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DYNAMIC LOADS OF A LAUNCH VEHICLE DUE TO INFLIGHT WINDS

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INTRODUCTION

Analysis of the stability and dynamic load environment of a launch vehicle resulting from atmospheric disturbances is a very complex problem. To determine the dynamic load environment of the vehicle requires an adequate description of the wind field, vehicle dynamics and control system. The essentials of such a study, namely, methods of analysis, wind field specification and representative vehicle response parameters for evaluation, are of equal importance. This paper is concerned with the mathematical foundations of the vehicle model and method of analysis.

Under the assumption that we have an exact trajectory solution for the vehicle, the nonlinear system of complete perturbation equations is derived. The vehicle trajectory equations are perturbed to obtain the small deviations of the vehicle from a reference trajectory as a system of linear differential equations with time varying coefficients. The system is expressed in vector-matrix form and solved by an algorithm based on extended continuation. The method is shown to be efficient for numerical computation and the form of solution provides physical insight into the evolutionary character of the response process.

Application of the technique is exemplified by an evaluation of Saturn V dynamic response to a synthetic wind profile representing a 95% quasi-steady wind, 99% wind build-up rate and superimposed discrete gust.

VEHICLE MODEL

Governing Equations of a Launch Vehicle

Although there are no basic generic differences, the dynamic variables of a launch vehicle are usually divided into two classes. The variables that are used to compute the boost trajectory of the vehicle are included in the first class, and those used to determine "local" stability and dynamic load environment are included in the second class. This dichotomy results from the evolutionary order involved in the design of a launch vehicle. In practice, a launch vehicle is regarded first as a point mass or rigid body with certain specific and undetermined characteristics. The undetermined characteristics are then established by detailed trajectory computations that fit the vehicle to the desired mission. Such calculations are sometimes performed with the complete nonlinear rigid body and control equations. On the other hand, it frequently happens that not all of the ideal trajectory equations are used, and often different systems are employed, the particular system being determined by the type and nature of problem to be solved by the trajectory calculations. Except with very special configurations or types of motion the differential equations which describe the deviation of the dynamic state of the vehicle from that described by the reference trajectory solution are nonlinear and do not admit exact solution.

Assume that the governing equations of the launch vehicle have been written as a system of ordinary differential equations (i.e., any partial differential equations have been reduced to a system of ordinary differential equations by the use of appropriate systems of orthogonal functions), and that these have been reduced to an equivalent system of first order equations.[1] Denote by $X(t)$ the N_1-element column matrix of dynamic variables that are used in the trajectory calculations, and by $Y(t)$ the N_2-element column matrix of dynamic variables that are not included in the trajectory calculations (i.e., the elements of $X(t)$ are the dynamic variables of the first class, and the elements of $Y(t)$ are those of the second class). Accordingly, the complete system of dynamic variables of a launch vehicle is given by the N-element column matrix

$$\begin{Bmatrix} X(t) \\ Y(t) \end{Bmatrix}, \quad N = N_1 + N_2 .$$

Thus, it follows that the governing equations of the launch vehicle can be written as

$$\frac{d}{dt}\begin{Bmatrix} X(t) \\ Y(t) \end{Bmatrix} = \begin{Bmatrix} F(X(t),\, Y(t),\, t) \\ G(X(t),\, Y(t),\, t) \end{Bmatrix} + \begin{Bmatrix} f(t) \\ g(t) \end{Bmatrix} \tag{1}$$

in which $F(X(t), Y(t), t)$, $G(X(t), Y(t), t)$, $f(t)$, $g(t)$ are known matrix functions of their indicated arguments.

Denote by $X^*(t)$ a known, exact solution of the trajectory equations. Ideally, the

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trajectory equations would be obtained from (1) by taking the first N_1 equations and setting $Y(t) = 0$. In practice, it frequently happens that not all of the ideal trajectory equations are used. To account for such a possibility, assume that the trajectory equations are given by

$$\frac{d}{dt} X^*(t) = E(X^*(t), t) + f^*(t) \tag{2}$$

where

$$\begin{aligned} F(X^*(t), 0, t) &= E(X^*(t), t) + K(X^*(t), t) \\ f(t) &= f^*(t) + k(t) \end{aligned} \tag{3}$$

The N_1-column matrices $K(X^*(t), t)$, $k(t)$ represent those terms in the complete rigid body equations and external excitations that are not included in the trajectory analysis. Since $X^*(t)$ is assumed to be a known, exact solution to the reference equations (2), we expect that the differences

$$\begin{Bmatrix} X^*(t) \\ 0 \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} X(t) \\ Y(t) \end{Bmatrix}$$

are small. Accordingly, we seek solutions to the complete system (1) in the form

$$\begin{aligned} X(t) &= X^*(t) + x(t) \\ Y(t) &= 0 + y(t) \end{aligned} \tag{4}$$

In passing, we should point out three obvious reasons for seeking solutions in the form (4). First, any and all nonlinearities that occur in the trajectory equations (2) will be subtracted out of the linear motion of deviation system obtained from substituting (4) into (1) and expanding about the trajectory solution. Second, since the values of the elements of the column matrices $x(t)$, $y(t)$ measure the deviation between solutions of the trajectory equations and the complete system of governing equations (1), they give a direct indication of stability (in the sense of Liapounov) of the launch vehicle about its rigid body trajectory. These column matrices also provide a direct means of determining the dynamic load environment of the structure, for if $a(t)$ and $b(t)$ are the column matrices giving the dynamic load environment of the structure at a particular point on the vehicle for unit values of the variables $X(t)$ and $Y(t)$, respectively, then the total load environment at that point is

$$a^T(X^*+x) + b^T y \tag{5}$$

Third, if we regard equations (4) as a system of transformations, these transformations take the solution of the trajectory equations into the origin. Hence $x(t)$ and $y(t)$ may be considered as components of a position vector in an N-dimensional space that starts at the origin and extends to the point with coordinates $(x_i(t), y_i(t))$.

