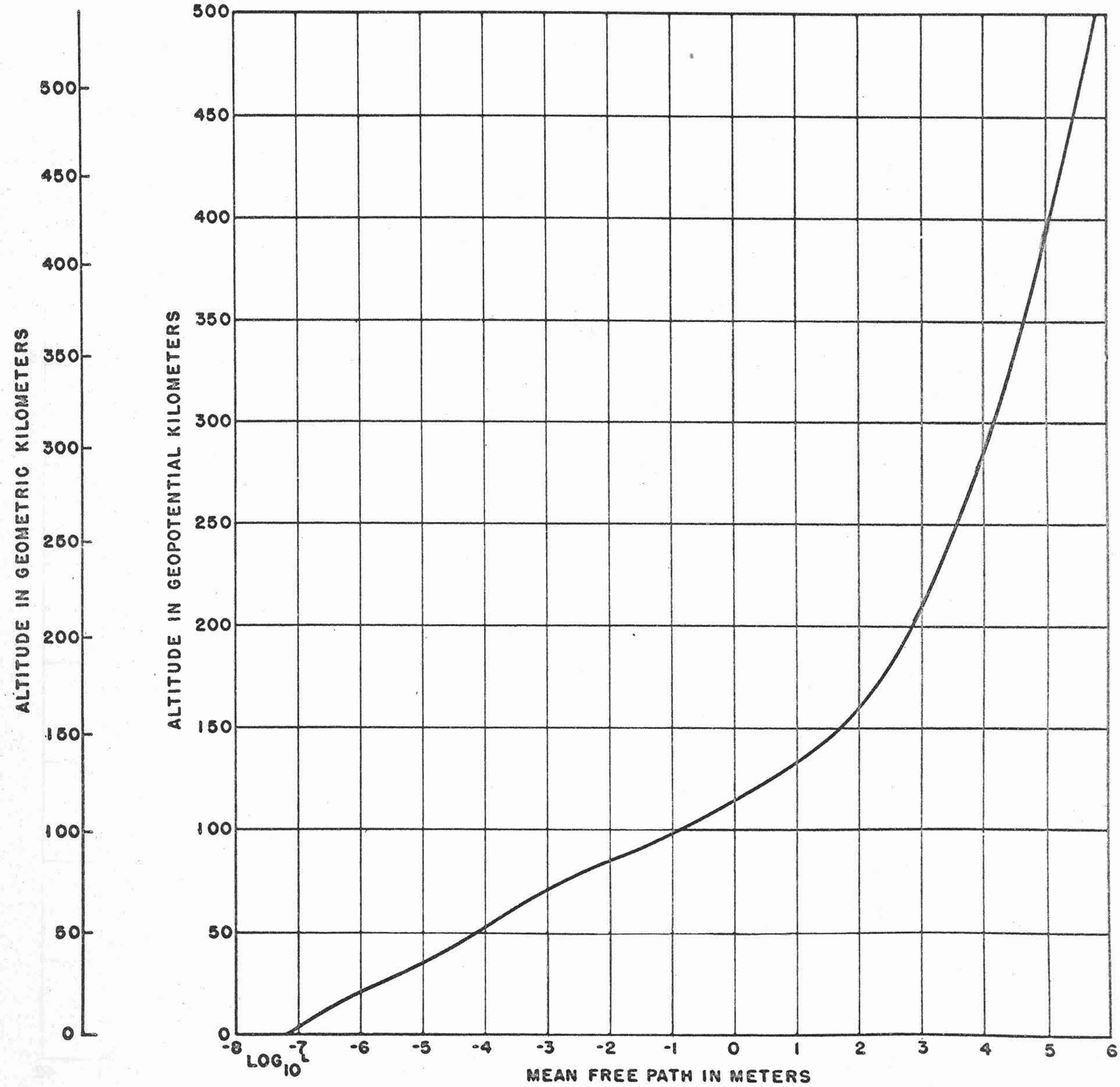
Figure 10.



SPECIFIC WEIGHT VS GEOPOTENTIAL ALTITUDE

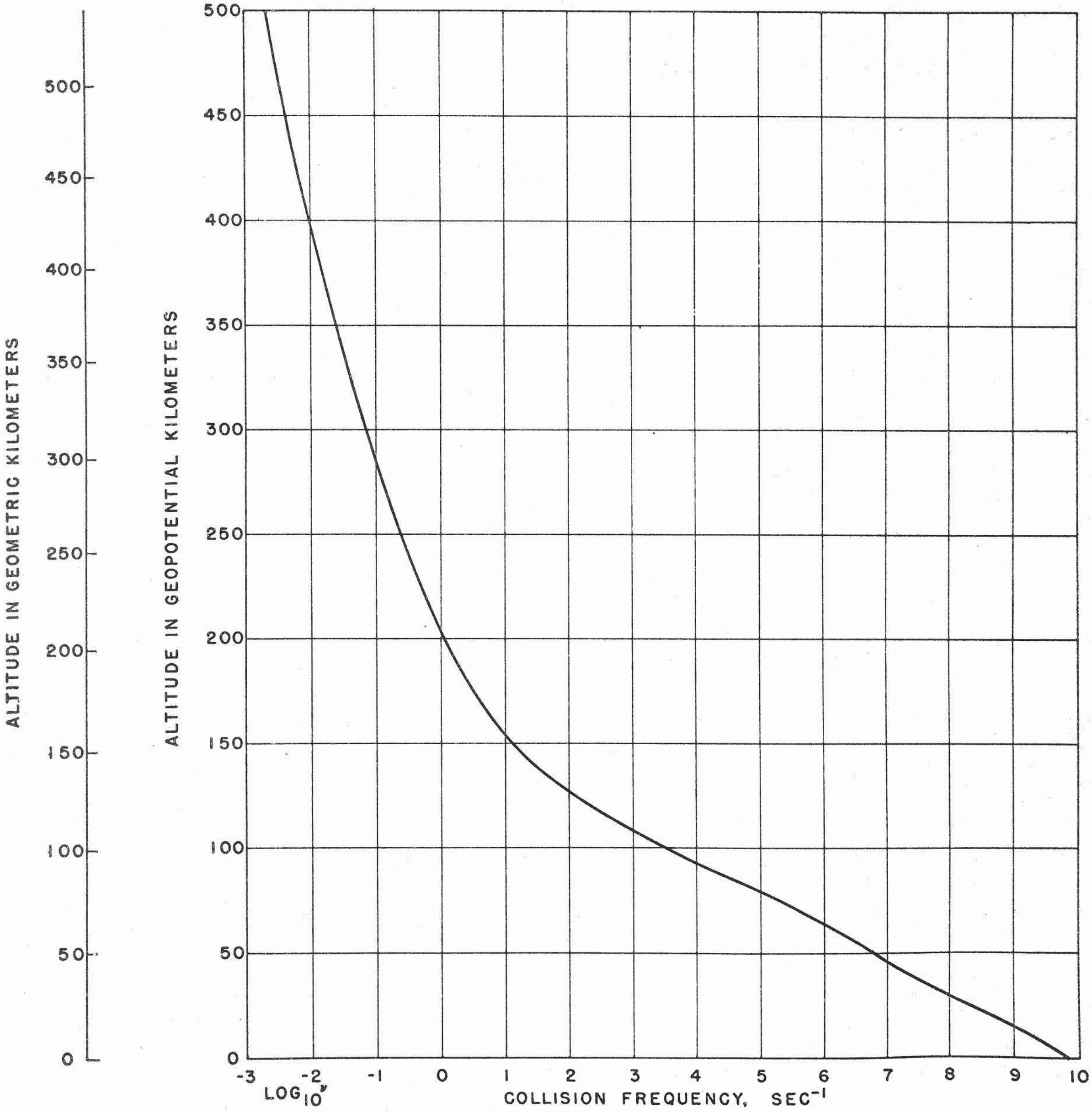
FIGURE 11.

GRD
FEB 1957



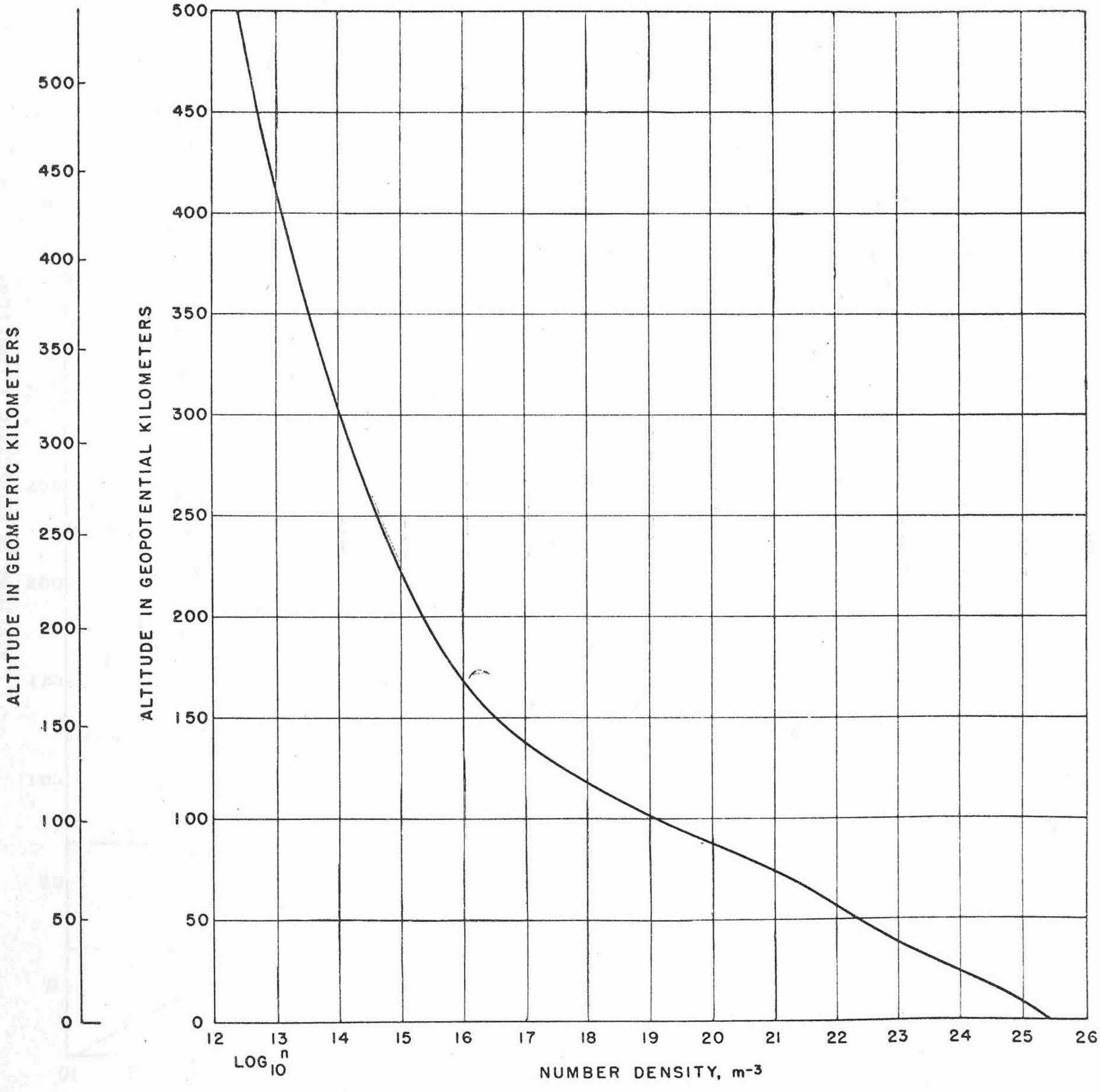
MEAN FREE PATH VS GEOPOTENTIAL ALTITUDE

FIGURE 12.



COLLISION FREQUENCY VS GEOPOTENTIAL ALTITUDE

FIGURE 13.



NUMBER DENSITY VS GEOPOTENTIAL ALTITUDE

GRD
MARCH 1957

FIGURE 14.

Section 11

ENGLISH TABLES

OF THE

ARDC MODEL ATMOSPHERE, 1956

NOTE: Superscripts appearing in the following tables indicate the power of ten by which each tabulated value should be multiplied.

ENGLISH TABLE I

TEMPERATURES, MOLECULAR WEIGHT, GRAVITATIONAL ACCELERATION AS FUNCTIONS OF GEOMETRIC AND GEOPOTENTIAL ALTITUDE

ALTITUDE		TEMPERATURE				MOLECULAR WEIGHT		GRAVT. ACCEL.
Z, ft	H, ft'	t, °C	t, °F	T, °R	T/T_o	M	M/M_o	g, ft sec^{-2}
-15000	-15011	44.739	112.531	572.22	1.10320	28.966	1.00000	32.2204
-14989	-15000	44.668	112.492	572.18	1.10313			32.2204
-12500	-12508	39.780	103.604	563.29	1.08599			32.2127
-12493	-12500	39.765	103.577	563.27	1.08594			32.2126
-10000	-10005	34.822	94.679	554.37	1.06879			32.2049
-9995.2	-10000	34.812	94.662	554.35	1.06875			32.2049
-7500	-7502.7	29.864	85.756	545.44	1.05158			32.1972
-7497.3	-7500	29.859	85.746	545.43	1.05157			32.1972
-5000	-5001.2	24.908	76.835	536.52	1.03439			32.1895
-4998.8	-5000	24.906	76.831	536.52	1.03438			32.1895
-2500	-2500.3	19.954	67.916	527.60	1.01719			32.1818
-2499.7	-2500	19.953	67.915	527.60	1.01719			32.1818
0	0	15.000	59.000	518.69	1.00000	constant at 28.966 for altitudes to 299,516 feet	1.00000 for altitudes to 299,516 feet	32.1741
2500	2499.7	10.048	50.086	509.77	.982814			32.1663
2500.3	2500	10.047	50.085	509.77	.982812			32.1663
5000	4998.8	5.096	41.174	500.86	.965632			32.1586
5001.2	5000	5.094	41.169	500.86	.965623			32.1586
7500	7497.3	0.146	32.263	491.95	.948453			32.1509
7502.7	7500	0.141	32.254	491.94	.948435			32.1509
10000	9995.2	-4.803	23.355	483.04	.931279			32.1432
10005	10000	-4.812	23.338	483.03	.931247			32.1432
12500	12493	-9.750	14.450	474.14	.914110			32.1355
12508	12500	-9.765	14.423	474.11	.914058			32.1355
15000	14989	-14.697	5.546	465.23	.896944			32.1278
15011	15000	-14.718	5.508	465.20	.896870			32.1278
17500	17485	-19.642	-3.356	456.33	.879782			32.1201
17515	17500	-19.671	-3.408	456.28	.879681			32.1201
20000	19981	-24.586	-12.255	447.43	.862625			32.1124
20019	20000	-24.624	-12.323	447.36	.862493			32.1124
22500	22476	-29.529	-21.152	438.54	.845471			32.1047
22524	22500	-29.577	-21.239	438.45	.845305			32.1047
25000	24970	-34.471	-30.047	429.64	.828322			32.0971
25030	25000	-34.530	-30.154	429.53	.828116			32.0970
27500	27464	-39.411	-38.940	420.75	.811177			32.0894
27536	27500	-39.483	-39.069	420.62	.810928	28.966	1.00000	32.0893

ALTITUDE		TEMPERATURE				MOLECULAR WEIGHT		GRAVT. ACCEL.
Z, ft	H, ft'	t, °C	t, °F	T, °R	$T/T_{\odot}$	M	$M/M_{\odot}$	g, ft sec^{-2}
30000	29957	-44.351	-47.831	411.86	.794036	28.966	1.00000	32.0817
30043	30000	-44.436	-47.985	411.70	.793740			32.0816
32500	32449	-49.289	-56.720	402.97	.776899			32.0740
32551	32500	-49.389	-56.900	402.79	.776551			32.0739
35000	34941	-54.226	-65.607	394.08	.759766			32.0663
35059	35000	-54.342	-65.816	393.87	.759363			32.0662
36152	36089	-56.500	-69.700	389.99	.751874			32.0628
37500	37433	-56.500	-69.700	389.99	.751874			32.0587
37568	37500	-56.500	-69.700	389.99	.751874			32.0585
40000	39923	-56.500	-69.700	389.99	.751874			32.0510
40077	40000	-56.500	-69.700	389.99	.751874			32.0508
42500	42414	-56.500	-69.700	389.99	.751874			32.0433
42587	42500	-56.500	-69.700	389.99	.751874			32.0433
45000	44903	-56.500	-69.700	389.99	.751874			32.0357
45097	45000	-56.500	-69.700	389.99	.751874			32.0354
47500	47392	-56.500	-69.700	389.99	.751874			32.0280
47608	47500	-56.500	-69.700	389.99	.751874			32.0277
50000	49880	-56.500	-69.700	389.99	.751874	constant at 28.966 for altitudes to 299,516 feet	1.00000 for altitudes to 299,516 feet	32.0203
50120	50000	-56.500	-69.700	389.99	.751874			32.0200
52500	52368	-56.500	-69.700	389.99	.751874			32.0127
52632	52500	-56.500	-69.700	389.99	.751874			32.0123
55000	54855	-56.500	-69.700	389.99	.751874			32.0050
55145	55000	-56.500	-69.700	389.99	.751874			32.0046
57500	57342	-56.500	-69.700	389.99	.751874			31.9974
57659	57500	-56.500	-69.700	389.99	.751874			31.9969
60000	59828	-56.500	-69.700	389.99	.751874			31.9897
60173	60000	-56.500	-69.700	389.99	.751874			31.9892
70000	69766	-56.500	-69.700	389.99	.751874			31.9592
70236	70000	-56.500	-69.700	389.99	.751874			31.9584
80000	79694	-56.500	-69.700	389.99	.751874			31.9286
80308	80000	-56.500	-69.700	389.99	.751874			31.9277
82345	82021	-56.500	-69.700	389.99	.751874			31.9215
90000	89613	-49.558	-57.204	402.48	.775966			31.8982
90390	90000	-49.204	-56.567	403.12	.777193			31.8970
100000	99523	-40.496	-40.893	418.79	.807411			31.8677
100482	100000	-40.060	-40.108	419.58	.808926			31.8663
110000	109423	-31.444	-24.599	435.09	.838827			31.8373
110583	110000	-30.916	-23.649	436.04	.840658	28.966	1.00000	31.8356

ENGLISH TABLE I CONTINUED

ALTITUDE		TEMPERATURE				MOLECULAR WEIGHT		GRAVT. ACCEL.
Z,ft	H,ft'	t, °C	t, °F	T, °R	T/T_0	M	M/M_0	g,ft sec^{-2}
120000	119313	-22.400	-8.320	451.37	.870212	28.966	1.00000	31.8070
120695	120000	-21.772	-7.190	452.50	.872390			31.8049
130000	129195	-13.364	7.944	467.63	.901567			31.7767
130815	130000	-12.628	9.270	468.96	.904123			31.7742
140000	139066	-4.338	24.192	483.88	.932893			31.7464
140946	140000	-3.484	25.729	485.42	.935855			31.7435
150000	148929	4.681	40.425	500.11	.964188			31.7162
151087	150000	5.660	42.188	501.88	.967587			31.7129
155348	154199	9.500	49.100	508.79	.980913			31.7000
160000	158782	9.500	49.100	508.79	.980913			31.6860
161237	160000	9.500	49.100	508.79	.980913			31.6823
170000	168626	9.500	49.100	508.79	.980913			31.6559
171397	170000	9.500	49.100	508.79	.980913			31.6517
175346	173885	9.500	49.100	508.79	.980913			31.6398
180000	178460	4.061	39.310	499.00	.962040			31.6258
181567	180000	2.230	36.015	495.70	.955686			31.6211
190000	188285	-7.618	18.288	477.98	.921510			31.5957
191747	190000	-9.657	14.618	474.31	.914434			31.5905
200000	198100	-19.297	-2.735	456.95	.880978	constant at 28.966 for altitudes to 299,516 feet	1.00000 for altitudes to 299,516 feet	31.5657
201937	200000	-21.544	-6.779	452.91	.873182			31.5599
210000	207907	-30.943	-23.697	435.99	.840565			31.5358
212136	210000	-33.431	-28.176	431.51	.831929			31.5294
220000	217704	-42.589	-44.659	415.03	.800151			31.5059
222345	220000	-45.318	-49.573	410.11	.790677			31.4988
230000	227491	-54.223	-65.602	394.09	.759775			31.4760
232565	230000	-57.206	-70.970	388.72	.749425			31.4683
240000	237270	-65.847	-86.525	373.16	.719437			31.4461
242794	240000	-69.093	-92.367	367.32	.708173			31.4378
249001	246063	-76.300	-105.340	354.35	.683162			31.4193
250000	247039	-76.300	-105.340	354.35	.683162			31.4164
253033	250000	-76.300	-105.340	354.35	.683162			31.4073
260000	256799	-76.300	-105.340	354.35	.683162			31.3866
263282	260000	-76.300	-105.340	354.35	.683162			31.3768
270000	266549	-76.300	-105.340	354.35	.683162			31.3569
273541	270000	-76.300	-105.340	354.35	.683162			31.3464
280000	276291	-76.300	-105.340	354.35	.683162			31.3272
283810	280000	-76.300	-105.340	354.35	.683162			31.3159
290000	286023	-76.300	-105.340	354.35	.683162			31.2976
294089	290000	-76.300	-105.340	354.35	.683162			31.2855
299516	295276	-76.300	-105.340	354.35	.683162	28.966	1.00000	31.2694

ENGLISH TABLE I CONTINUED

ALTITUDE		TEMPERATURE				MOLECULAR WEIGHT		GRAVT. ACCEL.
Z, ft	H, ft'	t, °C	t, °F	T, °R	T/T_o	M	M/M_o	g, ft sec^{-2}
300000	295746	-76.30	-105.3	354.4	.68318	28.89	.99748	31.2680
304378	300000	-75.84	-104.5	355.2	.68475	28.31	.97730	31.2551
325000	320013	-67.84	-90.12	369.6	.71251	26.64	.91967	31.1943
330145	325000	-65.00	-85.01	375.7	.72237	26.38	.91069	31.1791
350000	344223	-52.52	-62.53	397.2	.76569	25.66	.88583	31.1207
355974	350000	-48.45	-55.21	404.5	.77982	25.50	.88040	31.1032
375000	368376	-34.90	-30.81	428.9	.82685	25.11	.86691	31.0475
381866	375000	-29.85	-21.73	438.0	.84436	25.00	.86308	31.0274
400000	392473	-16.25	02.76	462.4	.89157	24.76	.85481	30.9745
407822	400000	-10.29	13.48	473.2	.91224	24.68	.85188	30.9517
421745	413386	0.42	22.75	492.4	.94940	24.54	.84736	30.9112
425000	416512	8.18	46.72	506.4	.97633	24.52	.84642	30.9018
433841	425000	29.22	84.60	544.3	1.0494	24.45	.84403	30.8761
450000	440495	67.55	153.6	613.3	1.1824	24.34	.84025	30.8293
459924	450000	91.02	195.8	655.5	1.2638	24.28	.83823	30.8006
475000	464422	126.6	259.8	719.5	1.3872	24.20	.83553	30.7571
486071	475000	152.6	306.7	766.4	1.4776	24.15	.83377	30.7252
500000	488293	185.3	365.6	825.2	1.5910	24.09	.83180	30.6851
512282	500000	214.1	417.3	877.0	1.6908	24.05	.83023	30.6498
550000	535868	302.1	575.7	1035	1.9963	23.93	.82627	30.5419
564897	550000	336.7	638.1	1098	2.1164	23.90	.82498	30.4995
590401	574147	395.9	744.5	1204	2.3217	23.84	.82303	30.4270
600000	583221	402.0	755.7	1215	2.3432	23.59	.81458	30.3997
617773	600000	413.6	776.5	1236	2.3833	23.17	.79991	30.3494
650000	630354	434.9	814.9	1275	2.4572	22.48	.77621	30.2585
670910	650000	448.9	840.0	1300	2.5058	22.09	.76252	30.1998
700000	677268	468.5	875.4	1335	2.5741	21.59	.74534	30.1183
724311	700000	485.1	905.3	1365	2.6316	21.22	.73242	30.0505
750000	723965	502.8	937.0	1397	2.6927	20.85	.71998	29.9791
777977	750000	522.1	971.7	1431	2.7597	20.50	.70766	29.9016
800000	770446	537.3	999.2	1459	2.8126	20.24	.69876	29.8408
831911	800000	559.5	1039	1499	2.8896	19.90	.68694	29.7531
850000	816714	572.1	1062	1521	2.9333	19.72	.68075	29.7035
886115	850000	597.3	1107	1567	3.0209	19.39	.66935	29.6049
900000	862768	607.1	1125	1584	3.0546	19.27	.66527	29.5671
940590	900000	635.5	1176	1636	3.1533	18.95	.65422	29.4571
950000	908611	642.1	1188	1647	3.1762	18.88	.65183	29.4317
995339	950000	673.9	1245	1705	3.2866	18.57	.64108	29.3097