To obtain the governing equations for $x(t)$ and $y(t)$, substitute (4) into (1),

$$\frac{d}{dt}\begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix} = \begin{Bmatrix} F(X^*+x, y, t) - E(X^*, t) \\ G(X^*+x, y, t) \end{Bmatrix} + \begin{Bmatrix} k(t) \\ g(t) \end{Bmatrix} \tag{6}$$

using (2) and (3). These equations are the "complete equations of disturbed motion", in that $x(t)$, $y(t)$ can be interpreted as perturbations of the trajectory solution $X^*(t)$, arising when the complete governing equations are used (i.e., the perturbations that arise from the inclusion of the dynamic variables of the second class and their interaction with those of the first class that are used in the trajectory calculations).

The disturbed equations (6) are, in general, nonlinear. We require the linear form of these equations. Expanding $F(X^*+x,y,t)$, $G(X^*+x,y,t)$ about

$$\begin{Bmatrix} X^*(t) \\ 0 \end{Bmatrix},$$

substituting the results into system (6), and using (3) we get, after discarding higher order terms in $x(t)$, $y(t)$,

$$\frac{d}{dt} Z(t) = A(t)\, Z(t) + \omega(t) \tag{7}$$

$$Z(t) = \begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix}, \quad \omega(t) = \begin{Bmatrix} K(X^*,t) + k(t) \\ G(X^*,0,t) + g(t) \end{Bmatrix}$$

$$A = \begin{bmatrix} A_{F_X} & A_{F_Y} \\ A_{G_X} & A_{G_Y} \end{bmatrix} \quad A_{F_X} = \left[\frac{\partial F}{\partial X}\right]_{\substack{X=X^* \\ Y=0}} \quad A_{F_Y} = \left[\frac{\partial F}{\partial Y}\right]_{\substack{X=X^* \\ Y=0}} \quad A_{G_X} = \left[\frac{\partial G}{\partial X}\right]_{\substack{X=X^* \\ Y=0}}$$

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$$A_{G_Y} = \left[\frac{\partial G}{\partial Y}\right]_{\substack{X=X^* \\ Y=0}}$$

System (7) is the linear equation of deviation associated with the systems (1) and (2). These are linear equations, but with variable coefficients, because the matrice $A_{F_X}, \ldots,$ are functions of $X^*(t)$, of which the elements are known functions of time.

In most applications, the linear equations of small motion of deviation (7) are derived directly without reference to the governing equations of the launch vehicle (1) and subsequent development of the complete equations of disturbed motion (6). This is usually accomplished by making certain simplifying assumptions. Such a procedure is valid only if the systems (6) and (7) exhibit the same behavior; ie., if the solutions to the linear system (7) and the corresponding solutions to the complete variational equations of disturbed motion (6) differ by a small amount. An acceptable measure of smallness has been established elsewhere, and will not be repeated here.

Frequently, another method of analysis of a launch vehicle is to use system (7), where A(t) is evaluated for a fixed value of t, and held constant during the solution of the system for a certain duration of time: this procedure is repeated for other values of t. If the results indicate that there is an adequate margin of stability and that the dynamic load environment is within acceptable limits, it is concluded that the design is adequate. Similarly, such a procedure is valid only if the solutions of system (7) and the solution to the system obtained from (7) by holding the matrix A(t) constant, exhibit the same behavior. In any event, the local dynamic state of a vehicle is described by a system of linear differential equations. An algorithm for the solution of such a system is derived in the following section.

Algorithm for Solution of the Linear Perturbation Equations

Several methods may be used to determine the vehicle response to atmospheric disturbances. These include numerical integration of the complete equations of motion by use of a digital computer, integration of the equations of motion by use of an analog computer and generalized harmonic analysis using the spectra of turbulence, or a combination of these methods. A digital simulation offers a high degree of accuracy and sophistication, but the amount of time required for one trajectory simulation is excessive and therefore economically undesirable. A high speed analog simulation affords a means of evaluating the vehicle response to many individual detailed wind profiles. However, computer limitations affect the accuracy and curtail the complexity of the analysis, necessitating correlation with the digital simulation.

That some of these difficulties can be obviated, was shown above. With the reference trajectory known, it was shown that the dynamic load environment consisted of two parts; one resulting from the reference trajectory and the other from the small deviation from that state. Thus to obtain the local dynamic environment of the vehicle, it simply becomes necessary to solve the linear system (7).

An algorithm for solution of the system (7) is next derived. The resulting form of solution appears highly efficient for digital computation and exhibits desirable features of present digital and analog methods. The approach is less time consuming than direct numerical integration and still permits parametric evaluation of a given time regime without reference to the entire flight time history. The form of solution will also be seen to provide physical insight into the vehicle response process. In order to illustrate the technique a few mathematical preliminaries are first reviewed.[1] The general solution of the inhomogeneous system (7) over the interval of flight (t_o, t) is given by

$$Z(t) = H(t,t_o)Z(t_o) + H(t,t_o)\int_{t_o}^{t} H^{-1}(\tau,t_o)\,\omega(\tau)d\tau \tag{8}$$

in which the fundamental normal matrix H and its inverse satisfy

$$\left.\begin{matrix} \frac{dH}{dt} = A(t)H \\ H(t_o,t_o) = I \end{matrix}\right\} \quad \text{and} \quad \left.\begin{matrix} \frac{dH^{-1}}{dt} = H^{-1}A(t) \\ H^{-1}(t_o,t_o) = I \end{matrix}\right\} \tag{9}$$

where I is the identity matrix. If we suppose that A(t) can be represented as a sum

$$A(t) = \alpha(t) + \beta(t) \tag{10}$$

then it can be shown that

$$\begin{aligned} H(t,t_o) &= \phi(t,t_o)\,\psi(t,t_o) \\ H^{-1}(t,t_o) &= \psi^{-1}(t,t_o)\,\phi^{-1}(t,t_o) \end{aligned} \tag{11}$$

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in which the fundamental normal matrices satisfy

$$\left.\begin{aligned}\frac{d\phi}{dt} &= \alpha(t)\phi \\ \phi(t_o,t_o) &= I\end{aligned}\right\} \quad \text{and} \quad \left.\begin{aligned}\frac{d\psi}{dt} &= \mu(t)\psi \\ \psi(t_o,t_o) &= I\end{aligned}\right\} \tag{12}$$

with

$$\mu(t) = \phi^{-1}\beta(t)\phi. \tag{13}$$

In particular, if α = const, we have

$$\phi(t,t_o) = \exp(t-t_o)\alpha \tag{14}$$

$$\psi(t,t_o) = I + \beta(t_o)\ (t-t_o) + [\beta^2(t_o) + \beta(t_o)\alpha - \alpha\beta(t_o) + \dot{\beta}(t_o)]\frac{(t-t_o)^2}{2} + \ldots\ldots$$

In view of (10-13), it follows that the solution for the state vector $Z(t)$ can be expressed as

$$Z(t) = Z^*(t) + w(t)$$

$$Z^*(t) = \phi(t,t_o)[Z(t_o) + \int_{t_o}^{t} \phi^{-1}(\tau,t_o)\omega(\tau)\ d\tau] \tag{15}$$

$$w(t) = \phi(t,t_o)\ \left\{ [\psi(t,t_o) - I]\ Z(t_o) + \int_{t_o}^{t} [\psi(t,t_o)\psi^{-1}(\tau,t_o) - I]\ \phi^{-1}(\tau,t_o)\omega(\tau)d\tau \right\}$$

with α constant, $Z^*(t)$ can be regarded as the constant, coefficient solution $Z(t)$ over the flight interval (t_o,t), and $w(t)$ as the difference between the variable coefficient solution and the corresponding constant coefficient approximation.

Suppose t_1 to be any point of the interval (t_o,t). Then

$$H(t,t_o) = H(t,t_1)\ H(t_1,t_o)$$

More generally, if the interval (t_o,t) is divided into many numbers of smaller intervals (t_o,t_1), $(t_1,t_2)\ldots$, (t_{n-1},t), we have

$$H(t,t_o) = H_n(t,t_{n-1})\ H_{n-1}(t_{n-1},t_{n-2})\ldots\ H_2(t_2,t_1)\ H_1(t_1,t_o) = \prod_{i=n}^{1} H_n \tag{16}$$

Suppose the solution of the system (7) to be required over the interval of flight (t_o,t). If all points of the interval are ordinary, it is theoretically possible to obtain the solution by use of the fundamental series relative to the initial point t_o. However, for purposes of practical computation, it is preferable to use the series relative to t_o only up to some intermediate point of the range, say t_1, and to continue the solution thereafter by the use of series appropriate to the point t_1 or to some other point of the interval (t_1,t). More generally, if n-1 successive intermediate points $t_1, t_2, \ldots, t_{n-1}$ are taken, it is possible to base the computation in a typical subinterval on the series relative to a suitable point of that subinterval. The solution can be carried over from subinterval to subinterval by identification of the initial conditions for any subinterval with the end condition for the preceding subinterval. A solution of (7), valid in each step, will be assumed known if t_i denotes some chosen point of the i^{th} subinterval (t_{i-1},t_i), then the solution appropriate to that subinterval may be supposed expressed either as a series of powers of $t-t_i$, or in terms of the fundamental normal matrix, or in any other convenient form. Moreover, if t_i is chosen at the initial point t_{i-1}, the procedure will be simplified considerably. With $t_i = t_{i-1}$, the solution to be used in the i^{th} subinterval will be denoted by

$$Z(t) = H_i(t,t_{i-1})\ Z(t_{i-1}) + H_i(t,t_{i-1}) \int_{t_{i-1}}^{t} H_i^{-1}(\tau,t_{i-1})\omega(\tau)d\tau \tag{17}$$

On application of (17) to the n assumed subintervals, we get the sequence of relations

$$Z(t_1) = H_1(t_1,t_o)\ Z(t_0) + H_1(t_1,t_o) \int_{t_o}^{t_1} H_1^{-1}(\tau,t_o)\ \omega(\tau)d\tau$$

$$Z(t_2) = H_2(t_2,t_1)\ Z(t_1) + H_2(t_2,t_1) \int_{t_1}^{t_2} H_2^{-1}(\tau,t_1)\ \omega(\tau)d\tau$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$Z(t_{n-1}) = H_{n-1}(t_{n-1},t_{n-2})\ Z(t_{n-2}) + H_{n-1}(t_{n-1},t_{n-2})\int_{t_{n-2}}^{t_{n-1}} H_{n-1}^{-1}(\tau,t_{n-2})\omega(\tau)d\tau$$

$$Z(t) = H_n(t,t_{n-1})\, Z(t_{n-1}) + H_n(t,t_{n-1}) \int_{t_{n-1}}^{t} H_n^{-1}(\tau,t_{n-1})\, \omega(\tau)d\tau$$