ENGLISH TABLE I CONTINUED

ALTITUDE		TEMPERATURE				MOLECULAR WEIGHT		GRAVT. ACCEL.
Z,ft	H,ft'	t,°C	t,°F	T,°R	T/T_0	M	M/M_0	g,ft sec^{-2}
1000000	954245	677.2	1251	1711	3.2980	18.54	.64004	29.2972
1050364	1000000	712.5	1315	1774	3.4206	18.24	.62955	29.1626
1100000	1044889	747.3	1377	1837	3.5415	17.97	.62034	29.0309
1161249	1100000	790.2	1454	1914	3.6903	17.68	.61028	28.8696
1200000	1134710	817.3	1503	1963	3.7844	17.51	.60454	28.7682
1273262	1200000	868.4	1595	2055	3.9617	17.23	.59481	28.5781
1300000	1223721	887.1	1629	2088	4.0263	17.14	.59158	28.5091
1386421	1300000	947.0	1737	2196	4.2343	16.86	.58213	28.2880
1400000	1311932	956.4	1754	2213	4.2669	16.82	.58076	28.2535
1500000	1399354	1025	1878	2337	4.5061	16.56	.57160	28.0013
1500743	1400000	1026	1879	2338	4.5079	16.56	.57153	27.9994
1600000	1485997	1094	2001	2461	4.7438	16.33	.56373	27.7525
1616246	1500000	1105	2021	2480	4.7822	16.29	.56255	27.7124
1700000	1571872	1162	2123	2583	4.9797	16.13	.55690	27.5069
1732949	1600000	1184	2163	2623	5.0571	16.07	.55484	27.4267
1780465	1640420	1216	2221	2681	5.1683	15.99	.55203	27.3117
1850870	1700000	1263	2306	2766	5.0457	15.88	.54815	27.1426

ENGLISH TABLE II

PRESSURE AND DENSITY AS FUNCTIONS OF GEOMETRIC AND GEOPOTENTIAL ALTITUDE

ALTITUDE		PRESSURE				DENSITY	
Z, ft	H, ft'	P, mb	P, in Hg	P, $\frac{lbf}{ft^2}$	P/P_o	ρ, $\frac{lbf sec^2}{ft^4}$	ρ/ρ_o
-15000	-15011	1.6979^{3}	5.0140^{1}	3.5462^{3}	1.67573^{0}	3.6105^{-3}	1.51897^{0}
-14989	-15000	1.6973	5.0122	3.5450	1.67514	3.6094	1.51853
-12500	-12508	1.5633	4.6163	3.2649	1.54281	3.3767	1.42064
-12493	-12500	1.5629	4.6151	3.2641	1.54242	3.3761	1.42036
-10000	-10005	1.4374	4.2446	3.0020	1.41858	3.1548	1.32728
-9995.2	-10000	1.4371	4.2439	3.0015	1.41835	3.1544	1.32711
-7500	-7502.7	1.3199	3.8976	2.7566	1.30261	2.9443	1.23871
-7497.3	-7500	1.3197	3.8972	2.7564	1.30249	2.9441	1.23862
-5000	-5001.2	1.2103	3.5740	2.5277	1.19446	2.7448	1.15475
-4998.8	-5000	1.2102	3.5738	2.5276	1.19441	2.7447	1.15471
-2500	-2500.3	1.1082	3.2726	2.3146	1.09372	2.5558	1.07524
-2499.7	-2500	1.1082	3.2725	2.3145	1.09371	2.5557	1.07523
0	0	1.01325^{3}	2.9921^{1}	2.1162^{3}	1.00000	2.3769^{-3}	1.00000
2500	2499.7	9.2501^{2}	2.7315	1.9319	9.12909^{-1}	2.2079	9.28873^{-1}
2500.3	2500	9.2500	2.7315	1.9319	9.12899	2.2078	9.28865
5000	4998.8	8.4311	2.4897	1.7609	8.32084	2.0482	8.61699
5001.2	5000	8.4307	2.4896	1.7608	8.32046	2.0481	8.61668
7500	7497.3	7.6720	2.2656	1.6023	7.57170	1.8975	7.98321
7502.7	7500	7.6712	2.2653	1.6022	7.57092	1.8974	7.98254
10000	9995.2	6.9694^{2}	2.05807^{1}	1.4556^{3}	6.87830^{-1}	1.7556^{-2}	7.38586^{-1}
10005	10000	6.9681	2.0577	1.4553	6.87702	1.7553	7.38474
12500	12493	6.3200	1.8663	1.3200	6.23738	1.6219	6.82345
12508	12500	6.3181	1.8657	1.3196	6.23552	1.6215	6.82180
15000	14989	5.7207	1.6893	1.1948	5.64584	1.4962	6.29453
15011	15000	5.7182	1.6886	1.1943	5.64339	1.4956	6.29232
17500	17485	5.1683	1.5262	1.0794	5.10069	1.3781	5.79768
17515	17500	5.1652	1.5253	1.0788	5.09762	1.3774	5.79485
20000	19981	4.6600^{2}	1.3761^{1}	9.7327^{2}	4.59909^{-1}	1.2673^{-3}	5.33151^{-1}
20019	20000	4.6563	1.3750	9.7249	4.59540	1.2664	5.32805
22500	22476	4.1931	1.2382	8.7576	4.13831	1.1634	4.89468
22524	22500	4.1888	1.2370	8.7485	4.13402	1.1624	4.89057
25000	24970	3.7650	1.1118	7.8633	3.71574	1.0663	4.48586
25030	25000	3.7601	1.1103	7.8531	3.71089	1.0651	4.48112
27500	27464	3.3730	9.9605^{0}	7.0447	3.32890	9.7544^{-4}	4.10379
27536	27500	3.3676	9.9444	7.0333	3.32353	9.7416	4.09843

ENGLISH TABLE II CONTINUED

ALTITUDE		PRESSURE				DENSITY	
Z,ft	H,ft'	P,mb	P,in Hg	P, $\frac{lbf}{ft^2}$	P/P_o	ρ, $\frac{lbf sec^2}{ft^4}$	ρ/ρ_o
30000	29957	3.0148^{2}	8.9028^{0}	6.2966^{2}	2.97541^{-1}	8.9068^{-4}	3.74720^{-1}
30043	30000	3.0089	8.8854	6.2843	2.96958	8.8927	3.74126
32500	32449	2.6882	7.9382	5.6144	2.65303	8.1169	3.41490
32551	32500	2.6819	7.9196	5.6012	2.64680	8.1015	3.40840
35000	34941	2.3909	7.0602	4.9934	2.35960	7.3820	3.10569
35050	35000	2.3842	7.0406	4.9795	2.35303	7.3653	3.09869
36152	36089	2.2632	6.6832	4.7268	2.23359	7.0611	2.97069
37500	37433	2.1217	6.2653	4.4312	2.09392	6.6196	2.78493
37568	37500	2.1148	6.2450	4.4169	2.08716	6.5982	2.77594
40000	39923	1.8823^{2}	5.5584^{0}	3.9312^{2}	1.85767^{-1}	5.8727^{-4}	2.47072^{-1}
40077	40000	1.8754	5.5380	3.9168	1.85085	5.8511	2.46165
42500	42414	1.6700	4.9314	3.4878	1.64813	5.2103	2.19203
42587	42500	1.6630	4.9110	3.4733	1.64130	5.1887	2.18294
45000	44903	1.4816	4.3753	3.0945	1.46226	4.6227	1.94483
45097	45000	1.4748	4.3549	3.0801	1.45547	4.6012	1.93579
47500	47392	1.3146	3.8820	2.7456	1.29739	4.1015	1.72555
47608	47500	1.3078	3.8619	2.7314	1.29068	4.0803	1.71662
50000	49880	1.1664^{2}	3.4444^{0}	2.4361^{2}	1.15115^{-1}	3.6391^{-4}	1.53104^{-1}
50120	50000	1.1597	3.4246	2.4221	1.14455	3.6183	1.52226
52500	52368	1.0349	3.0562	2.1615	1.02141	3.2290	1.35849
52632	52500	1.0284	3.0369	2.1479	1.01496	3.2086	1.34991
55000	54855	9.1834^{1}	2.7119	1.9180	9.06327^{-2}	2.8652	1.20542
55145	55000	9.1197	2.6931	1.9047	9.00048	2.8453	1.19707
57500	57342	8.1489	2.4064	1.7019	8.04231	2.5412	1.06911
57659	57500	8.0872	2.3882	1.6890	7.98144	2.5232	1.06154
60000	59827	7.2311^{1}	2.1354^{0}	1.5103^{2}	7.13658^{-2}	2.2561^{-4}	9.49172^{-2}
60173	60000	7.1716	2.1178	1.4978	7.07778	2.2375	9.41352
70000	69766	4.4850	1.3244	9.3672^{1}	4.42637	1.3993	5.88712
70236	70000	4.4348	1.3096	9.2623	4.37684	1.3837	5.82124
80000	79694	2.7831	8.2183^{-1}	5.8125	2.74666	8.6831^{-5}	3.65308
80308	80000	2.7425	8.0985	5.7278	2.70659	8.5564	3.59980
82345	82021	2.4886	7.3488	5.1975	2.45605	7.7644	3.26657
90000	89613	1.7376	5.1312	3.6291	1.71492	5.2531	2.21004
90390	90000	1.7067	5.0397	3.5644	1.68434	5.1513	2.16721
100000	99523	1.1053^{1}	3.2640^{-1}	2.3085^{1}	1.09087^{-2}	3.2114^{-5}	1.35107^{-2}
100482	100000	1.0820	3.1951	2.2598	1.06784	3.1377	1.32007
110000	109423	7.1565^{0}	2.1133	1.4947	7.06294^{-3}	2.0014	8.42003^{-3}
110583	110000	6.9810	2.0615	1.4580	6.88969	1.9480	8.19559

ALTITUDE		PRESSURE				DENSITY	
Z,ft	H,ft'	P,mb	P,in Hg	P, $\frac{lbf}{ft^2}$	P/P_o	ρ, $\frac{lbf\,sec^2}{ft^4}$	ρ/ρ_o
120000	119313	4.7101^{0}	1.3909^{-1}	9.8372^{0}	4.64848^{-3}	1.2697^{-5}	5.34178^{-3}
120695	120000	4.5779	1.3518	9.5611	4.51799	1.2310	5.17886
130000	129195	3.1474	9.2943^{-2}	6.5735	3.10626	8.1894^{-6}	3.44540
130815	130000	3.0476	8.9995	6.3650	3.00773	7.9072	3.32668
140000	139066	2.1332	6.2992	4.4552	2.10527	5.3640	2.25672
140946	140000	2.0575	6.0759	4.2972	2.03062	5.1574	2.16980
150000	148929	1.4650^{0}	4.3261^{-2}	3.0597^{0}	1.44582^{-3}	3.5642^{-6}	1.49952^{-3}
151087	150000	1.4074	4.1561	2.9395	1.38902	3.4122	1.43555
155348	154199	1.2044	3.5566	2.5155	1.18866	2.8803	1.21179
160000	158782	1.0173	3.0041	2.1247	1.00401	2.4329	1.02355
161237	160000	9.7267^{-1}	2.8723	2.0315	9.59953^{-4}	2.3261	9.78631^{-4}
170000	168626	7.0788	2.0904	1.4784	6.98625	1.6929	7.12218
171397	170000	6.7293	1.9872	1.4054	6.64128	1.6093	6.77051
175346	173885	5.8320	1.7222	1.2180	5.75573	1.3947	5.86773
180000	178460	4.9193	1.4527	1.0274	4.85495	1.1995	5.04652
181567	180000	4.6418	1.3707	9.6947^{-1}	4.58115	1.1394	4.79357
190000	188285	3.3740	9.9635^{-3}	7.0468	3.32989	8.5890^{-7}	3.61352
191747	190000	3.1537	9.3129	6.5866	3.11246	8.0903	3.40370
200000	198100	2.2752^{-1}	6.7186^{-3}	4.7518^{-1}	2.24543^{-4}	6.0583^{-7}	2.54879^{-4}
201937	200000	2.1047	6.2153	4.3958	2.07722	5.6545	2.37891
210000	207907	1.5079	4.4528	3.1493	1.48816	4.2082	1.77043
212136	210000	1.3775	4.0676	2.8769	1.35944	3.8841	1.63408
220000	217704	9.7927^{-2}	2.8918	2.0452	9.66460^{-5}	2.8710	1.20785
222345	220000	8.8224	2.6053	1.8426	8.70705	2.6175	1.10121
230000	227491	6.2217	1.8373	1.2994	6.14034	1.9210	8.08180^{-5}
232565	230000	5.5173	1.6293	1.1523	5.44519	1.7270	7.26582
240000	237270	3.8580	1.1393	8.0576^{-2}	3.80754	1.2580	5.29239
242794	240000	3.3599	9.9218^{-4}	7.0173	3.31596	1.1130	4.68242
249001	246063	2.452	7.241	5.121	2.4200	8.420^{-8}	3.5423
250000	247039	2.329^{-2}	6.877^{-4}	4.864^{-2}	2.2983^{-5}	7.996^{-8}	3.3641^{-5}
253033	250000	1.991	5.880	4.158	1.9650	6.837	2.8764
260000	256799	1.390	4.104	2.902	1.3715	4.772	2.0075
263282	260000	1.173	3.464	2.450	1.1578	4.028	1.6948
270000	266549	8.297^{-3}	2.450	1.733	8.1880^{-6}	2.849	1.1986
273541	270000	6.912	2.041	1.444	6.8219	2.374	9.9857^{-6}
280000	276291	4.956	1.463	1.035	4.8909	1.702	7.1592
283810	280000	4.073	1.203	8.506^{-3}	4.0195	1.399	5.8837
290000	286023	2.962	8.746^{-5}	6.185	2.9229	1.017	4.2784
294089	290000	2.400	7.086	5.012	2.3683	8.240^{-9}	3.4667
299516	295276	1.815	5.361	3.791	1.7916	6.234	2.6225

ENGLISH TABLE II CONTINUED

ALTITUDE		PRESSURE				DENSITY	
Z, ft	H, ft'	P, mb	P, in Hg	P, $\frac{lbf}{ft^2}$	P/P_o	$\rho, \frac{lbf sec^2}{ft^4}$	ρ/ρ_o
300000	295746	1.771^{-3}	5.229^{-5}	3.698^{-3}	1.7477^{-6}	6.065^{-9}	2.5517^{-6}
304378	300000	1.418	4.188	2.962	1.4000	4.749	1.9979
325000	320013	5.317^{-4}	1.570	1.110	5.2474^{-7}	1.610	6.7730^{-7}
330145	325000	4.225	1.248	8.824^{-4}	4.1697	1.250	5.2568
350000	344223	1.826	5.393^{-6}	3.814	1.8024	4.957^{-10}	2.0853
355974	350000	1.439	4.248	3.005	1.4199	3.810	1.6030
375000	368376	6.987^{-5}	2.063	1.459	6.8952^{-8}	1.718	7.2292^{-8}
381866	375000	5.453	1.610	1.139	5.3818	1.308	5.5011
400000	392473	2.919^{-5}	8.619^{-7}	6.096^{-5}	2.8807^{-8}	6.565^{-11}	2.7620^{-8}
407822	400000	2.257	6.664	4.713	2.2270	4.943	2.0797
421745	413386	1.451	4.285	3.030	1.4320	3.038	1.2781
425000	416512	1.314	3.879	2.744	1.2966	2.672	1.1240
433841	425000	1.017	3.003	2.124	1.0036	1.919	8.0726^{-9}
450000	440495	6.661^{-6}	1.967	1.391	6.5740^{-9}	1.111	4.6718
459924	450000	5.262	1.554	1.099	5.1933	8.187^{-12}	3.4445
475000	464422	3.785	1.118	7.906^{-6}	3.7358	5.348	2.2501
486071	475000	3.030	8.946^{-8}	6.327	2.9900	4.010	1.6872
500000	488293	2.334^{-6}	6.892^{-8}	4.875^{-6}	2.3034^{-9}	2.862^{-12}	1.2043^{-9}
512282	500000	2.052	6.058	4.285	2.0247	2.363	9.9417^{-10}
550000	535868	1.051	3.103	2.195	1.0371	1.020	4.2928
564897	550000	8.560^{-7}	2.528	1.788	8.4481^{-10}	7.827^{-13}	3.2930
590401	574147	6.189	1.828	1.293	6.1085	5.147	2.1655
600000	583221	5.516^{-7}	1.629^{-8}	1.152^{-6}	5.4443^{-10}	4.499^{-13}	1.8927^{-10}
617773	600000	4.485	1.324	9.366^{-7}	4.4258	3.531	1.4854
650000	630354	3.138	9.266^{-9}	6.553	3.0967	2.325	9.7822^{-11}
670910	650000	2.518	7.435	5.259	2.4849	1.797	7.5617
700000	677268	1.879^{-7}	5.550^{-9}	3.925^{-7}	1.8548^{-10}	1.277^{-13}	5.3707^{-11}
724311	700000	1.489	4.395	3.109	1.4690	9.718^{-14}	4.0885
750000	723965	1.175	3.471	2.455	1.1599	7.372	3.1014
777977	750000	9.187^{-8}	2.713	1.919	9.0665^{-11}	5.526	2.3249
800000	770446	7.624^{-8}	2.251^{-9}	1.592^{-7}	7.5340^{-11}	4.443^{-14}	1.8692^{-11}
831911	800000	5.881	1.737	1.228	5.8041	3.280	1.3798
850000	816714	5.103	1.507	1.066	5.0364	2.778	1.1688
886115	850000	3.885	1.147	8.114^{-8}	3.8342	2.019	8.4955^{-12}
900000	862768	3.511^{-8}	1.037^{-9}	7.332^{-8}	3.4646^{-11}	1.794^{-14}	7.5456^{-12}
940590	900000	2.637	7.788^{-10}	5.509	2.6029	1.284	5.4002
950000	908611	2.473	7.304	5.166	2.4410	1.191	5.0094
995339	950000	1.834	5.415	3.830	1.8098	8.391^{-15}	3.5301

ENGLISH TABLE II CONTINUED

ALTITUDE		PRESSURE				DENSITY	
Z,ft	H,ft'	P,mb	P,in Hg	P, $\frac{lbf}{ft^2}$	P/P_o	ρ, $\frac{lbfsec^2}{ft^4}$	ρ/ρ_o
1000000	954245	1.780^{-8}	5.256^{-10}	3.717^{-8}	1.7565^{-11}	8.103^{-15}	3.4089^{-12}
1050364	1000000	1.302	3.845	2.720	1.2852	5.622	2.3653
1100000	1044889	9.732^{-9}	2.874	2.033	9.6045^{-12}	3.999	1.6824
1161249	1100000	6.934	2.048	1.448	6.8436	2.690	1.1317
1200000	1134710	5.655	1.670	1.181	5.5814	2.119	8.9162^{-13}
1273262	1200000	3.925	1.159	8.197^{-9}	3.8733	1.382	5.8154
1300000	1223721	3.455	1.020	7.217	3.4102	1.191	5.0107
1386421	1300000	2.336	6.897^{-11}	4.878	2.3050	7.532^{-16}	3.1689
1400000	1311932	2.202	6.502	4.598	2.1730	7.030	2.9576
1500000	1399354	1.454^{-9}	4.293^{-11}	3.037^{-9}	1.4349^{-12}	4.326^{-16}	1.8201^{-13}
1500743	1400000	1.450	4.281	3.028	1.4306	4.311	1.8138
1600000	1485997	9.899^{-10}	2.921	2.068	9.7699^{-13}	2.760	1.1610
1616246	1500000	9.324	2.754	1.947	9.2025	2.573	1.0825
1700000	1571872	6.922	2.044	1.446	6.8319	1.816	7.6404^{-14}
1732949	1600000	6.185	1.827	1.292	6.1044	1.592	6.6976
1780465	1640420	5.281	1.559	1.103	5.2116	1.323	5.5666
1850870	1700000	4.214	1.244	8.802^{-10}	4.1591	1.016	4.2754

ENGLISH TABLE III

SOUND SPEED, VISCOSITY, AND KINEMATIC VISCOSITY AS FUNCTIONS OF GEOMETRIC AND GEOPOTENTIAL ALTITUDE

ALTITUDE		SOUND SPEED		VISCOSITY		KINEMATIC VISCOSITY	
Z, ft	H, ft'	$C_s, \frac{ft}{sec}$	C_s/C_{so}	$\mu, \frac{lbf\ sec}{ft^2}$	μ/μ_o	$\eta, \frac{ft^2}{sec}$	η/η_o
-15000	-15011	1172.6	1.05034	4.0298^{-7}	1.07828	1.1162^{-4}	$.709872^{-1}$
-14989	-15000	1172.6	1.05030	4.0296	1.07822	1.1164	.710040
-12500	-12508	1163.5	1.04211	3.9820	1.06548	1.1792	.749997
-12493	-12500	1163.4	1.04209	3.9819	1.06544	1.1794	.750122
-10000	-10005	1154.2	1.03382	3.9338	1.05258	1.2469	.793037
-9995.2	-10000	1154.2	1.03381	3.9337	1.05256	1.2471	.793123
-7500	-7502.7	1144.9	1.02547	3.8853	1.03959	1.3196	.839253
-7497.3	-7500	1144.9	1.02546	3.8852	1.03958	1.3197	.839305
-5000	-5001.2	1135.5	1.01705	3.8363	1.02650	1.3977	.888933
-4998.8	-5000	1135.5	1.01704	3.8363	1.02649	1.3977	.888958
-2500	-2500.3	1126.0	1.00856	3.7870	1.01330	1.4818	.942397
-2499.7	-2500	1126.0	1.00856	3.7870	1.01330	1.4818	.942403
0	0	1116.4	1.00000	3.7373^{-7}	1.00000	1.5723^{-4}	1.00000^{0}
2500	2499.7	1106.8	.991370	3.6872	.986591	1.6700	1.06214
2500.3	2500	1106.8	.991369	3.6872	.986589	1.6700	1.06215
5000	4998.8	1097.1	.982666	3.6367	.973073	1.7756	1.12925
5001.2	5000	1097.1	.982661	3.6366	.973066	1.7756	1.12928
7500	7497.3	1087.3	.973886	3.5857	.959443	1.8897	1.20183
7502.7	7500	1087.3	.973876	3.5857	.959428	1.8898	1.20191
10000	9995.2	1077.4	.965028	3.5344^{-7}	.945700	2.0132^{-4}	1.28042^{0}
10005	10000	1077.4	.965011	3.5343	.945673	2.0135	1.28058
12500	12492	1067.4	.956091	3.4826	.931840	2.1472	1.36564
12508	12500	1067.4	.956064	3.4824	.931798	2.1477	1.36591
15000	14989	1057.4	.947071	3.4303	.917861	2.2928	1.45819
15011	15000	1057.3	.947032	3.4301	.917801	2.2934	1.45861
17500	17485	1047.2	.937967	3.3776	.903762	2.4510	1.55883
17515	17500	1047.1	.937913	3.3773	.903679	2.4520	1.55945
20000	19981	1036.9	.928776	3.3245^{-7}	.889539	2.6234^{-4}	1.66846^{0}
20019	20000	1036.8	.928705	3.3241	.889429	2.6247	1.66933
22500	22476	1026.6	.919495	3.2708	.875190	2.8114	1.78804
22524	22500	1026.5	.919405	3.2703	.875050	2.8133	1.78926
25000	24970	1016.1	.910122	3.2167	.860712	3.0169	1.91872
25030	25000	1016.0	.910009	3.2161	.860537	3.0194	1.92036
27500	27464	1005.5	.900654	3.1621	.846102	3.2418	2.06176
27536	27500	1005.4	.900515	3.1613	.845889	3.2452	2.06393