Therefore, it follows that

$$Z(t) = \prod_{i=n}^{1} H_i Z(t_o) + \sum_{j=n}^{1} \prod_{i=n}^{j} H_i \int_0^{\tau_j} H_j^{-1}(t_{j-1}+\tau, t_{j-1})\omega(\tau+t_{j-1})d\tau \tag{18}$$

$$\tau_j = t_j - t_{j-1}$$

If we suppose A(t) to be represented as a sum in each subinterval

$$A_i(t) = \alpha_i(t) + \beta_i(t)$$

Then, analogous to (15), it follows

$$Z(t) = Z^*(t) + w(t)$$

$$Z^*(t) = \prod_{i=n}^{1} \phi_i\, Z(t_o) + \sum_{j=n}^{1} \prod_{i=n}^{j} \phi_i \int_0^{\tau_j} \phi_j^{-1}(t_{j-1}+\tau, t_{j-1})\, \omega(\tau+t_{j-1})d\tau \tag{19}$$

$$w(t) = \prod_{i=n}^{1} \phi_i[\psi_i - I_i]\, Z(t_o) + \sum_{j=n}^{1} \prod_{i=n}^{j} \phi_i \int_0^{\tau_j} [\psi_i\psi_j^{-1}(t_{j-1}+\tau,\ t_{j-1}) - I_j]\phi_j^{-1}(t_{j-1}+\tau,\ t_j-1)$$

$$\omega(t_{j-1}+\tau)d\tau$$

In order to appreciate the physical significance of the above formulae, consider the general solution, equation (18). The first term in (18) simply represents the state of the system at time t due to the initial conditions. Upon expanding the summation it can be seen that the first term in the sum represents the state of the system at time t due to the load applied in the time interval $t_{n-1}-t$, the second term represents the state of the system at time t due to the load applied in the interval $t_{n-2}-t_{n-1}$ and so forth. Thus, the "memory" characteristics of the system are explicitly demonstrated.

Formulae (19) afford us a simple method for stability and dynamic load environment studies of launch vehicles during atmospheric flight. As a first approximation to the solution we now substitute for each variable element A(t) in the typical subinterval $t_i - t_{i-1} = \tau_i$ an average value taken over that subinterval, and determine the fundamental matrix $H_i(t_i, t_{i-1})$ on the assumption that the elements of A(t) have these constant values, that is,

$$A_i(t) = \alpha_i,\ H_i(t_i,\ t_{i-1}) = \phi_i(t_i, t_{i-1})$$

Then, since α_i is a matrix of constants

$$\phi_i(t_i, t_{i-1}) = e^{\alpha_i\tau_i} \tag{20}$$

Thus, the first complete approximate solution is

$$Z(t) = Z^*(t) = \prod_{i=n}^{1} e^{\alpha_i\tau_i} Z(t_o) + \sum_{j=n}^{1} \prod_{i=n}^{j} e^{\alpha_i\tau_i} \int_0^{\tau_j} e^{-\alpha_j\tau}\, \omega(\tau+t_{j-1})\, d\tau \tag{21}$$

The choice of the average values for the elements of A(t) in any subinterval is provided with good accuracy by the arithmetic mean

$$\alpha_{i(jk)} = \frac{1}{\tau_i} \int_0^{\tau_i} A_{i(jk)}(\tau+t_{i-1})d\tau$$

which may be carried out by Simpson's rule of any other convenient scheme. The individual matrices may themselves be computed exactly by the use of Sylvester's theorem or a confluent form of Sylvester's theorem. In practice, the characteristic roots are known from prior stability studies, so that this is a simple computation to perform. The integral

$$\int_0^{\tau_j} e^{-\alpha_j\tau}\omega(\tau+t_{j-1})d\tau$$

may be evaluated by any convenient integration scheme, the choice, of course, being dictated by the behavior of $\omega(\tau+t_{j-1})$.

As a second approximation to the solution, we now assume

$$A_i(t) = \alpha_i + \beta_i(t)$$

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in which the elements of β_i are supposed small. In practice, the assumption is usually realized. We suppose ψ_i, ψ_i^{-1} to be expandable in series

$$\psi_i(t,t_{i-1}) = I + \beta_i(t_{i-1})(t-t_{i-1}) + [\beta_i^2(t_{i-1}) + \beta_i(t_{i-1})\alpha_i - \alpha_i\beta_i(t_{i-1}) + \dot{\beta}_i(t_{i-1})] \frac{(t-t_{i-1})^2}{2} + \ldots\ldots \quad (22)$$

$$\psi_i^{-1}(t,t_{i-1}) = I - \beta_i(t_{i-1})(t-t_{i-1}) + [\beta_i^2(t_{i-1}) - \dot{\beta}_i(t_{i-1}) - \beta_i(t_{i-1})\alpha_i + \alpha_i\beta_i(t_{i-1})] \frac{(t-t_{i-1})^2}{2}, + \ldots\ldots$$

Hence, the second complete approximate solution is

$$Z(t) = Z^*(t) + W(t)$$

in which $Z^*(t)$ is given by (21), and $W(t)$ by (19) after substitution of (20, 22) for ϕ_i, ψ_i. Any reasonable variation in β_i is acceptable. Thus, for example, if the coefficients vary linearly within the typical subinterval (t_i, t_{i-1}), we have $A_i(t_{i-1}+\tau)$

$$= \tfrac{1}{2}[A_i(t_i)+A_i(t_{i-1})] + [A_i(t_i)-A_i(t_{i-1})] \left\{ \frac{\tau-\tfrac{1}{2}(t_i-t_{i-1})}{(t_i-t_{i-1})} \right\}, \quad 0 \le \tau \le t_i - t_{i-1}$$

$$\alpha_i = \tfrac{1}{2}[A_i(t_i)+A_i(t_{i-1})] \qquad \beta_i(t_{i-1}+\tau) = [A_i(t_i)-A_i(t_{i-1})] \left\{ \frac{\tau-\tfrac{1}{2}(t_i-t_{i-1})}{(t_i-t_{i-1})} \right\} \quad (23)$$

$$\beta_i(t_{i-1}) = -\tfrac{1}{2}[A_i(t_i)-A_i(t_{i-1})], \quad \dot{\beta}_i(t_{i-1}) = \frac{A_i(t_i)-A_i(t_{i-1})}{t_i-t_{i-1}}$$

In many applications, (Saturn) it is not necessary to calculate the second approximation to the solution if the subintervals are kept within five to ten seconds during the first fifty to sixty seconds of flight and two to three seconds during the region of maximum dynamic pressure.

APPLICATION TO A SATURN V LAUNCH VEHICLE

To illustrate the above theory, consider the response of the Saturn V launch vehicle to scalar wind disturbances, as shown in Figures 1 and 2. For simplicity, assume the vehicle to be a rigid body subject to the assumptions stated in Reference (2). The equations of motion are

$$\ddot{y} + K_1\theta + K_2\gamma + K_3\delta = 0 \qquad \text{(Rigid body translation}$$

$$\ddot{\theta} + C_1\gamma + C_2\delta = 0 \qquad \text{(Rigid body rotation)}$$

$$\gamma = \theta - \frac{\dot{y}}{V} + \frac{V_w}{V} \qquad \text{(Angle of Attack)} \quad (24)$$

$$\delta = a_0\,\theta + a_1\dot{\theta} + b_0\gamma \qquad \text{(Control law)}$$

$$M(x,t) = M_\gamma'(x)\gamma(t) + M_\delta'(x)\delta(t) \qquad \text{(Rigid body bending moment)}$$

in which K_1, K_2, ... are known functions of time. In the above equations, the first four can be expressed in state variable form by introducing

$$\dot{\theta} = y_1$$

$$\dot{y} = y_2$$

and incorporating (24_3) and (24_4) in formulae (24_1) and (24_2). Thus, we have

$$\dot{Z}(t) = A(t)Z(t) + \omega(t) \quad (25)$$

with

$$Z(t) = \begin{Bmatrix} y_1 \\ \theta \\ y_2 \end{Bmatrix} \quad -\omega(t) = \begin{Bmatrix} \frac{C_1+C_2b_0}{V} \\ 0 \\ \frac{K_2+K_3b_0}{V} \end{Bmatrix} V_w \quad A(t) = \begin{bmatrix} -C_2a_1 & -[C_1+(a_0+b_0)C_2 & \frac{C_1+C_2b_0}{V} \\ 1 & 0 & 0 \\ -K_3a_1 & -[K_1+K_2+(a_0+b_0)K_3] & \frac{K_2+K_3b_0}{V} \end{bmatrix}$$

Moreover, it is seen that

$$U(t) = B(t)\,Z(t) + \omega_*(t) \qquad (26)$$

in which

$$U(t) = \begin{Bmatrix} \delta \\ \gamma \\ M \end{Bmatrix}, \qquad \omega_*(t) = \begin{Bmatrix} \frac{b_0}{V} \\ \frac{1}{V} \\ \frac{M'_\delta b_0 + M'_\gamma}{V} \end{Bmatrix} V_w$$

$$B(t) = \begin{bmatrix} a_1 & a_0+b_0 & \frac{-b_0}{V} \\ 0 & 1 & -\frac{1}{V} \\ M'_\delta a_1 & (a_0+b_0)M'_\delta + M'_\gamma & -\frac{1}{V}(M'_\gamma + b_0 M'_\delta) \end{bmatrix}$$

Note that the subsidiary system U may be solved directly for the desired quantities once the state vector Z is defined.

For this example, the wind disturbance is defined by the scalar wind profile illustrated in Figure 2. The wind profile represents a 95% (worst month) quasi-steady wind with a 99% wind build-up rate. A discrete gust may be seen superimposed on the profile. With the wind disturbance now defined and A(t) known, the system response is defined by substituting (25) into (23) and then (22) into (19). Substitution of the Z vector thus obtained into the auxiliary system (26) completes our solution for the vehicle state variables.

The above calculations were carried out on a digital computer for the first 80 seconds of vehicle flight. The computation was performed in five second intervals up to 60 seconds flight time and two second intervals thereafter. Figures 3-7 illustrate the results obtained. A comparison of these results with comprehensive analyses performed at MSFC indicates excellent agreement. This is a notable fact when considering relative computation times required. It is also notable that, for the particular vehicle model and wind used in the example, the solution process can be carried out over 15 to 20 second time intervals and possibly larger, up to approximately 60 seconds of flight with negligible loss in accuracy.