ENGLISH TABLE III CONTINUED

ALTITUDE		SOUND SPEED		VISCOSITY		KINEMATIC VISCOSITY	
Z, ft	H, ft'	$C_s, \frac{ft}{sec}$	C_s/C_{so}	$\mu, \frac{lbf\ sec}{ft^2}$	μ/μ_o	$\eta, \frac{ft^2}{sec}$	η/η_o
30000	29957	994.85	.891087	3.1070^{-7}	.831358	3.4884^{-4}	2.21861^{0}
30043	30000	994.66	.890921	3.1061	.831102	3.4929	2.22145
32500	32449	984.05	.881419	3.0514	.816477	3.7593	2.39093
32551	32500	983.83	.881221	3.0503	.816173	3.7651	2.39459
35000	34941	973.14	.871646	2.9953	.801455	4.0576	2.58060
35059	35000	972.89	.871414	2.9940	.801100	4.0649	2.58529
36152	36089	968.08	.867107	2.9692	.794486	4.2051	2.67441
37500	37433	968.08	.867107	2.9692	.794486	4.4855	2.85280
37568	37500	968.08	.867107	2.9692	.794486	4.5001	2.86204
40000	39923	968.08	.867107	2.9692^{-7}	.794486	5.0560^{-4}	3.21561^{0}
40077	40000	968.08	.867107	2.9692	.794486	5.0746	3.22746
42500	42414	968.08	.867107	2.9692	.794486	5.6988	3.62443
42587	42500	968.08	.867107	2.9692	.794486	5.7225	3.63953
45000	44903	968.08	.867107	2.9692	.794486	6.4232	4.08513
45097	45000	968.08	.867107	2.9692	.794486	6.4532	4.10420
47500	47392	968.08	.867107	2.9692	.794486	7.2394	4.60426
47608	47500	968.08	.867107	2.9692	.794486	7.2771	4.62821
50000	49880	968.08	.867107	2.9692^{-7}	.794486	8.1592^{-4}	5.18921^{0}
50120	50000	968.08	.867107	2.9692	.794486	8.2062	5.21912
52500	52368	968.08	.867107	2.9692	.794486	9.1955	5.84830
52632	52500	968.08	.867107	2.9692	.794486	9.2539	5.88548
55000	54855	968.08	.867107	2.9692	.794486	1.0363^{-3}	6.59093
55145	55000	968.08	.867107	2.9692	.794486	1.0435	6.63691
57500	57342	968.08	.867107	2.9692	.794486	1.1684	7.43128
57659	57500	968.08	.867107	2.9692	.794486	1.1768	7.48428
60000	59828	968.08	.867107	2.9692^{-7}	.794486	1.3161^{-3}	8.37031^{0}
60173	60000	968.08	.867107	2.9692	.794486	1.3270	8.43984
70000	69766	968.08	.867107	2.9692	.794486	2.1219	1.34953^{+1}
70236	70000	968.08	.867107	2.9692	.794486	2.1459	1.36481
80000	79694	968.08	.867107	2.9692	.794486	3.4196	2.17484
80308	80000	968.08	.867107	2.9692	.794486	3.4702	2.20703
82345	82021	968.08	.867107	2.9692	.794486	3.8242	2.43217
90000	89613	983.46	.880889	3.0484	.815663	5.8030	3.69071
90390	90000	984.24	.881586	3.0524	.816734	5.9255	3.76860
100000	99523	1003.2	.898561	3.1501^{-7}	.842875	9.8091^{-3}	6.23857^{+1}
100482	100000	1004.1	.899403	3.1549	.844174	1.0055^{-2}	6.39491
110000	109423	1022.5	.915875	3.2499	.869596	1.6239	1.03277^{+2}
110583	110000	1023.6	.916874	3.2557	.871140	1.6713	1.06294

ENGLISH TABLE III CONTINUED

ALTITUDE		SOUND SPEED		VISCOSITY		KINEMATIC VISCOSITY	
Z, ft	H, ft'	$C_s, \frac{ft}{sec}$	C_s/C_{so}	$\mu, \frac{lbf\ sec}{ft^2}$	μ/μ_o	$\eta, \frac{ft^2}{sec}$	η, η_o
120000	119313	1041.5	.932851	3.3480^{-7}	.895844	2.6369^{-2}	1.67705^{+2}
120695	120000	1042.8	.934018	3.3548	.897650	2.7253	1.73330
130000	129195	1060.1	.949509	3.4444	.921638	4.2060	2.67498
130815	130000	1061.6	.950854	3.4522	.923722	4.3659	2.77671
140000	139066	1078.3	.965864	3.5392	.946996	6.5980	4.19634
140946	140000	1080.0	.967396	3.5481	.949373	6.8796	4.37539
150000	148929	1096.3	.981931	3.6324^{-7}	.971932	1.0191^{-1}	6.48164^{+2}
151087	150000	1098.2	.983660	3.6424	.974617	1.0675	6.78916
155348	154199	1105.7	.990411	3.6816	.985101	1.2782	8.12933
160000	158782	1105.7	.990411	3.6816	.985101	1.5133	9.62436
161237	160000	1105.7	.990411	3.6816	.985101	1.5827	1.00661^{+3}
170000	168626	1105.7	.990411	3.6816	.985101	2.1748	1.38315
171397	170000	1105.7	.990411	3.6816	.985101	2.2877	1.45499
175346	173885	1105.7	.990411	3.6816	.985101	2.6397	1.67885
180000	178460	1095.1	.980836	3.6260	.970232	3.0229	1.92258
181567	180000	1091.4	.977592	3.6072	.965196	3.1659	2.01352
190000	188285	1071.7	.959953	3.5049	.937828	4.0807	2.59533
191747	190000	1067.6	.956260	3.4835	.932102	4.3058	2.73850
200000	198100	1047.9	.938604	3.3813^{-7}	.904748	5.5813^{-1}	3.54971^{+3}
201937	200000	1043.3	.934442	3.3572	.898305	5.9373	3.77613
210000	207907	1023.6	.916824	3.2554	.871062	7.7360	4.92007
212136	210000	1018.3	.912102	3.2282	.863768	8.3113^{0}	5.28597
220000	217704	998.67	.894512	3.1268	.836634	1.0891	6.92666
222345	220000	992.74	.889200	3.0962	.828453	1.1829	7.52308
230000	227491	973.15	.871651	2.9953	.801463	1.5593	9.91689
232565	230000	966.50	.865694	2.9611	.792318	1.7146	1.09047^{+4}
240000	237270	946.96	.848196	2.8609	.765510	2.2743	1.44644
242794	240000	939.52	.841530	2.8229	.755320	2.5363	1.61310
249001	246063	922.8	.82654	2.737	.73245	3.251	2.0677
250000	247039	922.8	.82654	2.737^{-7}	.73245	3.423^{0}	2.1772^{+4}
253033	250000	922.8	.82654	2.737	.73245	4.004	2.5464
260000	256799	922.8	.82654	2.737	.73245	5.737	3.6485
263282	260000	922.8	.82654	2.737	.73245	6.795	4.3218
270000	266549	922.8	.82654	2.737	.73245	9.609	6.1111
273541	270000	922.8	.82654	2.737	.73245	1.153^{+1}	7.3349
280000	276291	922.8	.82654	2.737	.73245	1.609	1.0231^{+5}
283810	280000	922.8	.82654	2.737	.73245	1.957	1.2449
290000	286023	922.8	.82654	2.737	.73245	2.692	1.7120
294089	290000	922.8	.82654	2.737	.73245	3.322	2.1128
299516	295276	922.8	.82654	2.737	.73245	4.391	2.7929

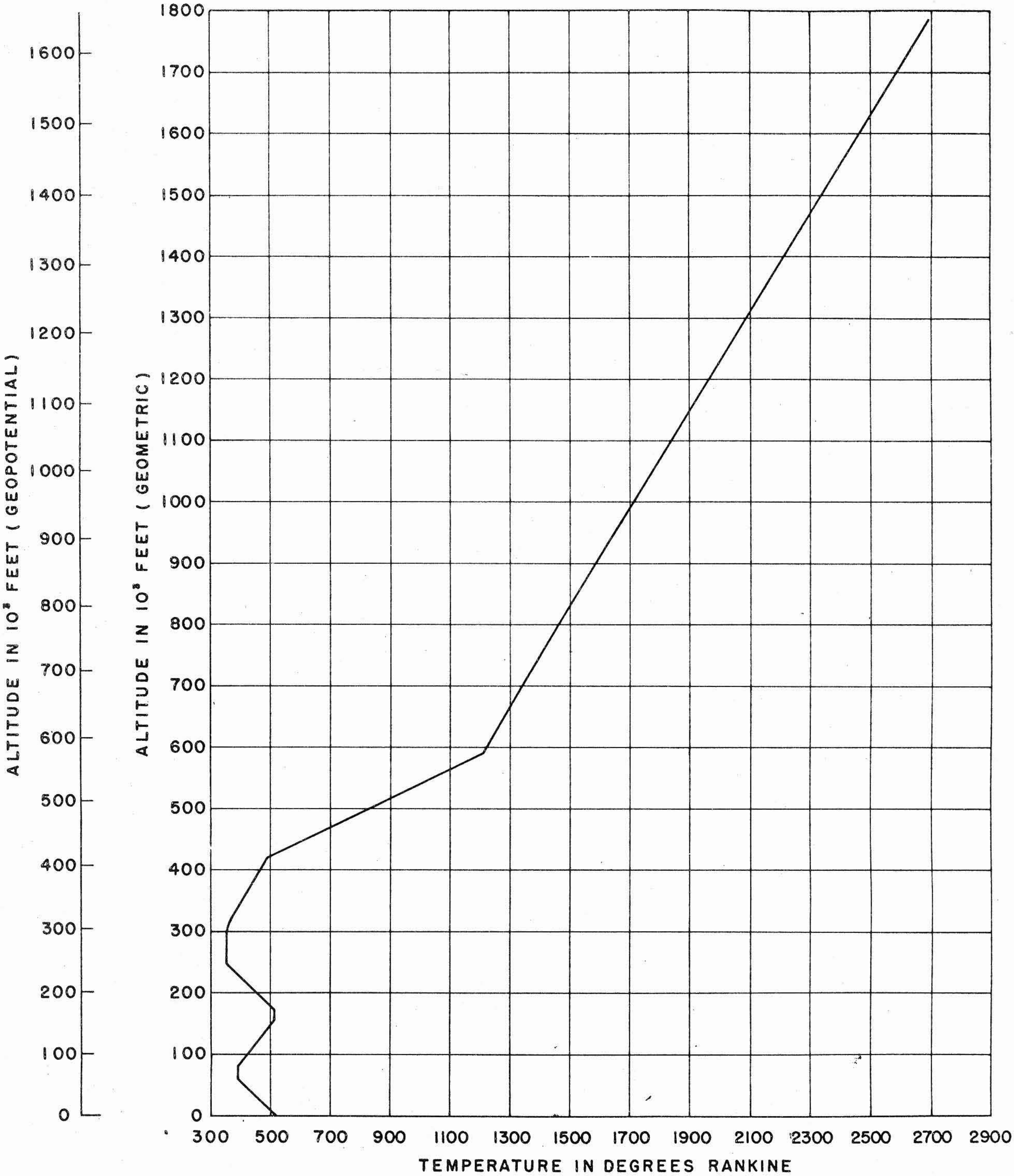


FIGURE 15. KINETIC TEMPERATURE VS GEOMETRIC ALTITUDE

GRD
DEC 1956

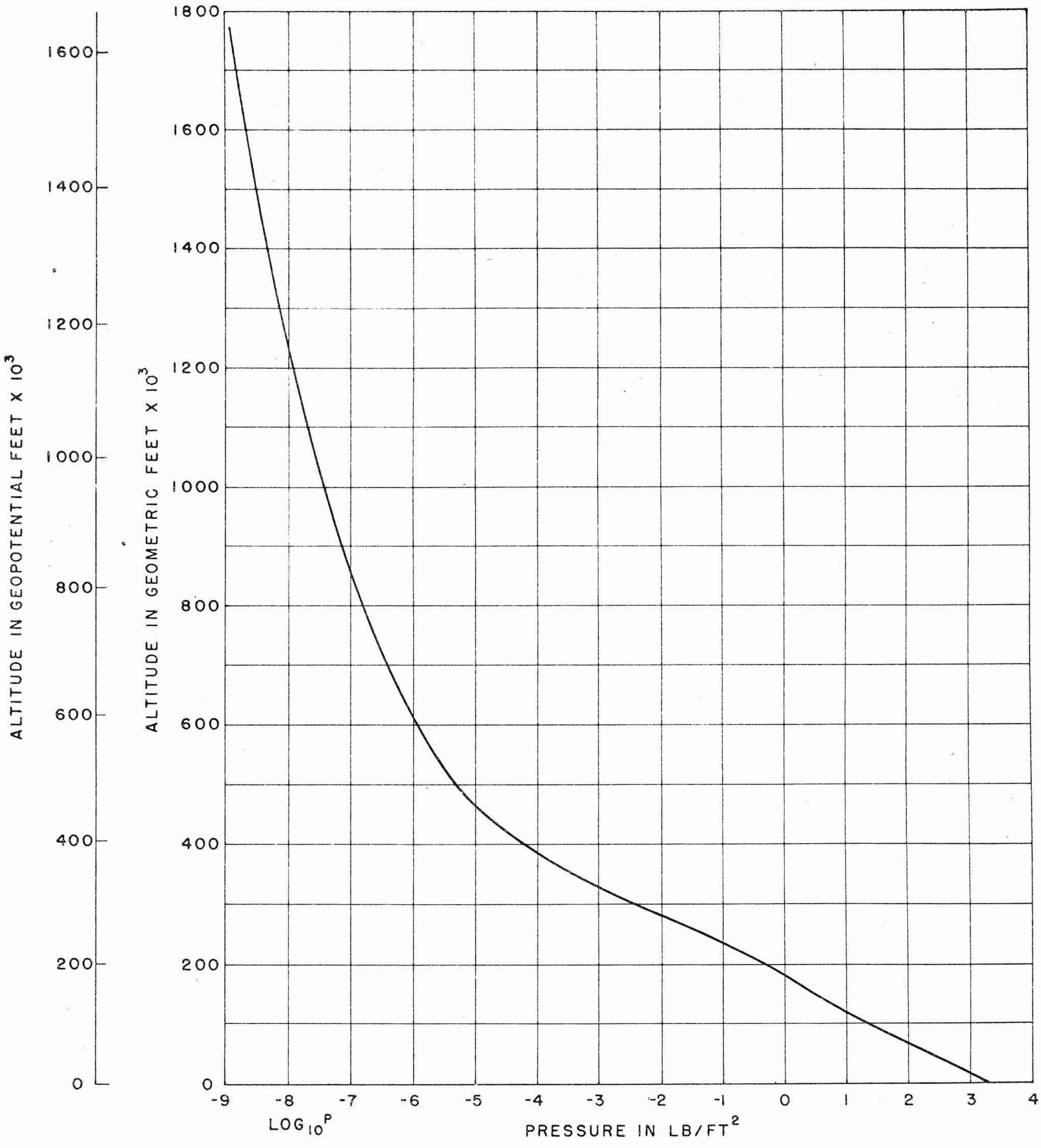


Figure 16. PRESSURE VS GEOMETRIC ALTITUDE

GRD
OCT. 1956

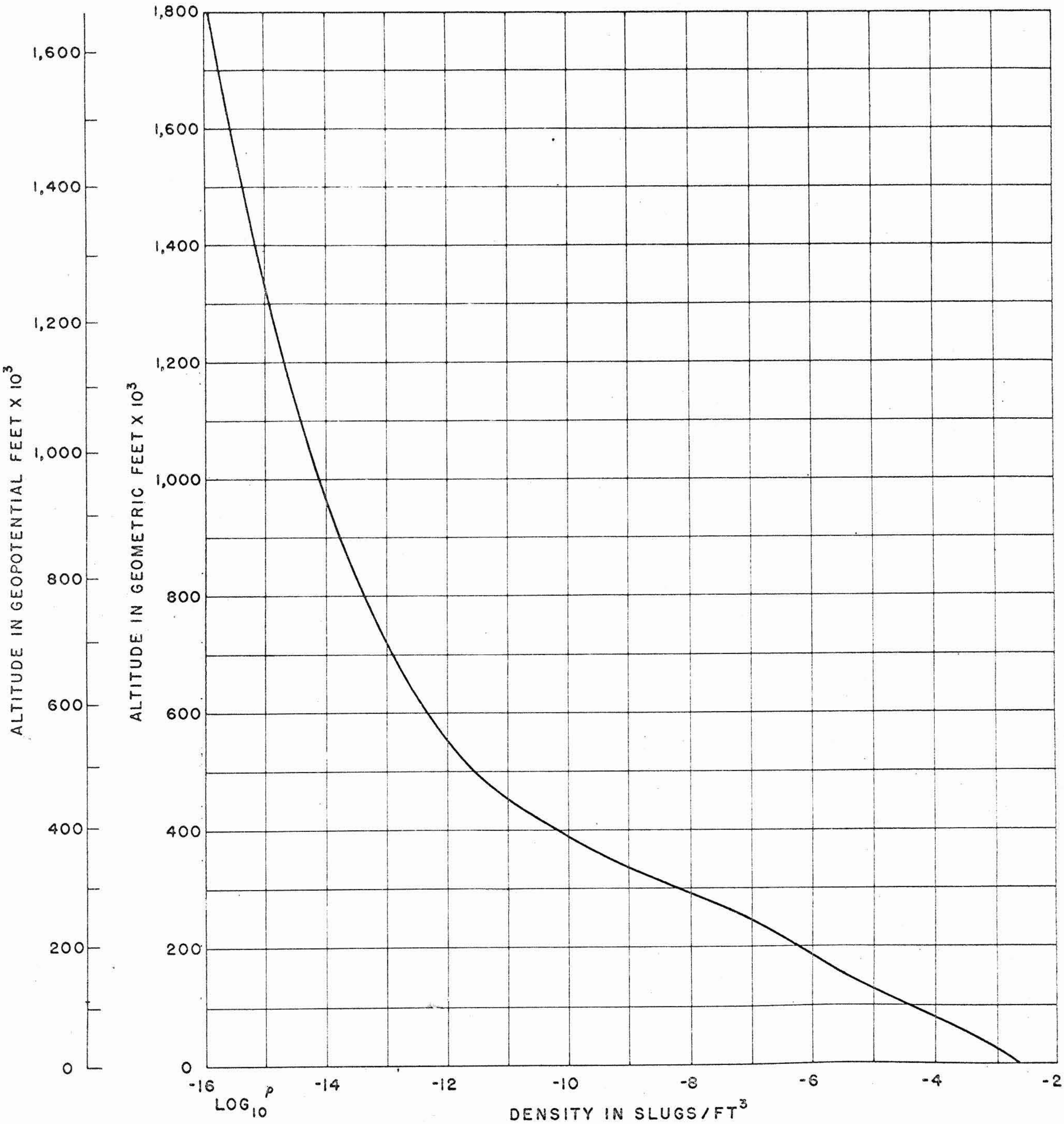


FIGURE 17. DENSITY VS GEOMETRIC ALTITUDE

GRD
SEPT. 1956

APPENDIX A

COMPARISON OF PROMINENT AERONAUTICAL STANDARD ATMOSPHERES

Constants Employed

Properties	Dimensions	Toussaint 1919 France	Gregg 1922 U.S.	ICAN 1924 Internat.	Diehl 1925 U.S.	Warfield 1947 U.S.	ICAO 1952 U.S. and Internat.	Minzner 1956 U.S.
P_o	mm			760	760		760	760
P_o	mb		1013.3	1013.2	1013.25	1013.25	1013.250	1013.250
P_o	kg m^{-3}	1.225	1.225	1.2256	1.2255	1.2255	1.2250	1.2250
$\frac{P_o M_o}{R^*}$	$\frac{°K\ kg}{m^3}$	352.8	352.8	352.969	352.945		353.000	353.000
R	$\frac{joules}{°K\ kg}$		2.8720	2.8705	2.87084		2.8704	2.8704
R^*	$\frac{joules}{°K\ kg}$							8.31439
T_o	°C				15.	15.	15.	15.
T_1	°K					273.	273.16	273.16
M_o						28.966	28.966	28.966
$(C_s)_o$	$\frac{m}{sec}$					340.22	340.43	340.292
γ						1.4	1.401119	1.4
S	°K					120	120	110.4
β	$\frac{kg}{sec\ m\ (°K)^{1/2}}$					1.488,82 x 10^{-5}	1.496,26 x 10^{-5}	1.458 x 10^{-5}
r	m					6,367,623		6,356,766

Sea Level Atmospheric Composition, Major Constituents by Per Cent

Constituent	Toussaint 1919 France	Gregg 1922 U.S.	ICAN 1924 Internat.	Diehl 1925 U.S.	Warfield 1947 U.S.	ICAO 1952 U.S. and Internat.	Minzner 1956 U.S.
H_2O			0.0	0.0		0.0	0.0
N_2			78.03			78.09	78.09
O_2			20.99			20.95	20.95
A			.94			.93	.93
CO_2			.04			.03	.03

APPENDIX A CONTINUED

COMPARISON OF PROMINENT AERONAUTICAL STANDARD ATMOSPHERES

Temperature-Altitude Profiles

	Toussaint 1919 France		Gregg 1922 U.S.		ICAN 1924 Internat.		Diehl 1925 U.S.		Warfield 1947 U.S.		ICAO 1952 U.S. and Internat.		Minzner 1956 U.S.	
Altitude km or km'	t	α	t	α	t	α	t	α	t	α	t	α'	t_M	L_M
0	15		15		15		15				15		15	
2	2		2		2		2				2		2	
4	-11		-11		-11		-11				-11		-11	
		-6.5		-6.5		-6.5		-6.5				-6.5		-6.5
6	-24		-24		-24		-24				-24		-24	
8	-37		-37		-37		-37				-37		-37	
10	-50		-50		-50		-50				-50		-50	
10.769,23							-55							
11	-57				-56.5		-55	0			-56.5		-56.5	
		0				0						0		
20	-57				-56.5		-55		-55		-56.5		-56.5	0
25									-55	0			-56.5	
32									-55				-35.5	+3
47										$+7\frac{1}{3}$			+9.5	
50									+77					0
53										0			+9.5	
60									+77					-3.9
75										$-6\frac{1}{9}$			-76.3	
78									-33					0
83									-33	0				
90										$+3\frac{24}{37}$			-76.3	
120									+102					+3.5
126													49.7	+10
175													539.7	
300													1264.7	+5.8
500													2424.7	

Footnotes:
t is in °C
α is in °C km^{-1}

t_M is in °C
α' is in °C km'^{-1}
L_M is in °C km'^{-1}

$t_M = \frac{t}{M} \cdot M_o$

Below 90 km', $t_M = t$

APPENDIX B

Constants

Defined Independent Physical Constants Adopted as Being Exact

mks absolute units	cgs units
g_o = 9.806,65 m sec^{-1}	980.665 cm-sec^{-1}
M_o = 28.966 (dimensionless)	28.966 (dimensionless)
N = 6.023,80 x 10^{26} (dimensionless) (for a kg-mol)	6.023,80 x 10^{23} (dimensionless) (for a gm-mol)
P_o = 1.013,250 x 10^5 nt m^{-2} or 0.76 m of mercury	1.013,250 x 10^6 dynes cm^{-2} or 76.0 cm of mercury
R* = 8.314,39 x 10^3 joules $(°K)^{-1}$ kg^{-1}	8.314,39 x 10^7 ergs $(°K)^{-1}$ gm^{-1}
r = 6.356,766 x 10^6 m	6.356,766 x 10^8 cm
S = 110.4°K	110.4°K
T_i = 273.16°K	273.16°K
t_o = 15°C	15°C
β = 1.458 x 10^{-6} kg sec^{-1} $m^{-1}(°K)^{-\frac{1}{2}}$	1.458 x 10^{-5} gm sec^{-1} $cm^{-1}(°K)^{-\frac{1}{2}}$ [or poise $(°K)^{-1/2}$]
γ = 1.4 (dimensionless)	1.4 (dimensionless)
σ = 3.65 x 10^{-10} m	3.65 x 10^{-8} cm

English Units

t_i = 32°F

Numerical Constants (not exact)

Log_{10} e = .434,294,481,9

π = 3.141,592,654

$\sqrt{2}$ = 1.414,213,562

APPENDIX C

Conversions

Defined and Derived Conversion Factors
for
Transformation of Units and Scales

1. Metric to English Conversions and Vice Versa

a. Defined relations

1 foot	= 0.304,8 meter (exact)
1 international nautical mile	= 1,852 meters (exact)
1 pound	= 0.453,592,3 kilogram (exact)

b. Derived relations

1 meter	= 3.280,839,895,013 feet
1 meter	= $5.399,568,034,557 \times 10^{-4}$ international nautical miles (i n mi)
1 kilogram	= 2.204,622,962,070 pounds
1 (i n mi)	= 6,076,115,48 feet
1 foot	= $1.645,788,33 \times 10^{-4}$ (i n mi)

c. Conversion factors

$$1 = 0.304,8 \text{ m ft}^{-1}$$

$$1 = 1,852 \text{ m (i n mi)}^{-1}$$

$$1 = .453,592,3 \text{ kg lb}^{-1}$$

$$1 = \frac{1,852.}{.304,8} \text{ ft (i n mi)}^{-1}$$

2. Geometric Altitude to Geopotential Altitude

a. Defined relations

1 standard geopotential meter = 9.806,65 joules kg^{-1} (exact);

geopotential altitude, $H(m') = \frac{1}{G}\int g \, dZ$

$$\left.\begin{aligned} \text{where } G &= \frac{9.806,65 \text{ joules kg}^{-1}}{1 \text{ m}'} \\ &= 9.806,65 \text{ m}^2 \text{ sec}^{-2} \text{ m}'^{-1} \end{aligned}\right\} \text{(exact)}$$

b. Derived relations

1 standard geopotential foot = 0.304,8 standard geopotential meter (exact)

1 standard geopotential meter = 3.280,839,895,013 standard geopotential feet

c. Conversion factors

$1 = 0.304,8 \text{ m}' \text{ ft}'^{-1}$

3. Temperature Unit and Scale Conversions

a. Defined relations

$t(°C) = T(°K) - T_i(°K)$

where $T_i(°K) = 273.16°K$

$T(°R) = 1.8\ T(°K)$

$t(°F) - t_i(°F) = T(°R) - T_i(°R)$

where $t_i(°F) = 32(°F)$

b. Derived relations

$t_i(°C) = 0°C$

$T_i(°R) = 491.688°R$

$1°K = 1.8°R$ (in magnitude)

$1°C = 1°K$ (in magnitude)

$1°F = 1°R$ (in magnitude)

$t(°C) = [T(°R) - T_i(°R)]/1.8$

$t(°C) = [t(°F) - t_i(°F)]/1.8$

$T(°R) = 1.8[t(°C) + 273.16(°C)]$

$T(°R) = [t(°F) - t_i(°F)] + 491.688°R$

$t(°F) - 32°F = 1.8\ t(°C)$

$t(°F) - 32°F = 1.8\ [T(°K) - 273.16(°K)]$

c. Conversion factors

$1 = 1.8°R\ (°K)^{-1}$

4. Absolute Systems to Absolute-Force, Gravitational Systems

a. Defined relations

1 kilogram (force), kgf = $9.806,65 \text{ m sec}^{-2}$ x 1 kilogram (mass), kg.

1 pound (force), lbf = $\frac{9.806,65}{.3048} \text{ ft sec}^{-2}$ x 1 pound (mass), lb.

b. Derived relations

$$1 \text{ kgf sec}^2 \text{ m}^{-1} = 9.806,65 \times 1 \text{ kg}$$

$$1 \text{ slug} = 1 \text{ lbf sec}^2 \text{ ft}^{-1} = \frac{9.806,65}{.304,8} \times 1 \text{ lb}$$

$$1 \text{ lbf} = .453,592,3 \text{ kgf}$$

c. Conversion factors

$$1 = 9.806,65 \text{ m sec}^{-2} \text{ kg kgf}^{-1}$$

$$1 = \frac{9.806,65}{.304,8} \text{ ft sec}^{-2} \text{ lb lbf}^{-1}$$

$$1 = .453,592,3 \text{ kgf lbf}^{-1}$$

APPENDIX D

Assumptions

$$g = g_o \left(\frac{r}{r + Z} \right)^2$$

$$dP = -g \; \rho \; dZ$$

$$\rho = \frac{PM}{R^*T}$$

$$T_M = \left(\frac{T}{M} \right) M_o$$

$$(T_M)_o = 288.16°K$$

$$T_M = (T_M)_b + L_M (H - H_b)$$

where L_M is given by the following table

L_M in °K m'$^{-1}$	Altitude Layer in m'
-0.0065 exact	-5,000. to 0.
-0.0065 exact	0. to 11,000.
0.0 exact	11,000. to 25,000.
+0.0030 exact	25,000. to 47,000.
0.0 exact	47,000. to 53,000.
-0.0039 exact	53,000. to 75,000.
0.0 exact	75,000. to 90,000.
+0.0035 exact	90,000. to 126,000.
+0.0100 exact	126,000. to 175,000.
+0.0058 exact	175,000. to 500,000.

$$H_s = \frac{R^*T}{gM}$$

$$C_s = \left(\frac{\gamma P}{\rho} \right)^{1/2}$$

$$\bar{V} = \left(\frac{8R^*T}{\pi M} \right)^{1/2}$$

$$\omega = \rho \, g, \text{ (not } \rho \, g_o)$$

For $-5{,}000.\ m' \leq H \leq +\ 90{,}000.\ m'$

$$M = 28.966 \text{ (exact)}$$

For $90{,}000.\ m' \leq H \leq 175{,}000.\ m'$

$$M = \frac{23.160{,}126{,}7\ H - 1{,}757{,}856.047}{H - 78{,}726.253}$$

For $175{,}000.\ m' \leq H \leq 500{,}000.\ m'$

$$M = \frac{13.139{,}119{,}0\ H + 514{,}492.021}{H - 56{,}969.889}$$

$$v = \frac{M'}{\rho}$$

$$n = \frac{N}{v}$$

$$L = \frac{1}{\sqrt{2}\pi\sigma^2 n}$$

$$\nu = \frac{\bar{V}}{L}$$

$$T = T_M \left(\frac{M}{M_o} \right)$$

$$\mu = \frac{\beta \, T^{3/2}}{T + S}$$

$$\eta = \frac{\mu}{\rho}$$

APPENDIX E

Sea-Level Values of the Atmospheric Properties in Metric Units

	mks units	cgs units
$(C_s)_o$	340.292,046 m sec^{-1}	34,029.204,6 cm sec^{-1}
g_o	9.806,65 m sec^{-2}	980.665 cm sec^{-2}
$(H_s)_o$	8.434,413,43 x 10^3 m	8.434,413,43 x 10^5 cm
L_o	6.631,722,29 x 10^{-8} m	6.631,722,29 x 10^{-6} cm
M_o	28.966 (dimensionless, exact)	28.966 (dimensionless, exact)
M'_o	28.966 kg (exact)	28.966 gm (exact)
n_o	2.547,552,07 x 10^{25} m^{-3}	2.547,552,07 x 10^{19} cm^{-3}
P_o	101,325 nt m^{-2}	1,013,250. dynes cm^{-2}
P_o	.76 m Hg	.76 cm Hg
P_o	10,332.274,5 kgf m^{-2}	———
T_o	288.16°K	288.16°K
$(T_M)_o$	288.16°K (exact)	288.16°K (exact)
$\bar{V}_o$	458.942,035 m sec^{-1}	45,894.203,5 cm sec^{-1}
v_o	23.645,444,1 m^3 for a kg-mol	23,645.444,1 cm^3 for a gm-mol
η_o	1.460,741,29 x 10^{-5} m^2 sec^{-1}	1.460,741,29 x 10^{-1} cm^2 sec^{-1}
μ_o	1.789,428,53 x 10^{-5} kg m^{-1} sec^{-1}	1.789,428,53 x 10^{-4} gm cm^{-1} sec^{-1}
μ_o	1.824,709,28 x 10^{-6} kgf sec m^{-2}	———

Sea-Level Values of the Atmospheric Properties in Metric Units

	mks units	cgs units
ν_o	$6.920,404,89 \times 10^9$ sec^{-1}	$6.920,404,89 \times 10^9$ sec^{-1}
ρ_o	$1.225,013,99$ kg m^{-3}	$1.225,013,99 \times 10^{-3}$ gm cm^{-3}
ρ_o	$.124,916,663$ kgf sec^2 m^{-4}	________
ω_o	$12.013,283.5$ kg m^{-2} sec^{-2}	$1.201,328,35$ gm cm^{-2} sec^{-2}
ω_o	$1.225,014,00$ kgf m^{-3}	________

Ice-Point Values of Some Atmospheric Properties

	mks units	cgs units
n_i	$2.687,445,47 \times 10^{25}$ m^{-3}	$2.687,445,47 \times 10^{19}$ cm^{-3}
v_i	$22.414,594,4$ m^3 for a kg-mol	$22,414.596,4$ cm^3 for a gm-mol
ρ_i	$1.292,283,037$ kg m^{-3}	$1.292,283,037 \times 10^{-3}$ gm cm^{-3}

APPENDIX F

Sea-Level Values of the Atmospheric Properties in English Units

$(C_s)_o$	$1.116,443,72 \times 10^3$ ft sec^{-1}
g_o	32.174,048,5 ft sec^{-2}
$(H_s)_o$	$2.767,196,00 \times 10^4$ ft
L_o	$2.175,761,91 \times 10^{-7}$ ft
M_o	28.966
M'_o	28.966 lbs
n_o	$7.213,864,1 \times 10^{23}$ ft^{-3}
P_o	68,087.267 lb ft^{-1} sec^{-2}
P_o	29.921,259,8 in Hg
P_o	2,116.216,95 lbf ft^{-2}
T_o	518.688°R
$(T_M)_o$	518.688°R
$\bar{V}_o$	$1.505,715,34 \times 10^3$ ft sec^{-1}
v_o	83.503,098 ft^3
η_o	$1.572,328,83 \times 10^{-4}$ ft^2 sec^{-1}
μ_o	$1.202,440,64 \times 10^{-5}$ lb ft^{-1} sec^{-1}
μ_o	$3.737,299,76 \times 10^{-7}$ lbf sec ft^{-2}
ν_o	$6.920,404,9 \times 10^9$ sec^{-1}
ρ_o	.076,475,137 lb ft^{-3}
ρ_o	$2.376,919,99 \times 10^{-3}$ lbf sec^2 ft^{-4}
ω_o	2.460,514,77 lb ft^{-2} sec^{-2}
ω_o	$7.647,513,7 \times 10^{-2}$ lbf ft^{-3}

APPENDIX G

Abbreviated Metric Tables of the ARDC Model Atmosphere (1956) to 542,686 m

H	Z	L_M	T_M	T	M	P	ρ
m'	m	°K m'$^{-1}$	°K	°K		mb	kg m^{-3}
-5,000	-4,996.070,27		320.66	320.66	28.966	1.777,6 x 10^{3}	1.931,2
		-0.0065					
0	0		288.16	288.16	28.966	1.013,25 x 10^{3}	1.225,0
		-0.0065					
11,000	11,019.067,83		216.66	216.66	28.966	2.263,2 x 10^{2}	3.639,1 x 10^{-1}
		0.0000					
*20,000	20,063.123,68		216.66	216.66	28.966	5.474,8 x 10^{1}	8.803,4 x 10^{-2}
		0.0000					
25,000	25,098.708,63		216.66	216.66	28.966	2.488,6 x 10^{1}	4.001,6 x 10^{-2}
		+0.0030					
32,000	32,161.903,22		237.66	237.66	28.966	8.677,6 x 10^{0}	1.272,1 x 10^{-2}
		+0.0030					
47,000	47,350.092,22		282.66	282.66	28.966	1.204,4 x 10^{0}	1.484,5 x 10^{-3}
		0.0000					
53,000	53,445.606,64		282.66	282.66	28.966	5.832,0 x 10^{-1}	7.188,1 x 10^{-4}
		-0.0039					
75,000	75,895.448,82		196.86	196.86	28.966	2.452,1 x 10^{-2}	4.339 x 10^{-5}
		0.0000					
90,000	91,292.532,70		196.86	196.86	28.966	1.815,4 x 10^{-3}	3.213 x 10^{-6}
		+0.0035					
126,000	128,548.001,3		322.86	273.6	24.54	1.451,0 x 10^{-5}	1.566 x 10^{-8}
		+0.0100					
175,000	179,954.085,9		812.86	669.0	23.84	6.189,5 x 10^{-7}	2.655 x 10^{-10}
		+0.0058					
**300,000	314,859.415,0		1,537.86	973.5	18.34	1.447,3 x 10^{-8}	3.279 x 10^{-12}
		+0.0058					
500,000	542,685.673,2		2,697.86	1,489.	15.99	7.698 x 10^{-9}	6.819 x 10^{-14}

* Top of 1952 United States (ICAO) Standard Atmosphere
** Top of 1956 United States Standard Atmosphere

APPENDIX H

Abbreviated English Tables of the ARDC Model Atmosphere to 1,780,465 Ft.

H	Z	L_M	T_M	T	M	P	ρ
ft'	ft	°R ft^{-1}	°R	°R		lbf ft^{-2}	slugs ft^{-3}
-16,404.199	-16,391.307		577.188	577.188	28.966	3.711,00 x 10^{3}	3.745,7 x 10^{-3}
		-.003,566,160					
0	0		518.688	518.688	28.966	2.116,22 x 10^{3}	2.376,9 x 10^{-3}
		-.003,566,160					
36,089.239	36,151.797		389.988	389.988	28.966	4.726,8 x 10^{2}	7.061,1 x 10^{-4}
		zero					
*65,616.798	65,823.897		389.988	389.988	28.966	1.154,8 x 10^{2}	1.725,1 x 10^{-4}
		zero					
82,020.997	82,344.844		389.988	389.988	28.966	3.197,5 x 10^{1}	7.764,4 x 10^{-5}
		+.001,645,920					
104,986.877	105,518.055		427.788	427.788	28.966	1.812,4 x 10^{1}	2.468,2 x 10^{-5}
		+.001,645,920					
154,199.475	155,348.071		508.788	508.788	28.966	2.515,5 x 10^{0}	2.880,3 x 10^{-6}
		zero					
173,884.514	175,346.478		508.788	508.788	28.966	1.218,0 x 10^{0}	1.394,7 x 10^{-6}
		-.002,139,696					
246,062.992	249,000.816		354.348	354.348	28.966	5.121,2 x 10^{-2}	8.419,8 x 10^{-8}
		zero					
295,275.590	299,516.183		354.348	354.348	28.966	3.791,5 x 10^{-3}	6.233,5 x 10^{-9}
		+.001,920,240					
413,386.826	421,745.411		581.148	492.4	24.54	3.030,5 x 10^{-5}	3.038,0 x 10^{-11}
		+.005,486,400					
574,146.981	590,400.544		1,463.148	1,204.000	23.84	1.292,7 x 10^{-6}	5.147,1 x 10^{-13}
		+.003,182,112					
**984,251.968	1,033,003.330		2,768.148	1,752.000	18.3	3.022,8 x 10^{-8}	6.361,8 x 10^{-15}
		+.003,182,112					
1,640,419.947	1,780,464.807		4,856.148	2,681,000	15.990	1.102,9 x 10^{-9}	1.323,1 x 10^{-16}

* Top of 1952 United States (ICAO) Standard Atmosphere

** Top of 1956 United States Standard Atmosphere

APPENDIX J

SYSTEMS OF MECHANICAL UNITS

		METRIC				ENGLISH		
		Absolute cgs	Absolute mks	Gravitational mks		Absolute fps	Gravitational fps	
		1	2	3 Type I	4 Type II	5	6 Type I	7 Type II
Property	Dimensions	$F = ma$	$F = ma$	$F = ma$	$gF = ma$	$F = ma$	$F = ma$	$gF = ma$
length (altitude) (scale height) (mean free path)	ℓ	centimeter (cm)	meter (m)	meter (m)	meter (m)	foot (ft)	foot (ft)	foot (ft)
mass	m	gram (gm)	kilogram (kg)	*kgf sec^{2} m^{-1}	kilogram (kg)	pound (lb)	**slug **or lbf sec^{2} ft^{-1}	pound (lb)
time	t	second (sec)	second (sec)	second (sec)	second (sec)	second (sec)	second (sec)	second (sec)
force	$m \ell t^{-2}$	dyne or gm cm sec^{-2}	newton (nt) or kg m sec^{-2}	*kilogram force (kgf)	*kilogram force (kgf)	poundal (pdl)	**pound force (lbf)	**pound force (lbf)
area	ℓ^{2}	cm^{2}	m^{2}	m^{2}	m^{2}	ft^{2}	ft^{2}	ft^{2}
volume	ℓ^{3}	cm^{3}	m^{3}	m^{3}	m^{3}	ft^{3}	ft^{3}	ft^{3}
speed (sound)	ℓt^{-1}	cm sec^{-1}	m sec^{-1}	m sec^{-1}	m sec^{-1}	ft sec^{-1}	ft sec^{-1}	ft sec^{-1}
acceleration	ℓt^{-2}	cm sec^{-2}	m sec^{-2}	m sec^{-2}	m sec^{-2}	ft sec^{-2}	ft sec^{-2}	ft sec^{-2}
energy	$m \ell^{2} t^{-1}$	erg = dyne cm	joule = nt m	*kgf m	*kgf m	pdl ft	**lbf ft	**lbf ft
geopotential	$\ell^{2} t^{-2}$	ergs gm^{-1} or cm^{2} sec^{-2}	joules kg^{-1} or m^{2} sec^{-2}	m^{2} sec^{-2}	*kgf m kg^{-1} *or m^{2}sec^{-2}g^{-1}	pdl ft lb^{-1} or ft^{2} sec^{-2}	lbf ft slug^{-1} or ft^{2} sec^{-2}	**lbf ft lb **or ft^{2} sec^{-2} g^{-1}
pressure	$m \ell^{-1} t^{-2}$	dyne cm^{-2} = 10^{-1} mb	nt m^{-2} = 10^{-2} mb	*kgf m^{-2}	*kgf m^{-2}	pdl ft^{-2}	**lbf ft^{-2}	**lbf ft^{-2}
density	$m \ell^{-3}$	gm cm^{-3}	kg m^{-3}	*kgf sec^{2} m^{-4}	kg m^{-3}	lb ft^{-3}	**slugs ft^{-3} **or lbf sec^{2} ft^{-4}	lb ft^{-3}
specific weight	$m \ell^{-2} t^{-2}$	gm cm^{-2} sec^{-2}	kg m^{-2} sec^{-2}	*kgf m^{-3}	kg m^{-2} sec^{-2}	lb ft^{-2} sec^{-2}	**slugs ft^{-2} sec^{-2} **lbf ft^{-3}	lb ft^{-2} sec^{-2}
number density	ℓ^{-3}	cm^{-3}	m^{-3}	m^{-3}	m^{-3}	ft^{-3}	ft^{-3}	ft^{-3}
collision frequency	t^{-1}	sec^{-1}	sec^{-1}	sec^{-1}	sec^{-1}	sec^{-1}	sec^{-1}	sec^{-1}
viscosity	$m \ell^{-1} t^{-1}$	poise or gm cm^{-1} sec^{-1}	kg m^{-1} sec^{-1}	*kgf sec m^{-2}	kg m^{-1} sec^{-1}	lb ft^{-1} sec^{-1}	**slugs ft^{-1} sec^{-1} **or lbf sec ft^{-2}	lb ft^{-1} sec^{-1}
kinematic viscosity	$\ell^{2} t^{-1}$	cm^{2} sec^{-1}	m^{2} sec^{-1}	m^{2} sec^{-1}	m^{2} sec^{-1}	ft^{2} sec^{-1}	ft^{2} sec^{-1}	ft^{2} sec^{-1}
		used by physicists	used by electrical engineers and physicists	used by European aerodynamicists			used by American aerodynamicists	used by some mechanical engineers

*At sea level and at a latitude of 45° 32' 40" the numbers associated with these units will be only 1/9.80665 (exact) as large as numbers associated with corresponding units of system 2.#

**At sea level and at a latitude of 45° 32' 40" the numbers associated with these units will be only 1/32.174,048,55 as large as numbers associated with corresponding units of system 5.#

#For the absolute-force versions of gravitational units as used in this MODEL, the same ratio applies at all altitudes.

APPENDIX K

Comparison of the Magnitudes of Comparable Units in the Metric Absolute cgs and mks Systems of Mechanical Measure

length	1 m = 10^2 cm
mass	1 kg = 10^3 gm
time	1 sec = 1 sec
force	1 nt = 10^5 dynes
area	1 m^2 = 10^4 cm^2
volume	1 m^3 = 10^6 cm^3
speed (sound)	1 m sec^{-1} = 10^2 cm sec^{-1}
acceleration	1 m sec^{-2} = 10^2 cm sec^{-2}
energy (work)	1 nt m = 10^7 dynes cm 1 joule = 10^7 ergs
geopotential	1 joule kg^{-1} = 10^4 ergs gm^{-1} 1 m^2 sec^{-2} = 10^4 cm^2 sec^{-2}
pressure	1 nt m^{-2} = 10^1 dynes cm^{-2}
density	1 kg m^{-3} = 10^{-3} gm cm^{-3}
specific weight	1 kg m^{-2} sec^{-2} = 10^{-1} gm cm^{-2} sec^{-2}
number density	1 m^{-3} = 10^{-6} cm^{-3}
collision frequency	1 sec^{-1} = 1 sec^{-1}
coefficient of viscosity	1 newton sec m^{-2} = 10^1 dynes sec cm^{-2} 1 kg m^{-1} sec^{-1} = 10 gm cm^{-1} sec^{-1} = 10 poise
kinematic viscosity	1 m^2 sec^{-1} = 10^4 cm^2 sec^{-1}

Pressure in Terms of the Bar or Millibar (mb)

1 bar = 10^3 millibars (mb) = 10^5 nt m^{-2} = 10^6 dynes cm^{-2}

APPENDIX L

Atmospheric Density Expressed as a Single Function of Altitude

At a recent Ad Hoc Conference on Units and Constants for Satellite Orbit Computations, this MODEL was adopted as a basis for initial calculations of IGY satellite orbits. Dr. Jacchia[33] who had received a prepublication copy of the MODEL, prepared and presented the following equations as closely representing the atmospheric density of this MODEL above 100 km altitude.

$$\log_{10}\rho = -10.919 - 0.004483Z + 7.321e^{-0.00685Z} + 3.400e^{-0.8\left[\frac{Z}{100}\right]^3} \quad (1)$$

$$\log_{10}\rho = -11.019 - 0.00481H + 7.300e^{-0.0067H} + 3.700e^{-0.87\left[\frac{H}{100}\right]^3} \quad (2)$$

where ρ is the atmospheric density in kg/m^3, Z is the geometric height above sea level in km, and H is the geopotential height in geopotential km. A comparison of densities computed from these equations with densities from the ARDC Model are tabulated on the next page.