CONCLUDING REMARKS

Under the assumption that there exists an exact reference trajectory solution for a launch vehicle, the system of complete perturbation equations have been derived. These equations were shown to describe the deviation of the dynamic state of the vehicle from the state described by the reference trajectory solution. Provision was made for the fact that the trajectory equations may not include all relevant terms when considered in conjunction with a more complete description of the vehicle's dynamic state. The complete equations of deviant motion were expanded about the trajectory solution, and the linearized equations of deviant motion were obtained. These equations were shown to have variable coefficients. Moreover, the dynamic load environment of the launch vehicle was seen to be comprised of that resulting from the reference trajectory solution and that stemming from the solution of the local deviant motion.

In order to validate the above procedure, an important question was posed: under what conditions do the linearized equations of deviant motion and the complete nonlinear equations of deviant motion exhibit the same behavior. This question has been resolved in part by the qualitative theory of differential equations.

An algorithm for determining the solution of the small deviant equations of motion was derived. The resulting form of solution was shown to be very efficient for digital computation and exhibited desirable features of present digital and analog methods. The approach was less time consuming than direct numerical integration and could be used for parametric evaluation of a given time regime without reference to the entire flight time history. The form of the solution was shown to give physical insight into vehicle response process not readily obtainable from other methods.

Application of the algorithm to a yaw plane, Saturn V vehicle was illustrated. The vehicle model analyzed was a simple rigid body system. However, the method used is applicable to systems of arbitrary order. Therefore, no intrinsic difficulties are involved in extending the model to include more degrees of freedom.

REFERENCES

[1]DeRusso, P.M., Roy, R.J., and Close, C.M., State Variables for Engineers: John Wiley & Sons, Inc., New York, (1966).

[2]Ryan, Robert S. and Scoggins, James R., "The Use of Wind Shears in the Design of Aerospace Vehicles"; Presented at the 23rd meeting of the Structures and Materials Panel, AGARD, October 4-11, 1966; ONERA, Paris, France.

Ryan

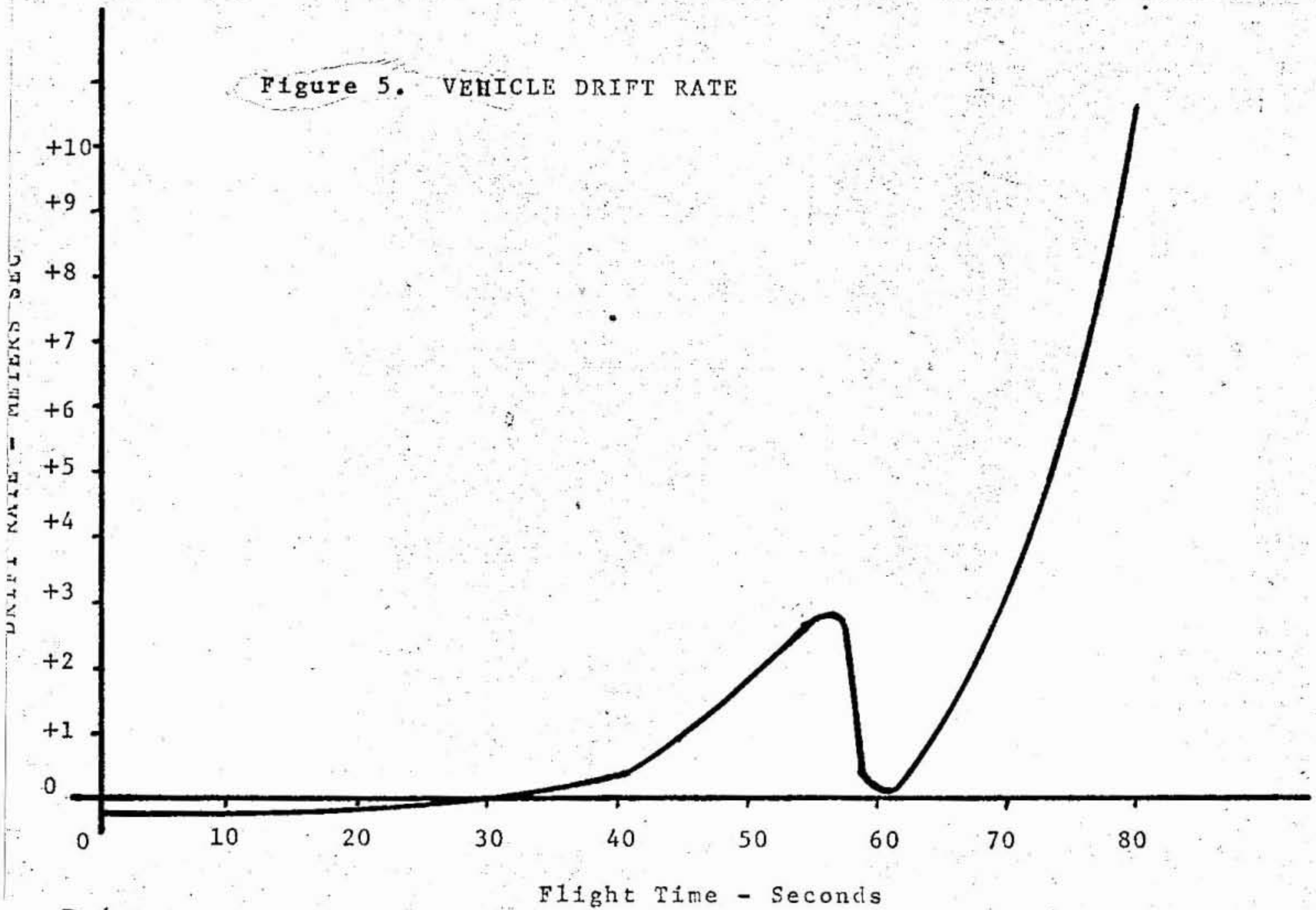
Figure 5. VEHICLE DRIFT RATE
DRIFT RATE - METERS SEC
+10
+9
+8
+7
+6
+5
+4
+3
+2
+1
0
0
10
20
30
40
50
60
70
80
Flight Time - Seconds