Residuals $\Delta \log_{10}\rho$ (ARDC Model Atmosphere densities minus interpolating formula) are given in the following table:

Z (km)	$\log_{10}\rho$ (kg/m^3)	$\Delta \log_{10}\rho$ [(from (1)]	H (km)	$\log_{10}\rho$ (kg/m^3)	$\Delta \log_{10}\rho$ [(from (2)]
0	+0.088	+.286	0	+0.088	+.107
25	-1.378	+.126	25	-1.398	-.184
50	-2.965	-.096	50	-2.986	-.267
75	-4.304	+.145	75	-4.363	+.037
100	-6.147	+.002	100	-6.258	-.043
125	-7.629	+.027	125	-7.754	+.031
150	-8.750	-.006	150	-8.871	+.001
175	-9.462	-.012	175	-9.576	-.010
200	-9.955	-.001	200	-10.068	-.001
250	-10.725	-.006	250	-10.859	-.005
300	-11.328	-.002	300	-11.484	.000
350	-11.822	.000	350	-12.001	+.002
400	-12.241	+.001	400	-12.442	+.001
450	-12.604	+.003	450	-12.826	-.001
500	-12.922	.000	500	-13.166	.000

Little effort was made to secure a good fit for heights smaller than 100 km.

APPENDIX M

Effective Radius of the Earth

The limitations of the inverse square law for determining the acceleration of gravity were discussed in Section 2.1 of this paper. A value of effective earth's radius was introduced as a means of offsetting some of these limitations.

The inverse square law for expressing the acceleration of gravity was given as

$$g = g_\phi \left(\frac{r_\phi}{r_\phi + Z} \right)^2 . \qquad \text{(M-1)}$$

The partial derivative of g with respect to Z is

$$\frac{\partial g}{\partial Z} = 2g_\phi \left(\frac{r_\phi}{r_\phi + Z} \right) \frac{(-r_\phi)}{(r_\phi + Z)^2} . \qquad \text{(M-2)}$$

This partial derivative evaluated at Z = 0 becomes

$$\left(\frac{\partial g}{\partial Z} \right)_{Z=0} = \frac{-2g_\phi}{r_\phi} . \qquad \text{(M-3)}$$

Thus, if the actual sea-level value of g_ϕ and the actual sea-level value of $(\partial g/\partial Z)$ for the particular latitude are introduced into Eq. (M-3), the value of r_ϕ consistent with these realistic quantities is the effective earth's radius at that latitude.

Harrison[23] presented the following expression for $(\partial g/\partial Z)_{Z=0}$ as a function of latitude ϕ, without indicating its derivation.*

$$- \frac{\partial g}{\partial Z} = 3.085,462 \times 10^{-6} + 2.27 \times 10^{-9} \cos 2\phi$$
$$- 2 \times 10^{-12} \cos 4\phi .$$

---

* This equation appears to be related to Lambert's[36,38] alternating power series expression for g in terms of ϕ and Z, which equation is discussed in Appendix O.

Using this expression, the effective earth's radius $\bar{r}_\phi$ at latitude ϕ is

$$\bar{r}_\phi = \frac{2g_\phi}{3.085,462 \times 10^{-6} + 2.27 \times 10^{-9} \cos 2\phi - 2 \times 10^{-12} \cos 4\phi} .$$

For $\phi = 45° \; 32' \; 40''$,

$$g_\phi = g_o = 9.806,65 \text{ m sec}^{-1}$$

and

$$\bar{r}_\phi = r = 6,356,766 \text{ m} .$$

APPENDIX N

Acceleration of Gravity

1. Background

The inverse square law employed in this MODEL for the computation of the acceleration of gravity has been adjusted at sea level to account for the effective sea level value and the vertical gradient of g at that point, by means of an effective earth's radius (see Appendix M). This correction accounts for the centrifugal acceleration which a body experiences at sea level, by virtue of the earth's rotation, but it does not account for the fact that this centrifugal acceleration increases rather than decreases with altitude. Since the centrifugal acceleration is opposite in direction to the gravitational acceleration, the net or effective value of g falls off more rapidly with altitude than even the adjusted inverse square law predicts. Because the actual earth's radius and the centrifugal acceleration both depend upon latitude, any general expression for a resultant or effective acceleration must be a function of both altitude Z and latitude ϕ.

Lambert[38] developed such a general expression* for g in the form of

$$
\begin{aligned}
g = c_1 &- (a_2 + b_2 \cos 2\phi)Z + (a_3 + b_3 \cos 2\phi)Z^2 \\
&- (a_4 + b_4 \cos 2\phi)Z^3 + (a_5 + b_5 \cos 2\phi)Z^4 \\
&- \ldots \qquad\qquad + \ldots
\end{aligned}
\tag{N-1}
$$

where

g = the acceleration of gravity in m sec^{-2},
Z = geometric altitude in m,
ϕ = latitude in degrees
$c_1 = g_\phi$, sea level value of g at latitude ϕ,

$a_2 = 3.085{,}462 \times 10^{-6}$ $\qquad b_2 = 2.27 \times 10^{-9}$
$a_3 = 7.254 \times 10^{-13}$ $\qquad b_3 = 1.0 \times 10^{-15}$
$a_4 = 1.517 \times 10^{-19}$ $\qquad b_4 = 6 \times 10^{-22}$
$a_5 = 2.97 \times 10^{-26}$ $\qquad b_5 = 2 \times 10^{-28}$

---

* The fifth term (in Z^4) has not been published, but was provided by Col. C. Spohn, of Air Weather Service USAF, who probably obtained it from Lambert or Harrison.

For the case when ϕ = 45° 32' 40", as in this MODEL, chosen to agree with g_o = 9.806,65 m sec^{-2},

$$\cos 2\phi = \cos 91° \; 5' \; 20'' = - \sin 1° \; 5' \; 20'' = - .019{,}003{,}7. \qquad \text{(N-2)}$$

For this value of ϕ, Eq. (N-1) becomes

$$g = c_1 - c_2 Z + c_3 Z^2 - c_4 Z^3 + c_5 Z^4 - \ldots \qquad \text{(N-3)}$$

where

$$\begin{aligned} c_1 &= 9.806{,}65 \text{ (exact)} & &m \; sec^{-2} \\ c_2 &= .308{,}541{,}8_8 \times 10^{-5} & &m^0 \; sec^{-2} \\ c_3 &= .007{,}253{,}_{81} \times 10^{-10} & &m^{-1} \; sec^{-2} \\ c_4 &= .000{,}151{,}6_{89} \times 10^{-15} & &m^{-2} \; sec^{-2} \\ c_5 &= .000{,}002{,}96_{96} \times 10^{-20} & &m^{-3} \; sec^{-2} \end{aligned}$$

The reliability of the limit of this series in expressing the true value of g at any altitude is unknown to the authors of this report. It is assumed that this function represents the best available analytical expression for g in terms of Z and ϕ. The small number of available terms and significant figures, however, places limitations on the evaluation of the series at high altitudes.

2. Problem

It is necessary to determine the limitations which the small number of terms and the small number of significant figures place upon the evaluation of the function at various altitudes. It is further necessary to compare the results of the adjusted, inverse-square-law function for g with the values obtained from the infinite series function for g.

The extent to which the availability of only five terms limits the value of g at various altitudes has been studied for the case where ϕ = 45° 32' 40" with the results indicated below. In the course of the analysis it was found that several additional terms were necessary to determine the value of g to the desired accuracies at altitudes above 150 km. The values of the additional terms were estimated by graphical extrapolation, and refined values of g were computed for various altitudes. These values of g were then compared with values from the inverse square law, using the effective earth's radius at 45° 32' 40" as determined in Appendix M.

3. Results, Concerning Required Number of Terms in Equation (N-3) For Various Degrees of Accuracy

Equation (N-3), limited to four terms as published, provides accuracies of one part in 9,800,000, or seven significant figures, for altitudes up to only about 60 km. The fifth term permits the equation to be used up to about 150 km with the same accuracy, provided that the coefficient of the third term has one additional significant figure. By means of extrapolation it was estimated that with five additional terms in Eq. (N-3), g could be determined to the stated accuracy for altitudes up to 1,140 km, provided a sufficient number of significant figures are added to all the terms beyond the first two. For other accuracies the maximum altitude to which g may be computed with a given number of terms in Eq. (N-3) is given in Table (N-I), neglecting significant figures in existing terms.

Number of Terms Available	Number of Significant Figures Required in g						
	2	3	4	5	6	7	8
2	260	80	25	8			
3	700	330	150	75	60	20	
4	1100	650	370	200	110	60	35
5		1000	640	400	250	150	100
6			950	610	420	260	180
7			1300	900	610	440	320
8				1100	860	610	480
9					1200	830	620
10						1140	800

Table N-I. Estimated maximum altitude in km for which a specified number of terms in Eq. (N-3) will yield accuracies of a specified number of significant figures in g, provided the various coefficients have a sufficient number of significant figures.

4. Results, Concerning Limitations Due to Available Significant Figures in Equations (N-1) and (N-3).

The number of significant figures in the coefficients of Eq. (N-3) stems directly from the number available in the coefficients of Eq. (N-1). An analysis of the limitations of these equations shows that for g accurate to four significant figures, these equations may be used up to 1,400 km.

For five-significant-figure accuracy in g, the accuracy of the coefficients limits the calculations to altitudes below 1,300 km; for six-significant-figure accuracy in g, the calculations are restricted to altitudes below 500 km; while for seven-and eight-significant-figure accuracy in g, the maximum permissible altitudes are only 150 and 50 km, respectively. (see figure N-6)

Applying these restrictions to Table N-I, one obtains Table N-II.

Number of Terms Available	Number of Significant Figures Required in g						
	2	3	4	5	6	7	8
2	260	80	25	8			
3	700	330	150	75	60	20	
4	1100	650	370	200	110	60	35
5		1000	640	400	250	150	50
6			950	610	420	150	50
7			1300	900	500	150	50
8				1300	500	150	50
9					500	150	50
10						150	50

Table N-II. Estimated maximum altitude in km for which a specified number of terms of Eq. (N-2) will yield a specified number of significant figures' accuracy in the value of g, with the significant figures of existing coefficients limiting the results.

NOTE: Underlined figures are those limited by the number of significant figures in coefficients.

5. Results of Comparison of Values of g from Equation (N-3) with Inverse-Square-Law Values of g

The inverse-square-law values of g, for $\phi = 45° 32' 40''$, when the effective earth's radius is used, are in good agreement with the values of Eq. (N-3), with no differences occurring in the fifth significant figure below 100 km. Above this altitude the differences increase rather rapidly to a peak at 500 km, after which they fall off to zero somewhere between 700 and 800 km and increase negatively above that altitude. This large fall-off is due principally to the omission of term six which becomes extremely significant in the series at this altitude. Since this term is negative, its presence would reduce the value of Eq. (N-3) at these altitudes and tend to retain the increasing difference with the inverse-square-law value.

Values of g were recalculated from Eq. (N-3) on the bases of four additional terms determined graphically, and these new values of g were then compared with the inverse-square-law values. In this latter comparison, the differences increased uniformly with altitude. Curves B and C of Fig. N-1 show the graphs of the two comparisons. Curve A in this figure shows the departure of the five-term-series value of g from the estimated nine-term-series value of g. Curves A and C are essentially the error curves of the five-term-series function and the inverse-square-law function, respectively, assuming the nine-term-series value of g to be the most correct. At 150 km, the five-term-series function provides two more significant figures than the inverse square law. As altitude increases, however, the differential in accuracy drops proportionately to one significant figure at 330 km, and no difference at 750 km. A comparison of the maximum altitudes to which the five-term-series function and the inverse-square-law function may each be used for various accuracies is given in Table N-III.

	Significant Figures				
	4	5	6	7	8
5 term series	640	400	250	150	50
inverse square	500	130	40	10	5

Table N-III. Comparison of maximum altitude to which each of two functions of g may be used for five different degrees of accuracy.

The numerical value of g by the several methods and the numerical differences between these values are given in Table N-VI.

6. Method of Analysis

The analysis was performed by using twenty-one values of Z between 1 and 1,000 km, and independently evaluating each of the five terms of Eq. (N-3). The logarithms of the absolute values of each term were plotted as a function of the number of the term, and points corresponding to the same value of Z were connected to form the solid line portion of Fig. N-2. The lines were then extrapolated to regions corresponding to higher order terms. The values indicated for these terms by the extrapolations then served as estimated values for these terms.

The values of the several terms were then plotted as a function of altitude, as in Fig. N-3, with solid lines connecting the computed terms, and broken lines connecting the estimated terms. The analysis of the contribution of varying numbers of terms to the value of the total function was then made visually from this graph.

The significant figure analysis was performed on tabulated values of the several terms (Table N-IV and Table N-V) and the net results are plotted on Figs. N-4, N-5, and N-6.

Alt. km.	2nd Term	3rd Term	4th Term	5th Term
1	.003,085,418,8	.000,000,725,38	.000,000,000,151	.000,000,000,000,029
5	.015,427,094	.000,018,134,52	.000,000,018,965	.000,000,000,018,56
10	.030,854,188	.000,072,538,1	.000,000,151,698	.000,000,000,296,96
20	.061,708,376	.000,290,152,4	.000,001,213,51	.000,000,004,711
30	.092,562,564	.000,652,843	.000,004,095,60	.000,000,024,054
40	.123,416,752	.001,160,609	.000,009,708,0	.000,000,076,022
50	.154,270,940	.001,813,452	.000,018,961,1	.000,000,185,60
60	.185,125,128	.002,611,372	.000,032,764,8	.000,000,384,86
70	.215,979,316	.003,554,367	.000,052,029,3	.000,000,713,00
80	.246,833,504	.004,642,44	.000,077,665	.000,001,216,35
90	.277,687,692	.005,875,59	.000,110,581	.000,001,948,3
100	.308,541,880	.007,253,81	.000,151,689	.000,002,969,6
200	.617,083,760	.029,015,24	.001,213,51	.000,047,513
300	.925,625,64	.065,284,3	.004,095,60	.000,240,54
400	1.234,167,52	.116,060,9	.009,708,1	.000,760,22
500	1.542,709,40	.181,345,2	.018,961,1	.001,856,0
600	1.851,251,28	.261,137,2	.032,764,8	.003,848,6
700	2.159,793,16	.355,436,7	.052,029,3	.007,130,0
800	2.468,335,04	.464,244	.077,664	.012,163,4
900	2.776,876,92	.587,559	.110,581	.019,483
1000	3.085,418,80	.725,381	.151,689	.029,696

Table N-IV. Values of the first four variable terms of Eq. (N-3) for various altitudes from 1 km to 1,000 km.

NOTE: The underlined figures are beyond the limit of significance but are carried for smoothness.

	6th Term	7th Term	8th Term	9th Term
100	.000,000,05	.000,000,001	.000,000,000	.000,000,000
200	.000,001,8	.000,000,08	.000,000,002	.000,000,000
300	.000,012	.000,000,8	.000,000,04	.000,000,000
400	.000,055	.000,003,5	.000,000,2	.000,000,001
500	.000,15	.000,012	.000,001	.000,000,05
600	.000,42	.000,045	.000,004	.000,000,3
700	.000,9	.000,11	.000,014	.000,001,5
800	.001,7	.000,24	.000,03	.000,003,5
900	.002,6	.000,4	.000,05	.000,006,5
1000	.004,5	.000,7	.000,10	.000,013

Table N-V. Estimated values of terms 6 through 9 of Eq. (N-3) for altitudes between 100 and 1,000 km.

Alt. km	$g = g_o\left[\frac{r}{r+z}\right]^2$	g* from 5 terms of Eq. (N-3)	g** from estimated 9 terms of Eq. (N-3)	g - g*	g - g**	g* - g**
1	9.803,565,30	9.803,565,306	identical to adjacent column	.000,000,00	same as g - g*	zero within the available number of significant figures.
5	9.791,241,06	9.791,241,021		.000,000,04		
10	9.775,868,42	9.775,868,19		.000,000,23		
20	9.745,231,56	9.745,230,56		.000,001,00		
30	9.714,738,52	9.714,736,2		.000,002,32		
40	9.684,388,35	9.684,384,2		.000,004,1		
50	9.654,180,19	9.654,173,7	departures from g* are underlined below	.000,006,5		
60	9.624,113,15	9.624,103,8		.000,009,3		
70	9.594,186,36	9.594,173,7		.000,012,6		
80	9.564,398,93	9.564,382		.000,016		
90	9.534,750,01	9.534,729		.000,021		
100	9.505,238,75	9.505,213		.000,026		
200	9.217,512,92	9.217,415	9.217,414	.000,098	.000,099	.000,001
300	8.942,656,38	8.942,45	8.942,44	.000,20	.000,21	.000,01
400	8.679,912,89	8.679,59	8.679,54	.000,32	.000,38	.000,05
500	8.428,581,04	8.428,18	8.428,04	.000,40	.000,54	.000,14
600	8.188,009,42	8.187,61	8.187,24	.000,39	.000,76	.000,37
700	7.957,592,42	7.957,39	7.956,59	.000,20	.001,00	.000,80
800	7.736,766,50	7.737,0	7.735,6	-.000,3	.001,2	.001,4
900	7.525,006,62	7.526,2	7.524,0	-.001,2	.001,0	.002,2
1000	7.321,823,24	7.324,6	7.320,7	-.002,8	.001,1	.003,9

Table N-VI. Values of the acceleration of gravity for various altitudes computed from three different equations as indicated, and the differences between these values of the acceleration of gravity.

NOTE: Underlined numbers in Column g* indicate figures of questionable significance.

Underlined numbers in Column g** indicate figures differing from Column g*.

7. Conclusions

a. For most engineering purposes, the adjusted inverse-square-law function for g provides adequate accuracy.

b. For the standard atmosphere, and for future editions of this MODEL, the values of g should be computed on the basis of an expanded version of Eq. (N-3) in which a minimum of three, and preferably five, additional terms are employed, and in which sufficient additional significant figures are provided for the various limiting coefficients, particularly coefficients of terms 3, 4, and 5.

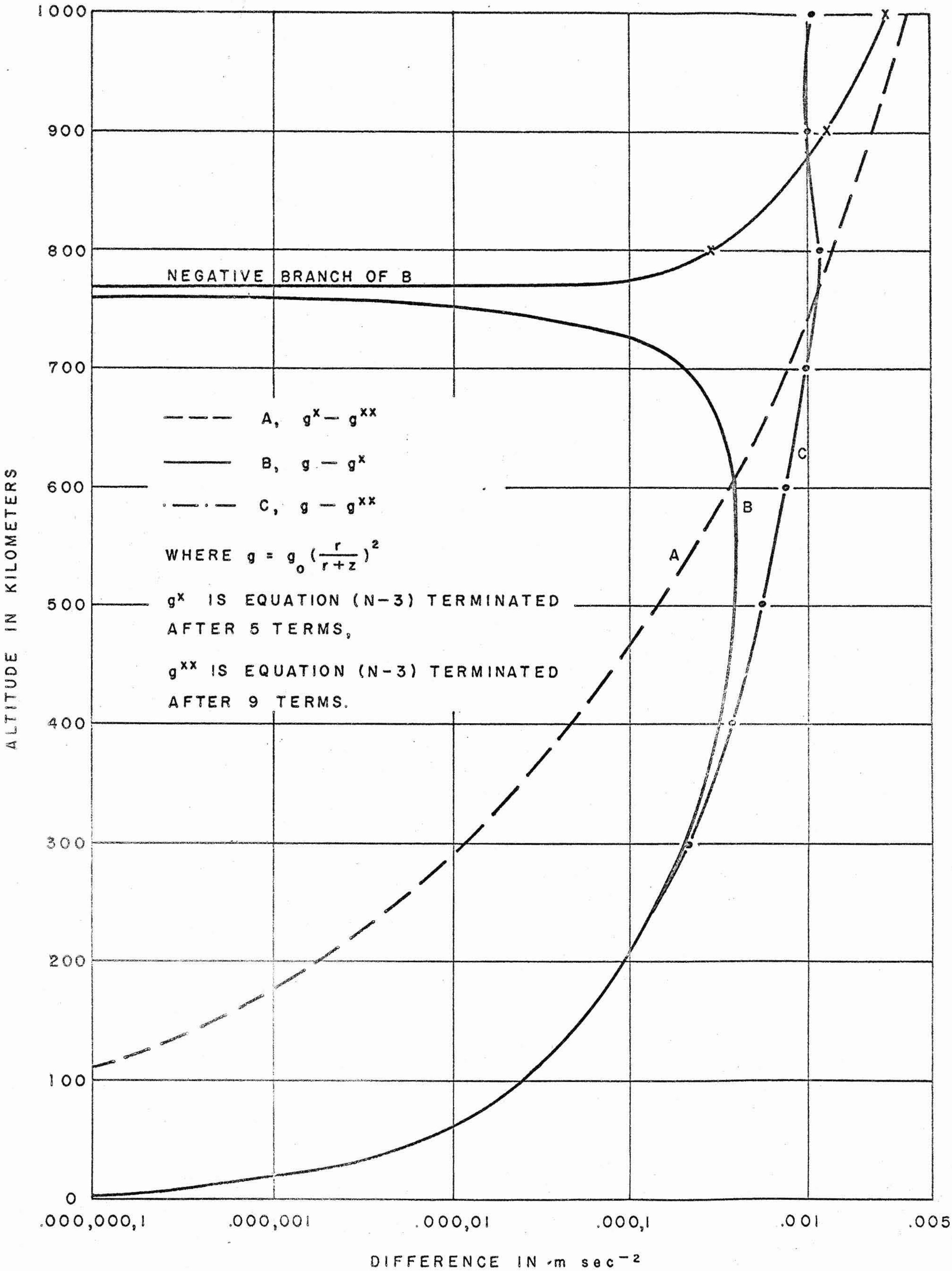
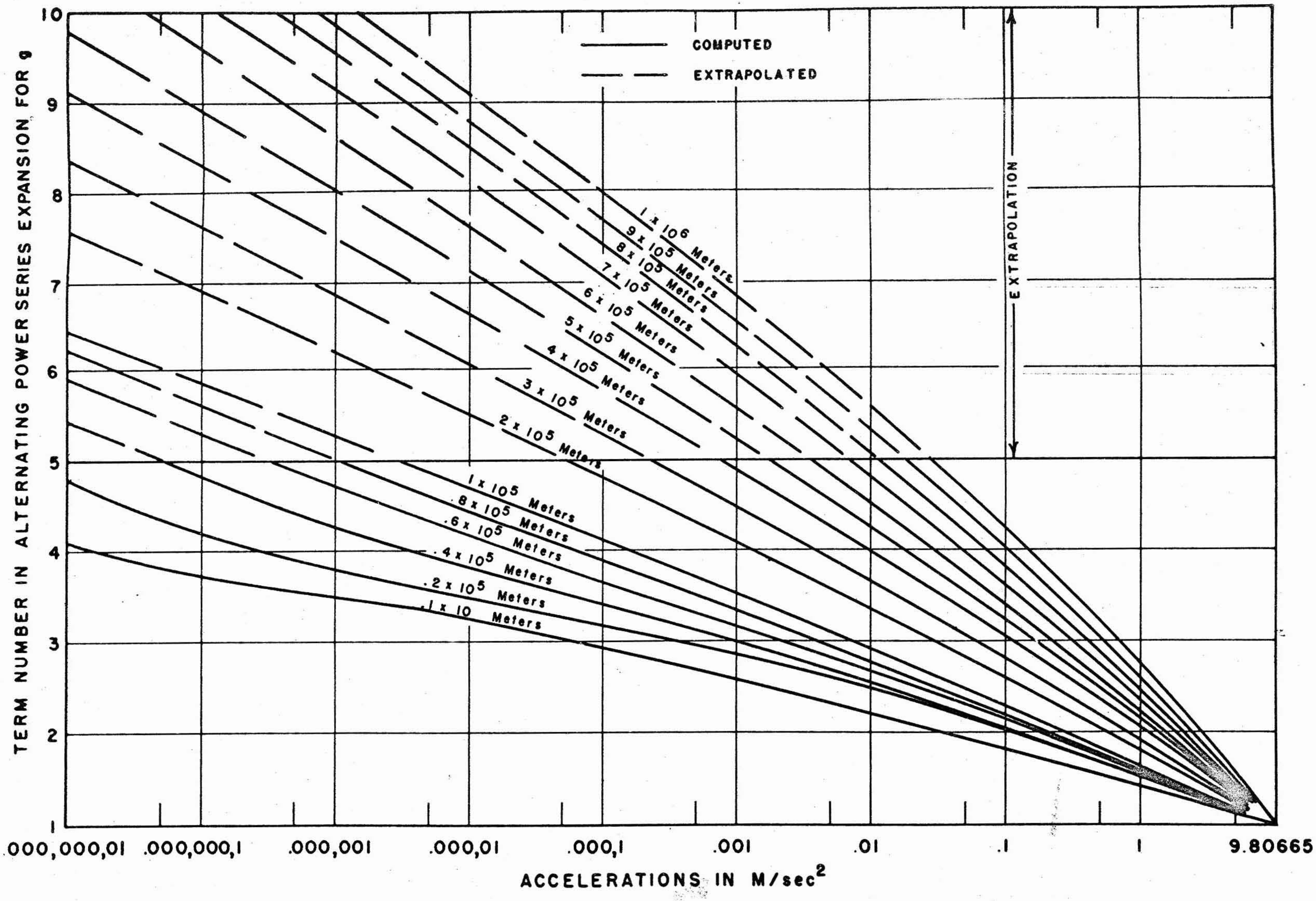


FIGURE N-1 DIFFERENCES BETWEEN THE VALUES OF THE ACCELERATION OF GRAVITY COMPUTED FROM THREE DIFFERENT EQUATIONS.