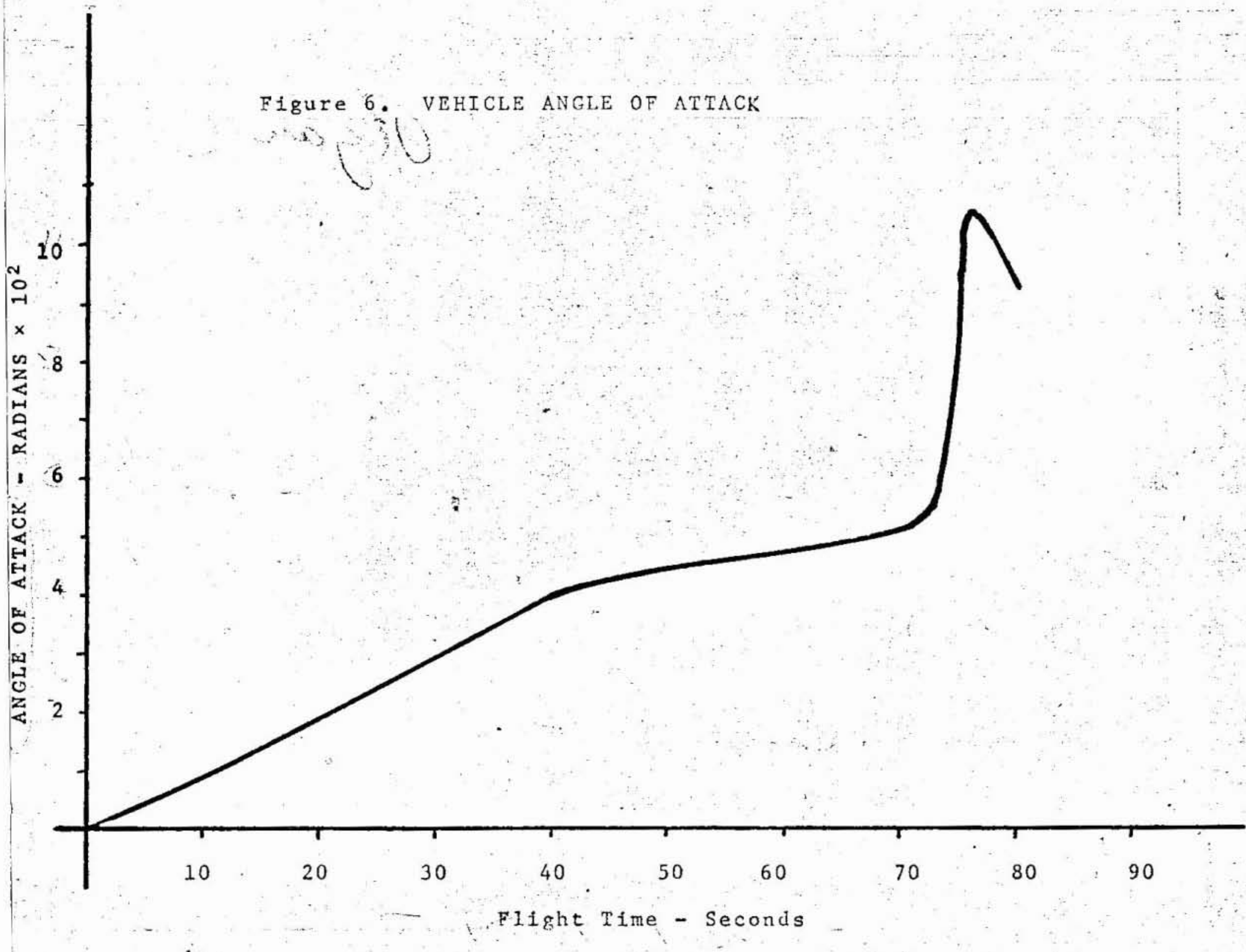

Figure 6. VEHICLE ANGLE OF ATTACK

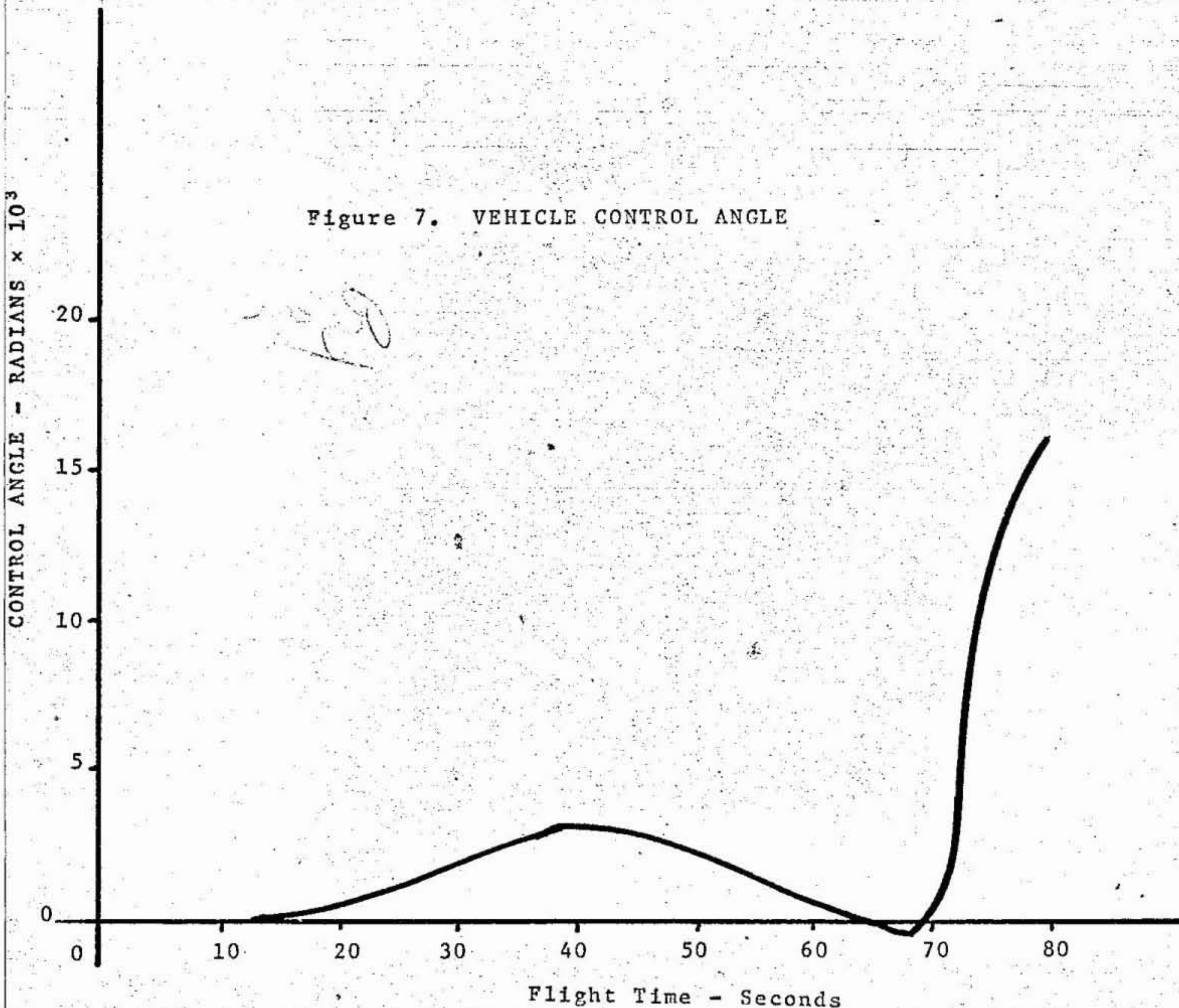