GRD
MARCH 1957



MAGNITUDE OF SUCCESSIVE TERMS OF LAMBERT'S ALTERNATING POWER SERIES FOR g WHEN EVALUATED FOR VARIOUS ALTITUDES

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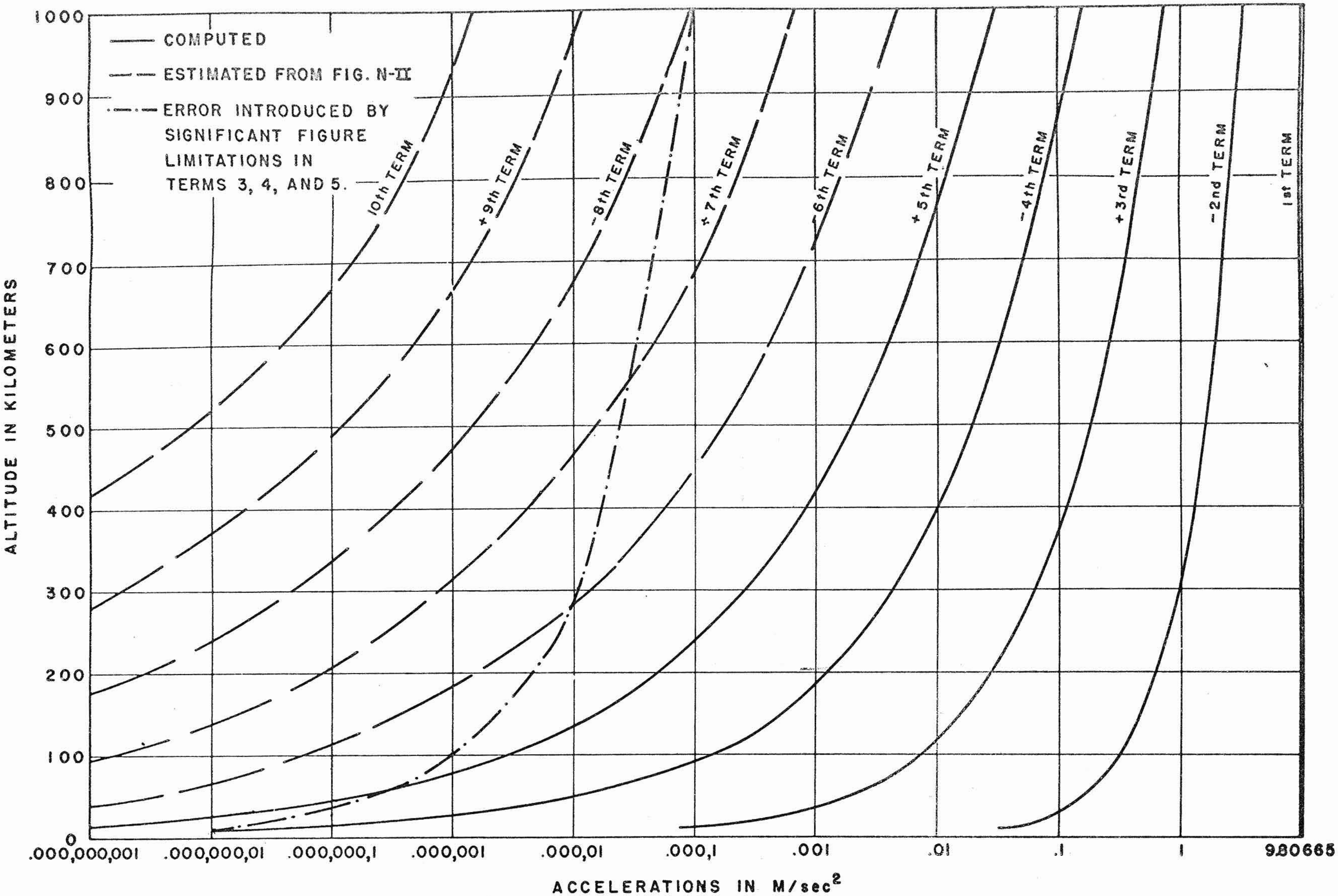
MARCH 1957



MAGNITUDES OF EACH OF THE FIRST TEN TERMS OF LAMBERT'S ALTERNATING POWER SERIES FOR g, FOR VARIOUS ALTITUDES, BETWEEN 10 AND 1000 km.

GRD
MARCH 1957

FIGURE N-3

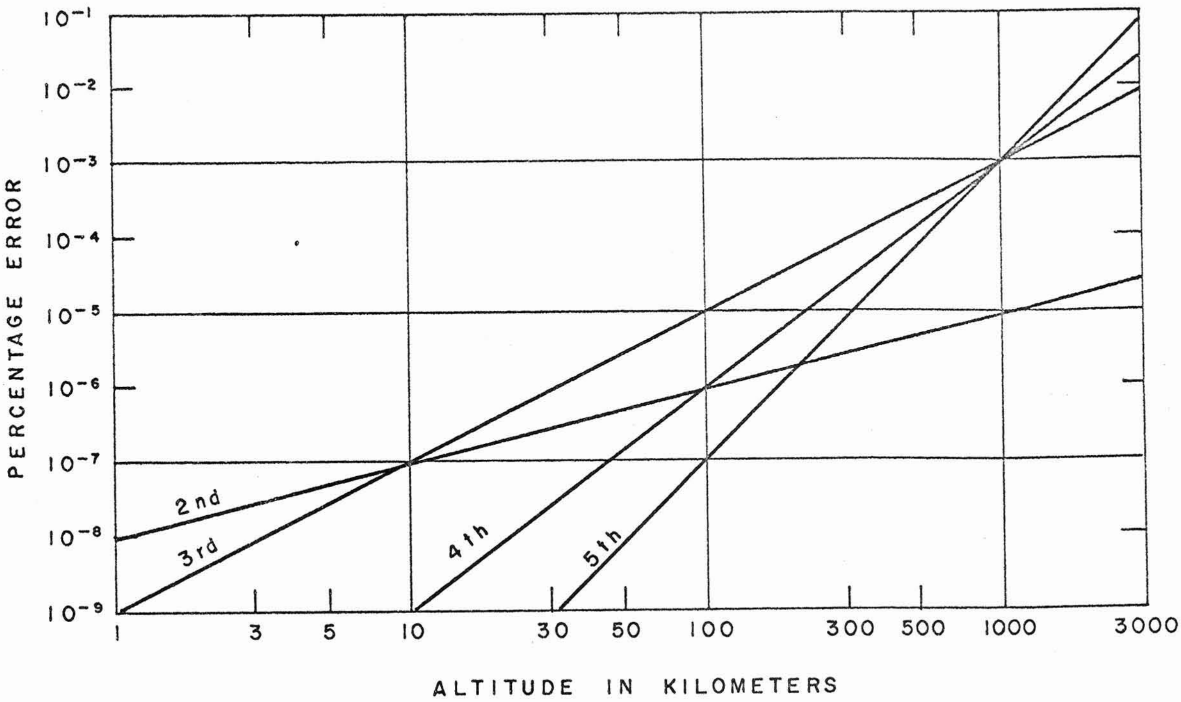


FIGURE N-4 PERCENTAGE ERROR IN THE VALVE OF THE ACCELERATION OF GRAVITY AT VARIOUS ALTITUDES INTRODUCED BY THE SIGNIFICANT FIGURE LIMITATIONS OF THE SEVERAL TERMS OF EQUATION N-3.

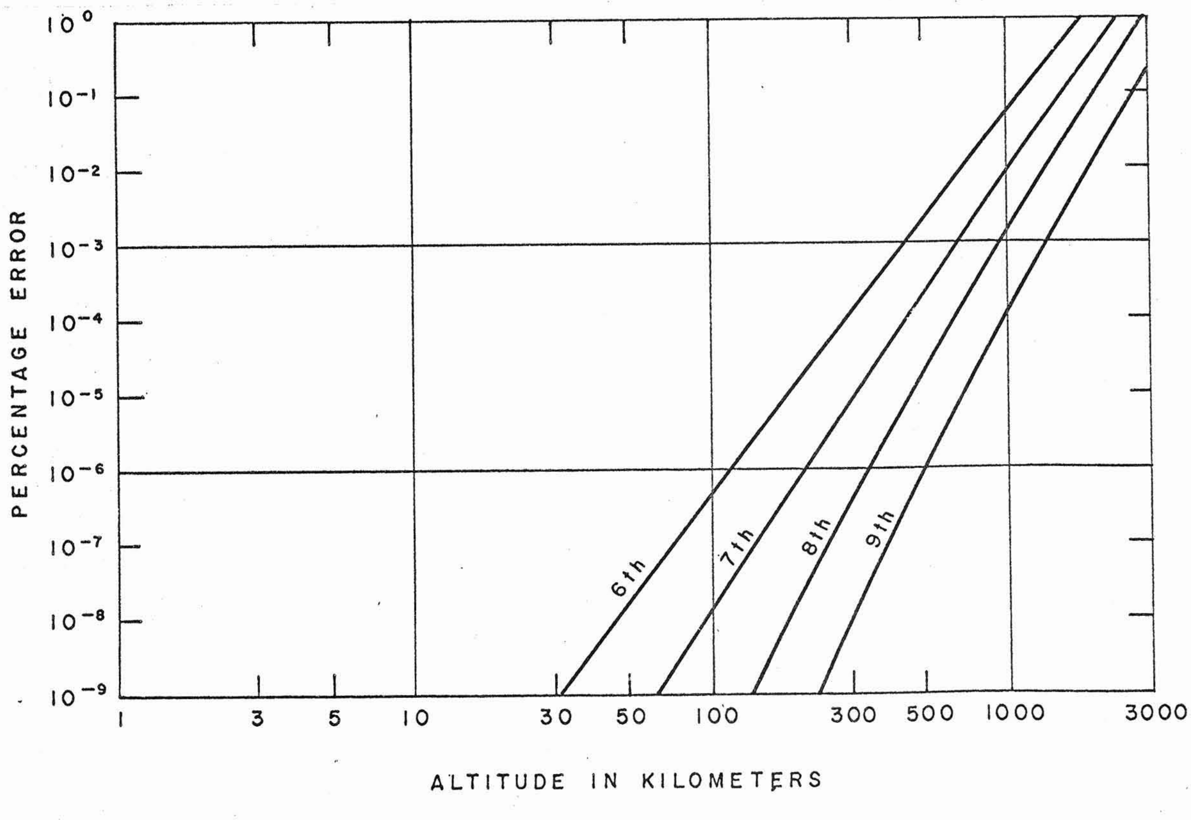


FIGURE N-5 ESTIMATED PERCENTAGE ERROR IN THE VALUE OF THE ACCELERATION OF GRAVITY AT VARIOUS ALTITUDES INTRODUCED BY THE OMISSIONS OF TERMS 6, 7, 8 AND 9 OF EQUATION (N-3).

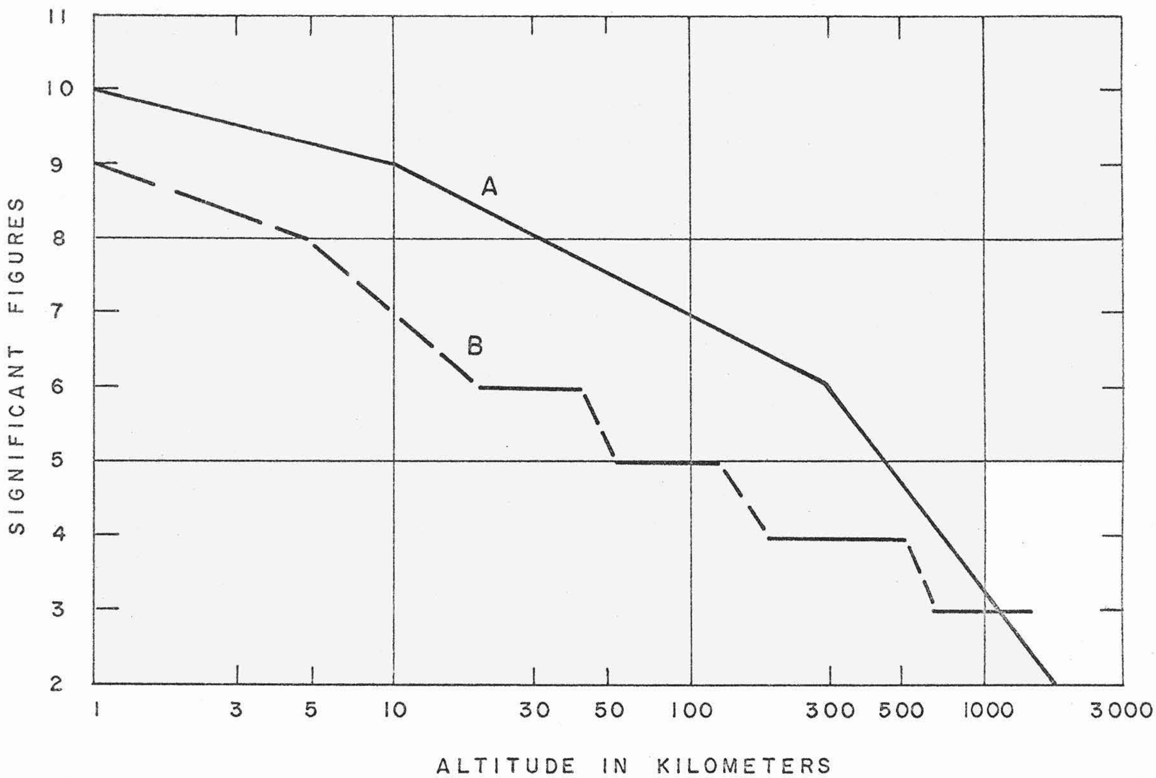


FIGURE N-6 (A) MAXIMUM NUMBER OF SIGNIFICANT FIGURES AVAILABLE FROM THE EXISTING 5 TERM VERSION OF EQUATION N-3, FOR VARIOUS ALTITUDES.

(B) THE MAXIMUM NUMBER OF SIGNIFICANT FIGURES OF THE VALUE OF g AT VARIOUS ALTITUDES, COMPUTED FROM THE ADJUSTED INVERSE SQUARE LAW, WHICH ARE IN AGREEMENT WITH VALUES COMPUTED FROM EITHER THE 5 TERM OR 9 TERM VERSION OF EQUATION (N-3).

APPENDIX O

Scale Height

1. Geometric Scale Height

First Concept - Scale height is equal to the height above any reference altitude at which the atmospheric pressure falls to 1/e of the pressure at the reference altitude in a constant gravity, isothermal atmosphere.

In a manner analogous to the development of Eq. (15) in terms of H (Section 3.2.1), the following equation is developed in terms of Z:

$$\ell n \frac{P}{P_b} = \frac{M_o}{R^*}\int_{Z_b}^{Z} \frac{g dZ}{T_M} . \tag{O-1}$$

For the case of an isothermal layer in a constant gravity atmosphere, Eq. (O-1) upon integration leads to

$$P = P_b \text{ exponential} - \frac{g_o M_o}{R^* (T_M)_b} (Z - Z_b). \tag{O-2}$$

It is noted that in a constant gravity atmosphere:

$$\frac{R^* (T_M)_b}{g_o M_o} = (H_s)_b , \tag{O-3}$$

and it follows that

$$P = P_b \text{ exponential} - \frac{(Z - Z_b)}{(H_s)_b} . \tag{O-4}$$

For the case that

$$(Z - Z_b) = (H_s)_b , \tag{O-5}$$

Eq. (O-4) simplifies to

$$P = P_b\, e^{-1} = P_b / e . \tag{O-6}$$

It appears, therefore, that in a constant gravity atmosphere and in a layer of constant T_M, the scale height at any reference level is the increment in geometric altitude required for the pressure to fall to 1/e of the value at the reference level. Since this MODEL does not assume constant gravity, the above concept does not apply rigorously in these tables. In the special case, where sea level is the reference altitude the same concept would apply but only if the isothermal layer is assumed to extend down to there, and only for a constant gravity atmosphere.

Second Concept - In an atmosphere of constant g and constant T_M, the scale height at any altitude Z_b is equal to the total mass of air in a unit column extending upward from that altitude to infinity, divided by the density at the reference altitude.

From Eq. (33) one obtains

$$\frac{P}{P_b} = \frac{\rho}{\rho_b} \cdot \frac{T_M}{(T_M)_b} . \tag{0-7}$$

In a constant T_M atmosphere, $T_M = (T_M)_b$ and thus,

$$\frac{P}{P_b} = \frac{\rho}{\rho_b} . \tag{0-8}$$

Equation (0-2) may then be rewritten as

$$\rho = \rho_b \text{ exponential } - \frac{g_o M_o}{R^*(T_M)_b} . \tag{0-9}$$

The total mass in a unit column from the reference level to infinity is:

$$\int_{Z_b}^{\infty} \rho dZ = \rho_b \int_{Z_b}^{\infty} \text{exponential } - \frac{g_o M_o}{R^*(T_M)_b} \quad (Z - Z_b) \tag{0-10}$$

$$= \rho_b \left[\frac{R^*(T_M)_b}{-g_o M_o} \right] \left[\text{exponential } - \frac{g_o M_o}{R^*(T_M)_b} \quad (Z - Z_b) \right]_{Z_b}^{\infty} \tag{0-10a}$$

$$= \rho_b \left[\frac{R^*(T_M)_b}{-g_o M_o} \right] \left[e^{-\infty} - e^{0} \right] \tag{0-10b}$$

$$= \rho_b \cdot \frac{R^*(T_M)_b}{g_o M_o} \tag{0-10c}$$

Since $\frac{R^*(T_M)_b}{g_o M_o}$ = scale height at H_b in a constant gravity atmosphere, it follows that

$$(H_s)_b = \frac{1}{\rho_b} \int_{Z_b}^{\infty} \rho dZ. \tag{0-11}$$

Thus the assertion of Concept 2 is demonstrated.

Third Concept - In a constant-g, constant-T_M, constant-M atmosphere, the scale height at any altitude is equal to the total number of particles in a column of unit cross section extending from a reference level to infinity, divided by the number density at that altitude.

From Eqs. (26) and (27) of Sections 5.2.1 and 5.3.1, respectively, it follows that:

$$n = \rho \frac{N}{M'} \tag{0-12}$$

but

$$\frac{M'}{N} = m \tag{0-13}$$

where m = the mass of a single air particle.

Thus

$$\rho = n\, m \tag{0-14}$$

and

$$\rho_b = n_b m_b \tag{0-15}$$

Thus it follows directly from Eq. (0-11) that

$$(H_s)_b = \frac{1}{n_b m_b} \int_{Z_b}^{\infty} \rho dZ, \tag{0-16}$$

$$(H_s)_b = \frac{1}{n_b m_b} \int_{Z_b}^{\infty} n m dZ. \qquad (0\text{-}17)$$

The right-hand side of this equation would not strictly equal the total number of atmospheric particles in the column, unless the molecular weight were constant. Thus, for the assertion of the third concept to be rigorously correct, it was necessary to make the restriction of constant molecular weight in addition to the restrictions made in the first and second concepts. With this constant-M restriction, Eq. (0-17) becomes

$$(H_s)_b = \frac{1}{n_b} \int_{Z_b}^{\infty} n dZ, \qquad (0\text{-}18)$$

and the assertion is demonstrated. It is noted that a corollary to the third concept is that scale height is the length of the unit column necessary to enclose all the atmospheric particles normally present in an infinitely long unit column, extending vertically above the reference altitude, when these particles are compressed to the number density at the reference level. Hence, this quantity is the basis for computing reduced thickness of the atmosphere. Such computations are limited by the fact that constant gravity, constant T_M, and constant molecular weights are assumed in the derivation of the expression.

2. Geopotential Scale Height

Geopotential scale height was defined in Section 4.1.3 of this paper as

$$H_s' = \frac{GM_o}{R^* T_M} .$$

In terms of this property the several concepts developed above do not have the restriction of a constant gravity atmosphere. Thus Eq. (15) of Section 3.2.2 may be rewritten as

$$P = P_b \text{ exponential } - \frac{GM_o}{R^*(T_M)_b} (H - H_b). \qquad (0\text{-}19)$$

For a geopotential altitude increment equal to the geopotential scale height

$$(H - H_b) = \frac{R^*(T_M)_b}{GM_o} = H_s' ,$$

and hence Eq. (0-19) reduces to

$$P = P_b/e. \tag{0-19a}$$

Note that no assumption of constant gravity is made, only constant T_M. Hence, a revision of Concept 1, eliminating the constant gravity restrictions, will apply rigorously in this MODEL in isothermal layers. For example, H_s' at 11 km' is $6.341,615,82 \times 10^3$ m'. Thus, at 17.341,615,82 km', the pressure will be P_{11}/e, where P_{11} is the pressure at 11 km. At 14 km', H_s' has the same value; hence at $20.341,615,82 \times 10^3$ m' altitude, the pressure will be P_{14}/e. The geometric altitude increment, however, will be different in the two instances, accounting for the effect of variable g on the pressure.

In geopotential form, Eq. (0-10) may be rewritten as

$$\int_{H_b}^{\infty} \rho dH = \rho_b \int_{H_b}^{\infty} \text{exponential} - \frac{GM_o}{R^*(T_M)_b} (H - H_b). \tag{0-20}$$

By analogy this reduces to

$$(H_s')_b = \frac{1}{\rho_b} \int_{H_b}^{\infty} \rho dH. \tag{0-21}$$

This equation and concept rigorously apply to isothermal layers of this MODEL.

Equation (0-16) is converted by analogy to

$$(H_s')_b = \frac{\int_{H_b}^{\infty} \rho dH}{n_b m_b}. \tag{0-22}$$

If constant molecular weight is assumed, this equation becomes:

$$(H_s') = \frac{1}{n_b} \int_{H_b}^{\infty} n dH. \tag{0-23}$$

This equation would provide a better basis for computing reduced thickness for this MODEL than Eq. (0-18), but Eq. (0-23) is similarly limited by constant M and constant T_M assumptions. Thus, for still greater accuracy of reduced-thickness calculations consistent with this MODEL, additional equations accounting for variable M and T_M must be developed.

APPENDIX P

More Accurate Method for Computing Geopotential in this Model

1. Adjusted Classical Approach

Equation (2d) of this paper indicates the rigorous relationship between geopotential H, geometric altitude Z, and the acceleration of gravity g to be

$$H = \frac{1}{G}\int_{0}^{Z} g\,dZ. \qquad \text{(P-1)}$$

When g is expressed by the classical, inverse-square law, adjusted for 45° 32' 40" latitude,

$$g = g_o\left(\frac{r}{r+Z}\right)^2, \qquad \text{(P-2)}$$

the expression for geopotential becomes

$$H = \frac{g_o}{G}\left(\frac{rZ}{r+Z}\right), \qquad \text{(P-3)}$$

where g_o and r have the values 9.80665 m sec^{-2} and 6,356,766 m, respectively, as indicated in Section 2.1.

2. Lambert Series Method

In Appendix N, another expression for g in terms of Z for latitude 45° 32' 40" was developed from Lambert's general alternating power series.[38] This specific expression is

$$g = c_1 - c_2Z + c_3Z^2 - c_4Z^3 + c_5Z^4 - \ldots \qquad \text{(P-4)}$$

where

$c_1 = 9.806{,}65$ (exact)	m sec^{-2},
$c_2 = 30{,}854.18_8 \times 10^{-10}$	m^o sec^{-2},
$c_3 = 725.3_{81} \times 10^{-15}$	m^{-1} sec^{-2},
$c_4 = 15.16_{89} \times 10^{-20}$	m^{-2} sec^{-2},
$c_5 = .296_{96} \times 10^{-25}$	m^{-3} sec^{-2},

Z is in meters, and
g is in meters sec^{-2}.

When this expression for g is introduced into Eq. (P-1) the expression for H becomes

$$H = \frac{1}{G}\left[c_1\int_o^Z dZ - c_2\int_o^Z ZdZ + c_3\int_o^Z Z^2dZ - c_4\int_o^Z Z^3dZ + c_5\int_o^Z Z^4dZ - \ldots\right] \quad \text{(P-5)}$$

where H is in standard geopotential meters.

Performing the indicated integration one obtains

$$H = \frac{c_1}{G}Z - \frac{c_2}{2G}Z^2 + \frac{c_3}{3G}Z^3 - \frac{c_4}{4G}Z^4 + \frac{c_5}{5G}Z^5 - \ldots, \quad \text{(P-6)}$$

where the coefficients of the various powers of Z have the following numerical values:

$$\frac{c_1}{G} = \frac{9.806{,}65}{9.806{,}65} \qquad = 1.0 \text{ exact}$$

$$\frac{c_2}{2G} = \frac{30{,}854.18_8 \times 10^{-10}}{2 \times 9.806{,}65} \qquad = 1{,}573.125_{78} \times 10^{-10}$$

$$\frac{c_3}{3G} = \frac{725.3_{81} \times 10^{-15}}{3 \times 9.806{,}65} \qquad = 24.65_{61} \times 10^{-15}$$

$$\frac{c_4}{4G} = \frac{15.16_{89} \times 10^{-20}}{4 \times 9.806{,}65} \qquad = .386{,}6_{99} \times 10^{-20}$$

$$\frac{c_5}{5G} = \frac{.296_{96} \times 10^{-25}}{5 \times 9.806{,}65} \qquad = .006{,}05_{63} \times 10^{-25}$$

Hence one obtains

$$H = Z - 1{,}573.125_{78} \times 10^{-10}Z^2 + 24.65_{61} \times 10^{-15}Z^3$$

$$- .386{,}6_{99} \times 10^{-20}Z^4 + .006{,}05_{63} \times 10^{-25}Z^5 - \ldots \quad \text{(P-7)}$$

(where the exponents have been selected for convenience when Z is expressed in units of 10^5 meters).

Evaluating the five defined terms of Eq. (P-7) for various altitudes yields the data presented in Table P-I. An examination of the logarithms of successive terms of the series evaluated for particular altitudes shows that the absolute magnitudes of successive terms fall off very nearly at a constant rate, or, in other words, the logarithmic decrement of successive terms is very nearly constant. Examples of this nearly constant logarithmic decrement, Δ log, are given for 1,000, 300, and 100 km.

Alt.	1,000,000 m		300,000 m		100,000 m	
Term #	Log_{10} Term	Δ log	Log_{10} Term	Δ log	Log_{10} Term	Δ log
1	6.000,00		5.477,12		5.000,00	
		.803,24		1.326,11		1.803,24
2	5.196,76		4.151,01		3.196,76	
		.804,84		1.327,72		1.804,84
3	4.391,92		2.823,29		1.391,92	
		.804,55		1.327,43		1.804,55
4	3.587,37		1.495,86		9.587,37	
		.805,16		1.331,64		1.805,16
5	2.782,21		.164,22		7.782,21	

NOTE: Underline indicates non-significant digits.

3. Extension of the Lambert Series

The departure of the logarithmic decrement from linearity is less than one half of one percent over the five available terms for the altitudes discussed. On the average, the differences between the logarithms of successive terms increase very slightly with increasing term number. It is not unreasonable to assume that this pattern of logarithmic decrement with slowly increasing differences might continue for a considerable number of additional terms in the series. Employing this pattern, the values of the ninth term of Eq. (P-7) for 1,000, 300, and 100 km are 3.6×10^{-1}, 4.9×10^{-6}, and 3.6×10^{-10}, respectively, in standard geopotential meters.

Estimated values of the 6th, 7th, 8th, and 9th terms of Eq. (P-7) for various altitudes may also be determined graphically by plotting the logarithms of the various terms as functions of term number, and connecting those points corresponding to each specific altitude as in Fig. P-1. These lines are then extended linearly to higher term numbers as in the dashed line portion of Fig. P-1. The estimated values of terms 6, 7, 8, and 9 of Eq. (P-7) determined graphically on a figure three times as large as Fig. P-1 are given in Table P-II. Graphically determined values of the ninth term of Eq. (P-7) for altitudes of 1,000, 300, and 100 km differ from the three computed values given above by less than 10 per cent.

A replotting of the data of Table P-I in terms of the value of each term of Eq. (P-7) as a function of altitude is given in Fig. P-2. The estimated values for the 6th, 7th, 8th, and 9th terms of the equation come from Fig. P-1. Figure P-2 clearly shows the contribution which each term in the series makes to the value of geopotential of a given geometric altitude. Figure P-2 demonstrates that for errors in geopotential of less than .1 m', the five term version of Eq. (P-7) may be used only to altitudes of about 280 km, neglecting the possible limitations due to significant figures.

4. Comparison of the Three Methods

The values of geopotential in standard geopotential meters for various geometric altitudes are given in Table P-III. Values designated by H are computed from the simple Eq. (P-3). Values designated by H* are computed from the five defined terms of Eq. (P-7). Values designated by H** are those resulting from the estimated nine-term version of Eq. (P-7). The values of the differences H - H*, H - H**, and H* - H** are also given in Table P-III. The difference H - H** is of particular interest, since it indicates the amount of error in geopotential altitude incurred by using the simple Eq. (P-3) instead of the nine-term version of Eq. (P-7). (Below 100 km altitude the error is less than 0.1 m'.)

5. Limitation of the Five Term Lambert Series Due to Numbers of Terms

Because of the increase of centrifugal acceleration with altitude which is not accounted for in Eq. (P-3), the departure between the value of H from Eq. (P-3) and the value from Eq. (P-7) is expected to increase with altitude. The reversal of the trend resulting in smaller departures (i.e. smaller values in H - H*) above 800 km suggests the inadequacy of the five-term version of Eq. (P-7). The difference H - H** involving the nine-term version of Eq. (P-7) continues to increase to altitudes well over 1000 km. A graph of the various differences is given in Fig. P-3.

6. Limitations of the Five Term Lambert Series Due to Significant Figures

An analysis of the values and number of significant figures of terms 2, 3, 4, and 5 of Eq. (P-7) as listed in Table P-I indicates the limitations which the number of significant figures of each term place upon the computed value of geopotential. The results of this analysis are presented in Fig. P-4. Below 10 km altitude, the number of significant figures in term number 2 is seen to limit the accuracy of Eq. (P-7). From 10 km to about 3,200 km altitude, term number 3 limits the accuracy of the equation, provided a sufficient number of terms is employed so that the number of terms does not limit the accuracy at some altitude below 3,200 km.

7. Combined Limitations of the Lambert Series

The minimum numerical error obtainable with the existing five-term version of Eq. (P-7) is given as the three-segment curve A of Fig. P-5.

Segment a represents the limitation due to significant figures of term 2; segment b represents the limitation due to significant figures of term 3; while segment c represents the limitation due to the termination of the series after term 5. Line B of that same graph represents the minimum numerical error incurred in using the simple equation for geopotential, Eq. (P-3). This error is determined from the values of H - H**. The difference between these two curves (given more accurately by values of H - H* in Table P-III) shows that for altitudes between 10 and 500 km, an improvement of only one significant figure in geopotential altitude is obtained by switching from Eq. (P-3) to the presently available form of Eq. (P-7).

8. Requirements Which the Extended Lambert Series Must Meet

In order to obtain the ten significant figure accuracy desirable for standard atmosphere computations at altitudes of 300, 500, and 1,000 km; three, four, and eight additional terms, respectively, must be developed for Eq. (P-7). Also, the following numbers of significant figures should be available for the several coefficients:

Alt.	300 km	500 km	1,000 km
Term #	Number of Sig. Fig.	Number of Sig. Fig.	Number of Sig. Fig.
2	9	9	10
3	7	8	9
4	6	7	8
5	5	6	7
6	3	5	7
7	2	4	6
8	1	2	5
9		1	4
10			3
11			2
12			2
13			1

These requirements reflect back directly upon Lambert's general expression for g as a function of Z and ϕ; i.e.,

$$g = c_1 - (a_2 + b_2 \cos 2\phi)Z + (a_3 + b_3 \cos 2\phi)Z^2 - (a_4 + b_4 \cos 2\phi)Z^3 + (a_5 + b_5 \cos 2\phi)Z^4 - \dots + \dots \quad \text{(ref. 38)(P-8)}$$

To meet the above requirements for latitude 90°, the coefficients a_2, a_3, a_4, ... a_n and b_2, b_3, b_4, ... b_n of Eq. (P-8) must have numbers of significant figures graphically estimated to be the following:

Alt.	300 km		500 km		1,000 km	
n	a_n	b_n	a_n	b_n	a_n	b_n
2	9	7	9	7	10	7
3	7	5	8	6	9	7
4	6	3	7	4	8	5
5	5	3	6	4	7	5
6	3	1	5	3	7	5
7	2		4	2	6	4
8	1		2	1	5	4
9			1		4	3
10					3	3
11					2	2
12					2	2
13					1	1

To meet standard atmosphere requirements at latitude 45° 32' 40", the number of significant figures required for b_n would be one to two less than required for the case when ϕ = 90°. In any case, b_n must have enough significant figures so as not to invalidate the accuracy of a_n.

9. Conclusions

This analysis is strictly mathematical and does not consider whether it is physically possible to obtain the required number of terms or the necessary accuracy in Eq. (P-4) or Eq. (P-8). If no substantial improvement of Eq. (P-7) is physically possible through a better expression for the acceleration of gravity in Eq. (P-4) or Eq. (P-8) and if one must resort to arbitrary definitions as in the standard sea level pressure, then it is suggested that Eq. (P-2) for g be retained by definition, in which case geopotential is given by the simple Eq. (P-3), sufficiently accurate for most engineering purposes. Only a study of Lambert's unpublished method for the development of Eq. (P-8) will suggest the course to follow.

1st Term	2nd Term	3rd Term	4th Term	5th Term
1,000	.157,312,58	.000,024,656	.000,000,003,866,9	.000,000,000,000,6
5,000	3.932,814,45	.003,082,01	.000,002,416,87	.000,000,001,892,5
10,000	15.731,257,8	.024,656,1	.000,038,669,9	.000,000,060,563
20,000	62.925,031,2	.197,249	.000,618,718	.000,001,938,02
30,000	141.581,320	.665,715	.003,132,26	.000,014,716,8
40,000	251.700,125	1.577,99	.009,899,49	.000,062,016
50,000	393.281,445	3.082,01	.024,168,7	.000,189,259
60,000	566.325,281	5.325,72	.050,116,2	.000,470,94
70,000	770.831,632	8.457,04	.092,846,4	.001,017,88
80,000	1,006.800,499	12.623,9	.158,391,9	.001,984,53
90,000	1,274.231,88	17.974,3	.253,713	.003,576,2
100,000	1,573.125,78	24.656,1	.386,699	.006,056,3
200,000	6,292.503,12	197.249	6.187,18	.193,802
300,000	14,158.132,0	665.715	31.322,6	1.471,68
400,000	25,170.012,5	1,577.99	98.994,9	6.201,6
500,000	39,328.144,5	3,082.01	241.687	18.925,9
600,000	56,632.528,1	5,325.72	501.162	47.094
700,000	77,083.163,2	8,457.04	928.464	101.788
800,000	100,680.049,9	12,623.9	1,583.919	198.452
900,000	127,423.188	17,974.3	2,537.13	357.62
1000,000	157,312.578	24,656.1	3,866.99	605.63

Table P-I. Values of the First Five Terms of Eq.(P-7)for Various Geometric Altitudes as Indicated by the Value of the First Term. (Value of terms in m^1)

NOTE: The underlined figures are beyond the limit of significance, but are carried for smoothness.

Alt. km	6th Term	7th Term	8th Term	9th Term
1	.000,000,000,000			
5	.000,000,000,001			
10	.000,000,000,095	.000,000,000,000		
20	.000,000,006	.000,000,000,018	.000,000,000,000	
30	.000,000,064	.000,000,000,27	.000,000,000,001	
40	.000,000,4	.000,000,002,4	.000,000,000,014	
50	.000,001,4	.000,000,011	.000,000,000,082	.000,000,000,000
60	.000,004,4	.000,000,04	.000,000,000,33	.000,000,000,003
70	.000,011,2	.000,000,12	.000,000,001,35	.000,000,000,014
80	.000,023	.000,000,28	.000,000,003,4	.000,000,000,042
90	.000,048	.000,000,7	.000,000,010	.000,000,000,14
100	.000,095	.000,001,5	.000,000,024	.000,000,000,36
200	.006	.000,18	.000,054	.000,000,160
300	.064	.002,7	.000,125	.000,005,1
400	.40	.024	.001,43	.000,089
500	1.4	.11	.008,2	.000,63
600	4.4	.4	.033	.003,4
700	11.2	1.2	.135	.014
800	23.	2.8	.34	.042
900	48.	7.0	1.0	.14
1,000	95.	15.	2.4	.36

Table P-II. Estimated Values of Terms 6, 7, 8, and 9 of Eq.(P-7) for Various Altitudes (estimated from expanded version of Fig. P-1). (Values of Terms in m^{-1})

Alt. km	H = $\frac{rZ}{r+Z}$	H*, 5 Terms of Eq. (P-7)	H**, 9 Terms of Eq. (P-7)	H - H*	H - H**	H* - H**
1	999.842,712,0	999.842,712,0	↑	.000,000	.000,000	
5	4,996.070,273,5	4,996.070,265		.000,008	.000,008	
10	9,984.293,438	9,984.293,360		.000,078	.000,078	.000,000,000
20	19,937.272,278	19,937.271,60		.000,68	.000,68	.000,000,005
30	29,859.083,61	29,859.081,27		.002,3	.002,3	.000,000,063
40	39,749.873,60	39,749.868,0		.005,6	.005,6	.000,000,397
50	49,609.787,52	49,609.776,6	same as H*	.010,9	.010,9	.000,001,389
60	59,438.969,72	59,438.950,8		.018,9	.018,9	.000,004,36
70	69,237.563,65	69,237.533,6		.030,1	.030,1	.000,011,08
80	79,005.711,87	79,005.667		.045	.045	.000,022,7
90	88,743.556,07	88,743.492		.064	.064	.000,047,3
100	98,451.237,0	98,451.149	↓	.088	.087,4	.000,093,5
200	193,899.431,5	193,898.75		.68	.685	.005,874
300	286,479.921,3	286,477.72	286,477.6	2.2	2.26	.061,42
400	376,320.032,3	376,315.2	376,314.8	4.8	5.22	.377,3
500	463,539.662,8	463,531.1	463,529.8	8.6	9.85	1.297,6
600	548,251.817,0	548,239.1	548,235.1	12.7	16.7	4.030
700	630,563.093,6	630,547.2	630,537.1	15.9	26.0	10.12
800	710,574.133,6	710,558.4	710,537.9	15.7	36.2	20.50
900	788,380.030,4	788,371.6	788,329.7	8.4	50.3	41.86
1,000	864,070.707	864,082.2	864,000.1	-11.5	70.6	82.04

Table P-III. Values of Geopotential in Standard Geopotential Meters for Various Geometric Altitudes at Latitude 45° 32' 40" Computed from Three Different Equations as Indicated, and the Differences Between Those Values of Geopotential, also in Standard Geopotential Meters.

NOTE: The underlined portion of values of H indicates the degree of departure from values of H* and H**.

Nonsignificant figures in values of H* and H** are depressed.

The difference tabulations are reliable to not more than three significant figures and usually only to two.

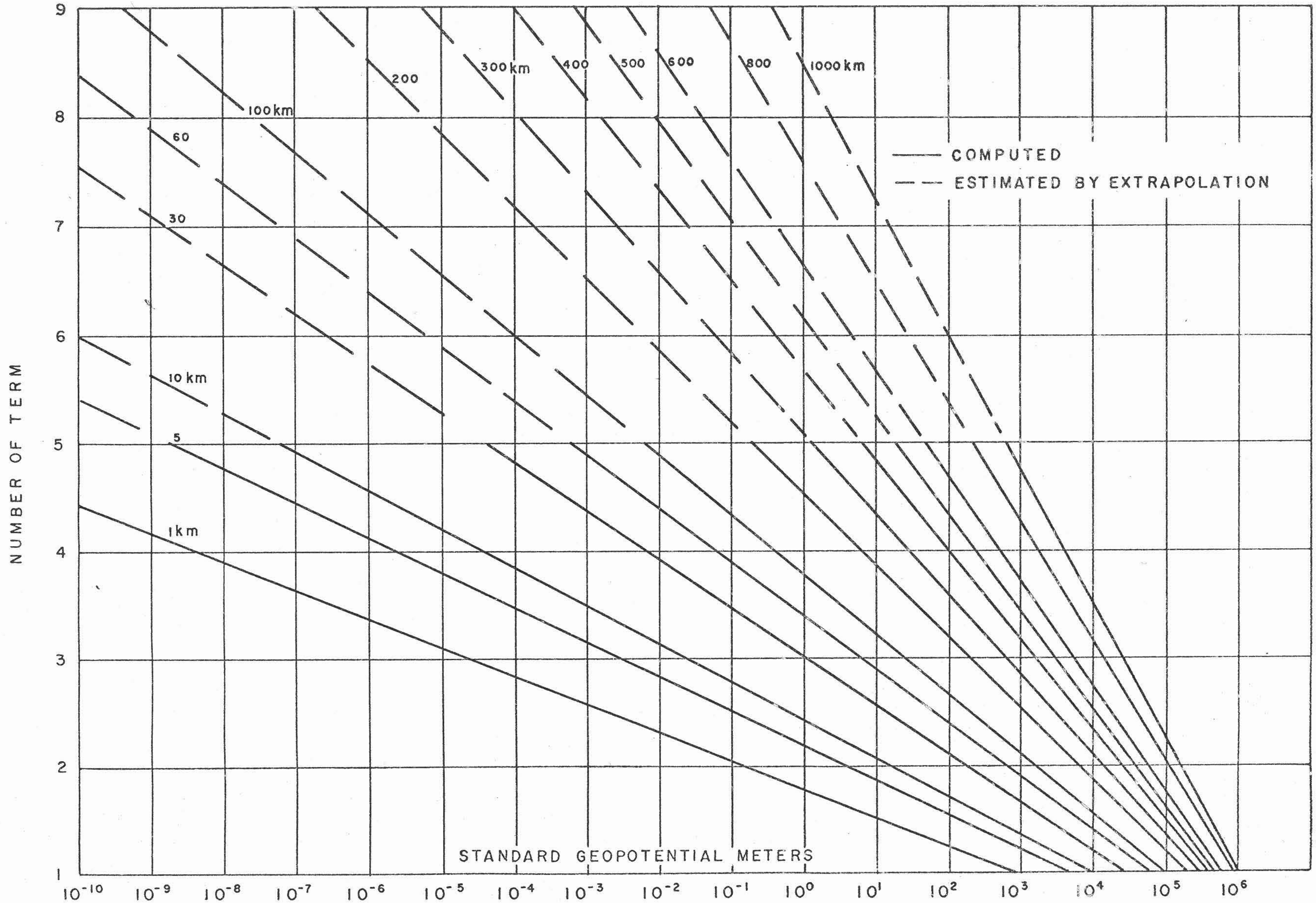


FIGURE P-1 COMPUTED AND ESTIMATED ABSOLUTE VALUES OF 9 TERMS OF EQ.(P-7) FOR VARIOUS ALTITUDES.

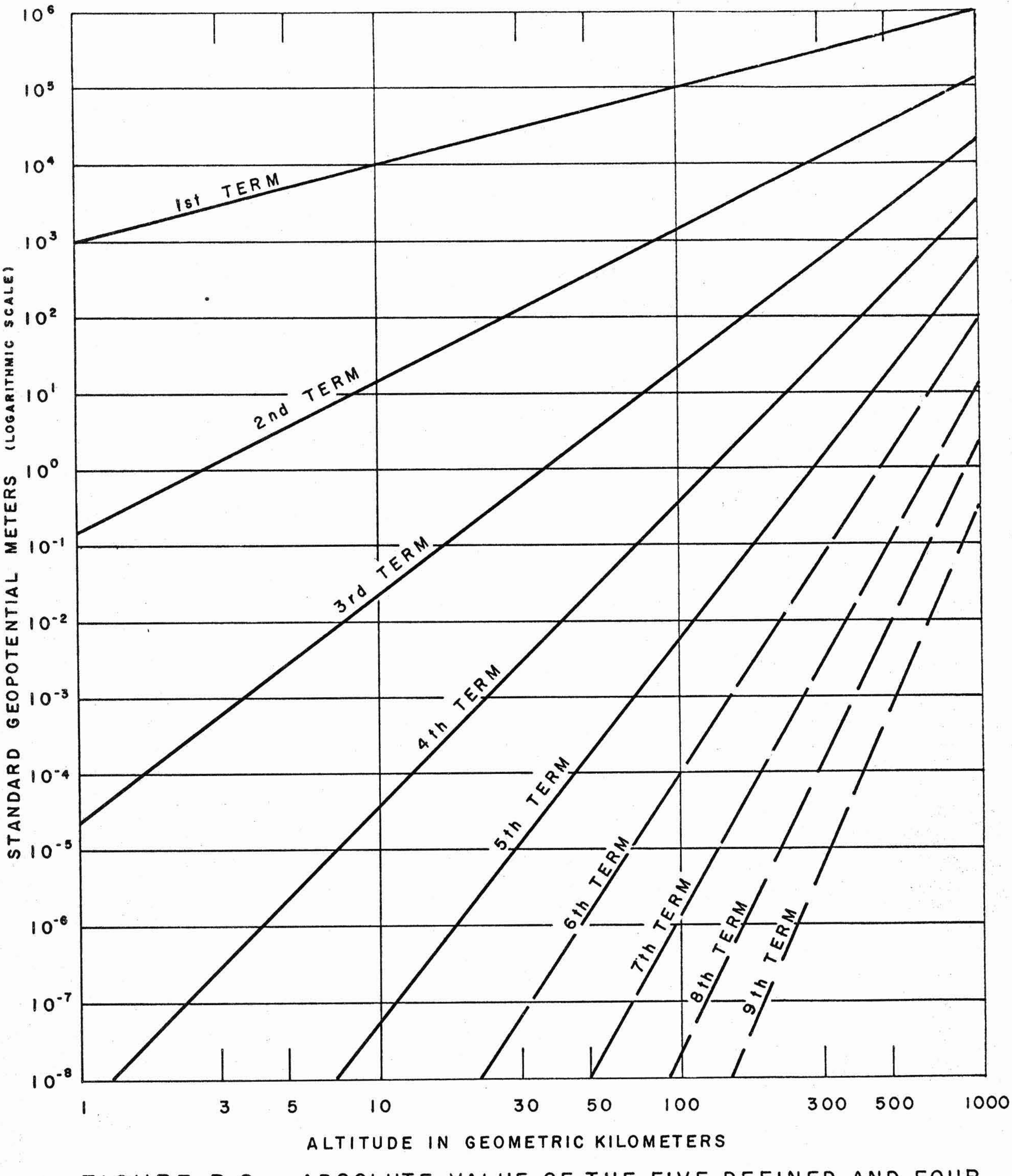


FIGURE P-2 ABSOLUTE VALUE OF THE FIVE DEFINED AND FOUR ESTIMATED TERMS OF EQUATION P-7 AS A FUNCTION OF ALTITUDE

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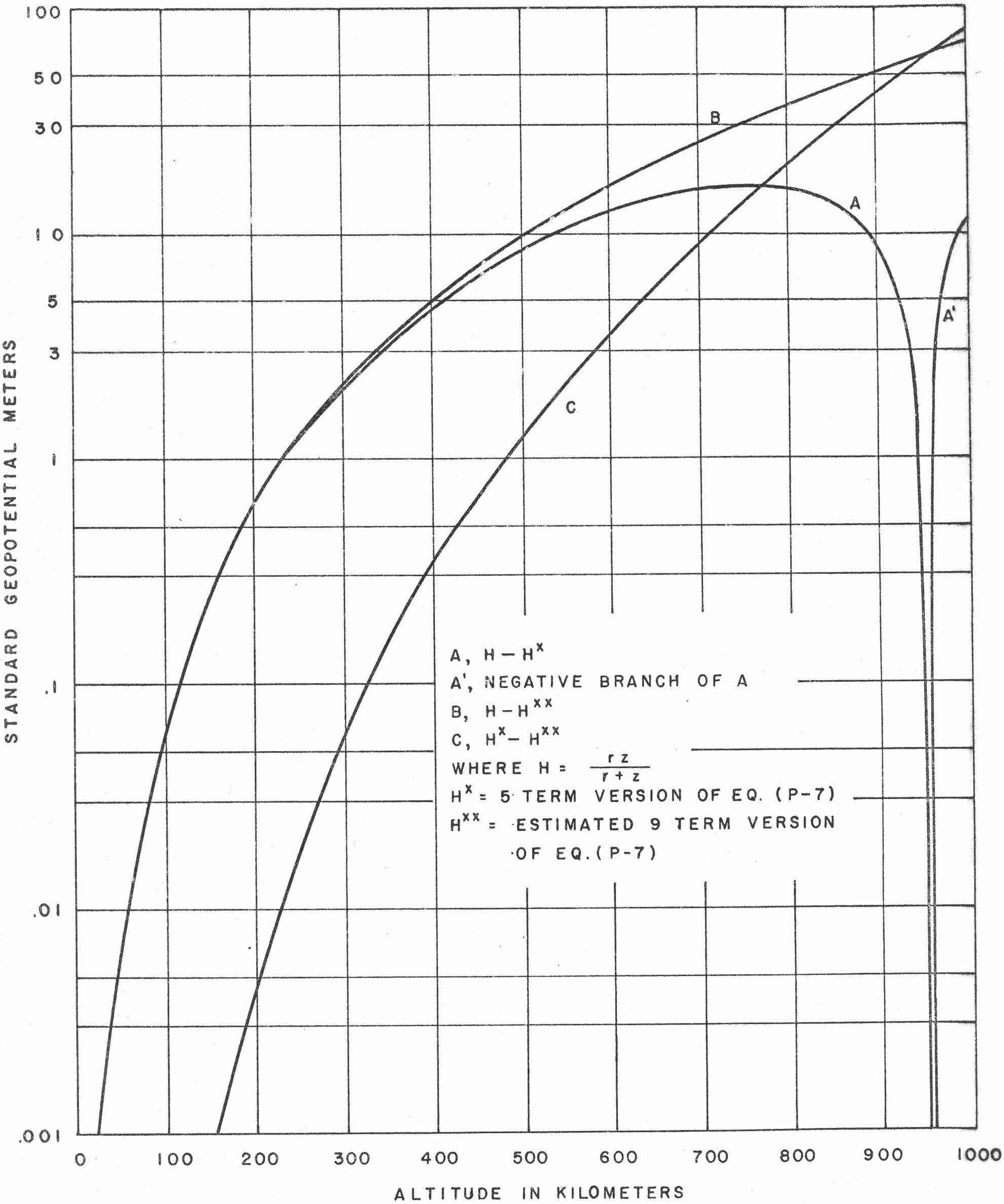

FIGURE P-3 DIFFERENCES BETWEEN VALUES OF GEOPOTENTIAL FROM THREE DIFFERENT EQUATIONS AS SPECIFIED, FOR VARIOUS ALTITUDES.

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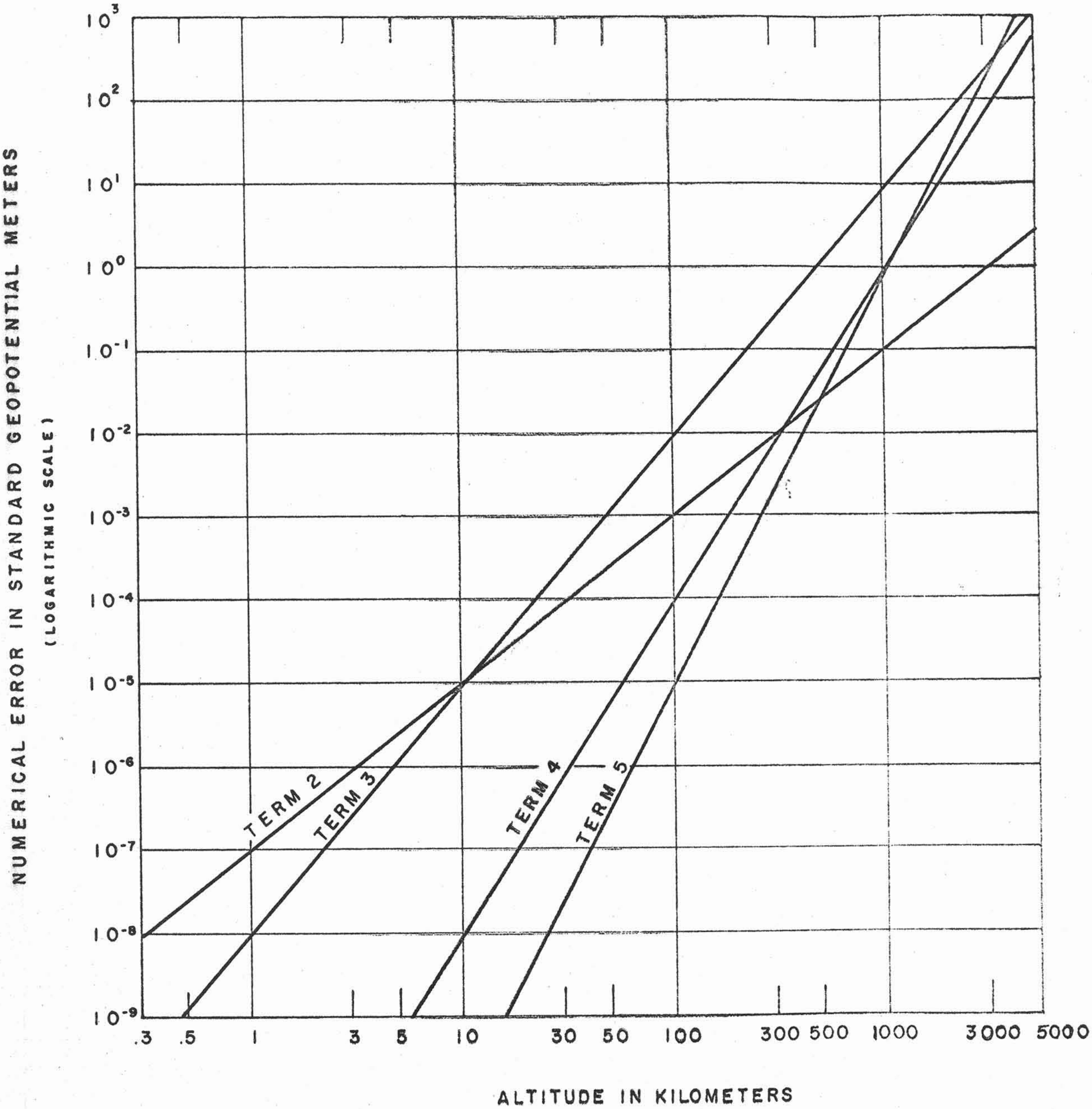

FIGURE P-4 NUMERICAL ERROR CONTRIBUTED BY SIGNIFICANT FIGURE LIMITATIONS IN EACH OF TERMS 2, 3, 4 AND 5 OF EQUATION (P-7) FOR VARIOUS ALTITUDES.

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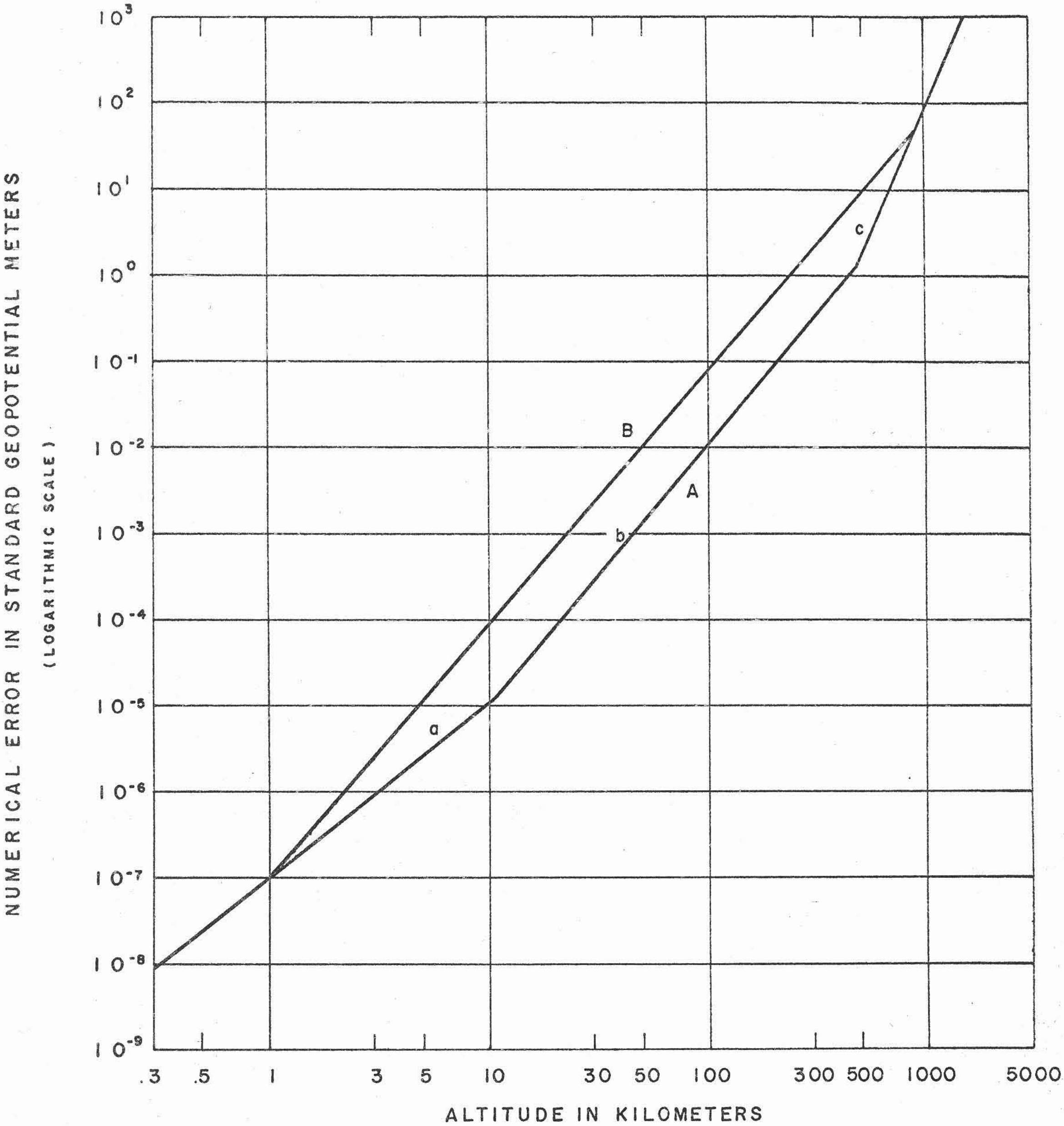

FIGURE P-5 THE ALTITUDE VARIATION OF (A), MINIMUM NUMERICAL ERROR ASSOCIATED WITH THE EXISTING 5 TERM VERSION OF EQUATION P-7 FROM BOTH SIGNIFICANT FIGURE CONSIDERATIONS, AND A LACK OF SUFFICIENT NUMBER OF TERMS.

(B), MINIMUM NUMERICAL ERROR ASSOCIATED WITH THE USE OF THE ADJUSTED VERSION OF $H = \frac{r z}{r + z}$ AT VARIOUS ALTITUDES AT 45° 32' 40" L.

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LIST OF AIR FORCE SURVEYS IN GEOPHYSICS*

(Unclassified)

Number	Title	Author	Date	Security Class.
1		W. K. Widger, Jr.	Mar 52	S-RD
2	Methods of Weather Presentation for Air Defense Operations	W. K. Widger, Jr.	Jun 52	C
3	Some Aspects of Thermal Radiation From the Atomic Bomb	R. M. Chapman	Jun 52	S
4	Final Report on Project 8-52M-1 Tropopause	S. Coroniti	Jul 52	S
5	Infrared as a Means of Identification	N. Oliver J. W. Chamberlain	Jul 52	S
6	Heights of Atomic Bomb Results Relative to Basic Thermal Effects Produced on the Ground	R. M. Chapman G. W. Wares	Jul 52	S-RD
7	Peak Over-Pressure at Ground Zero From High Altitude Bursts	N. A. Haskell	Jul 52	S
8	Preliminary Data From Parachute Pressure Gauges. Operation Snapper. Project 1.1 Shots No. 5 and 8	N. A. Haskell	Jul 52	S-RD
9	Determination of the Horizontal	R. M. Chapman M. H. Seavey	Sep 52	S
10	Soil Stabilization Report	C. Molineux	Sep 52	U
11	Geodesy and Gravimetry, Preliminary Report	R. J. Ford, Maj., USAF	Sep 52	S
12	The Application of Weather Modification Techniques to Problems of Special Interest to the Strategic Air Command	C. E. Anderson	Sep 52	S
13	Efficiency of Precipitation as a Scavenger	C. E. Anderson	Aug 52	S-RD
14	Forecasting Diffusion in the Lower Layers of the Atmosphere	B. Davidson	Sep 52	C
15	Forecasting the Mountain Wave	C. F. Jenkins	Sep 52	U
16	A Preliminary Estimate of the Effect of Fog and Rain on the Peak Shock Pressure From an Atomic Bomb	J. H. Healy H. P. Gauvin	Sep 52	S-RD

*Titles that are omitted are classified.

Number	Title	Author	Date	Security Class.
17	Operation Tumbler-Snapper Project 1.1A. Thermal Radiation Measurements With a Vacuum Capacitor Microphone	M. O'Day J. L. Bohn F. H. Nadig R. J. Cowie, Jr.	Sep 52	C-RD
18	Operation Snapper Project 1.1. The Measurement of Free Air Atomic Blast Pressures	J. O. Vann, Lt Col., USAF N. A. Haskell	Sep 52	S-RD
19	The Construction and Application of Contingency Tables in Weather Forecasting	E. W. Wahl R. M. White H. A. Salmela	Nov 52	U
20	Peak Overpressure in Air Due to a Deep Under-water Explosion	N. A. Haskell	Nov 52	S
21	Slant Visibility	R. Penndorf B. Goldberg D. Lufkin	Dec 52	U
22	Geodesy and Gravimetry	R. J. Ford, Maj., USAF	Dec 52	S
23	Weather Effect on Radar	D. Atlas V. G. Plank W. H. Paulsen A. C. Chmela J. S. Marshall T. W. R. East K. L. S. Gunn	Dec 52	U
24	A Survey of Available Information on Winds Above 30,000 Ft.	C. F. Jenkins	Dec 52	U
25	A Survey of Available Information on the Wind Fields Between the Surface and the Lower Stratosphere	W. K. Widger, Jr.	Dec 52	U
26		A. L. Aden L. Katz	Dec 52	S
27		N. A. Haskell	Dec 52	S
28	A-Bomb Thermal Radiation Damage Envelopes for Aircraft	R. H. Chapman G. W. Wares M. H. Seavey	Dec 52	S-RD
29	A Note on High Level Turbulence Encountered by a Glider	J. Kuettner	Dec 52	U

Number	Title	Author	Date	Security Class.
30	Results of Controlled-Altitude Balloon Flights at 50,000 to 70,000 Feet During September 1952	T. O. Haig Maj., USAF R. A. Craig	Feb 53	U
31	Conference: Weather Effects on Nuclear Detonation	B. Grossman, Ed.	Feb 53	S-RD
32	Operation IVY Project 6.11. Free Air Atomic Blast Pressure and Thermal Measurements	N. A. Haskell P. R. Gast	Mar 53	S-RD
33	Variability of Subjective Cloud Observations -- 1	A. M. Galligan	Mar 53	U
34	Feasibility of Detecting Atmospheric Inversions by Electromagnetic Probing	A. L. Aden	Mar 53	U
35	Flight Aspects of the Mountain Wave	C. F. Jenkins J. Kuettner	Apr 53	U
36	Report on Particle Precipitation Measurements Performed During the Buster Tests at Nevada	A. J. Parziale	Apr 53	S-RD
37	Critical Envelope Study for the XB-63, 13-52A, and F-89	N. A. Haskell M. H. Seavey R. M. Chapman	Apr 53	S
38	Notes on the Prediction of Overpressures From Very Large Thermo-Nuclear Bombs	N. A. Haskell	Apr 53	S
39	Atmospheric Attenuation of Infrared Oxygen Afterglow Emission	N. J. Oliver J. W. Chamberlain	Apr 53	S
40		R. E. Hanson, Capt, USAF	May 53	S
41	The Silent Area Forecasting Problem	W. K. Widger, Jr.	May 53	S
42	An Analysis of the Contrail Problem	R. A. Craig	Jun 53	C
43	Sodium in the Upper Atmosphere	L. E. Miller	Jun 53	U
44	Silver Iodide Diffusion Experiments Conducted at Camp Wellfleet, Mass., During July-August 1952	P. Goldberg A. J. Parziale G. Faucher B. Manning H. Lettau	Jun 53	U
45	The Vertical Distribution of Water Vapor in the Stratosphere and the Upper Atmosphere	L. E. Miller	Sep 53	U
46	Operation IVY Project 6.11. Free Air Atomic Blast Pressure and Thermal Measurements -- Final Report	N. A. Haskell J. O. Vann, Lt Col, USAF P. R. Gast	Sep 53	S-RD

Number	Title	Author	Date	Security Class.
47	Critical Envelope Study for the B61-A	N. A. Haskell R. M. Chapman M. H. Seavey	Sep 53	S-RD
48	Operation Upshot-Knothole Project 1.3. Free Air Atomic Blast Pressure Measurements. Revised Report	N. A. Haskell R. M. Brubaker, Maj., USAF	Nov 53	S-RD
49	Maximum Humidity in Engineering Design	N. Sissenwine	Oct 53	U
50	Probable Ice Island Locations in the Arctic Basin, January 1954	A. P. Crary I. Browne	May 54	U
51	Investigation of TRAC for Active Air Defense Purposes	G. W. Wares R. Penndorf V. G. Plank B. H. Grossman	Dec 53	S-RD
52	Radio Noise Emissions During Thermonuclear Reactions	T. J. Keneshea	Jun 54	C
53	A Method of Correcting Tabulated Rawinsonde Wind Speeds for Curvature of the Earth	R. Leviton	Jun 54	U
54	A Proposed Radar Storm Warning Service for Army Combat Operations	M. G. H. Ligda	Aug 54	U
55	A Comparison of Altitude Corrections for Blast Overpressure	N. A. Haskell	Sep 54	S
56	Attenuating Effects of Atmospheric Liquid Water on Peak Overpressures from Blast Waves	H. P. Gauvin J. H. Healy M. A. Bennet	Oct 54	S
57	Windspeed Profile, Windshear, and Gusts for Design of Guidance Systems for Vertical Rising Air Vehicles	N. Sissenwine	Nov 54	U
58	The Suppression of Aircraft Exhaust Trails	C. E. Anderson	Nov 54	U
59	Preliminary Report on the Attenuation of Thermal Radiation From Atomic or Thermonuclear Weapons	R. M. Chapman M. H. Seavey	Nov 54	S-RD
60	Height Errors in a Rawin System	R. Leviton	Dec 54	U
61	Meteorological Aspects of Constant Level Balloon Operations	W. K. Widger, Jr. M. L. Haas E. A. Doty, Lt Col. E. M. Darling, Jr. S. B. Solot	Dec 54	S

Number	Title	Author	Date	Security Class.
62	Variations in Geometric Height of 30 to 60,000 Ft. Pressure Altitudes	N. Sissenwine A. E. Cole W. Baginsky	Dec 54	C-MA
63	Review of Time and Space Wind Fluctuations Applicable to Conventional Ballistic Determinations	W. Baginsky N. Sissenwine B. Davidson H. Lettau	Dec 54	U
64	Cloudiness Above 20,000 Feet for Certain Stellar Navigation	A. E. Cole	Jan 55	S
65	The Feasibility of the Identification of Hail and Severe Storms	D. Atlas R. Donaldson	Jan 55	U
66	The Rate of Rainfall Frequencies Over Selected Air Routes and Destinations	A. E. Cole N. Sissenwine	Mar 55	S
67	Some Considerations on the Modelling of Cratering Phenomena in Earth	N. A. Haskell	Apr 55	S-RD
68	The Preparation of Extended Forecasts of the Pressure Height Distribution in the Free Atmosphere Over North America by Use of Empirical Influence Functions	R. M. White	May 55	U
69	Cold Weather Effects on B-62 Launching Personnel	N. Sissenwine	Jun 55	S
70	Atmospheric Pressure Pulse Measurements: Operation Castle	E. Flauraud	Aug 55	S-RD
71	Refraction of Shock Waves in the Atmosphere	N. A. Haskell	Aug 55	S
72	Wind Variability as a Function of Time at Muroc, California	B. Singer	Sep 55	U
73	The Atmosphere	N. C. Gerson	Sep 55	U
74	Areal Variation of Ceiling Height	W. Baginsky A. E. Cole	Oct 55	C-MA
75	An Objective System for Preparing Operational Weather Forecasts	I. A. Lund	Nov 55	U
76	The Practical Aspect of Tropical Meteorology	C. E. Palmer C. W. Wise L. J. Stempson G. H. Duncan	Sep 55	U
77	Remote Determination of Soil Trafficability by Aerial Penetrometer	C. Molineux	Oct 55	U

Number	Title	Author	Date	Security Class.
78	Effects of the Primary Cosmic Radiation on Matter	H. O. Curtis	Jan 56	U
79	Tropospheric Variations of Refractive Index at Microwave Frequencies	C. F. Campen A. E. Cole	Oct 55	U
80	A Program to Test Skill in Terminal Forecasting	I. I. Gringorten I. A. Lund M. A. Miller	Jun 55	U
81	Extreme Atmospheres and Ballistic Densities	N. Sissenwine A. E. Cole	Jul 55	U
82	Rotational Frequencies and Absorption Coefficients of Atmospheric Gases	S. N. Ghosh H. D. Edwards	Mar 56	U
83	Ionospheric Effects on Positioning of Vehicles at High Altitudes	W. Pfister T.J. Keneshea	Mar 56	U
84	Pre-Trough Winter Precipitation Forecasting	P. W. Funke	Feb 57	U
85	Geomagnetic Field Extrapolation Techniques -- An Evaluation of the Poisson Integral for a Plane	J. F. McClay P. Fougere	Feb 57	S