Figure 7. VEHICLE CONTROL ANGLE

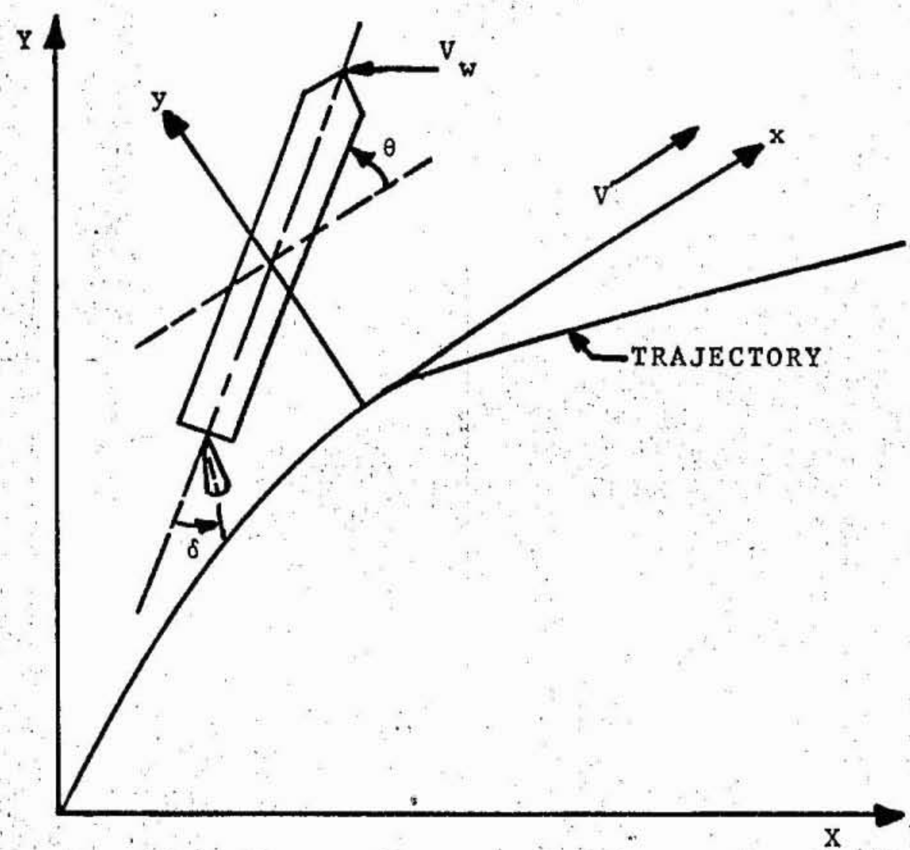

Figure 1. VEHICLE MODEL

Figure 2. SYNTHETIC WIND PROFILE

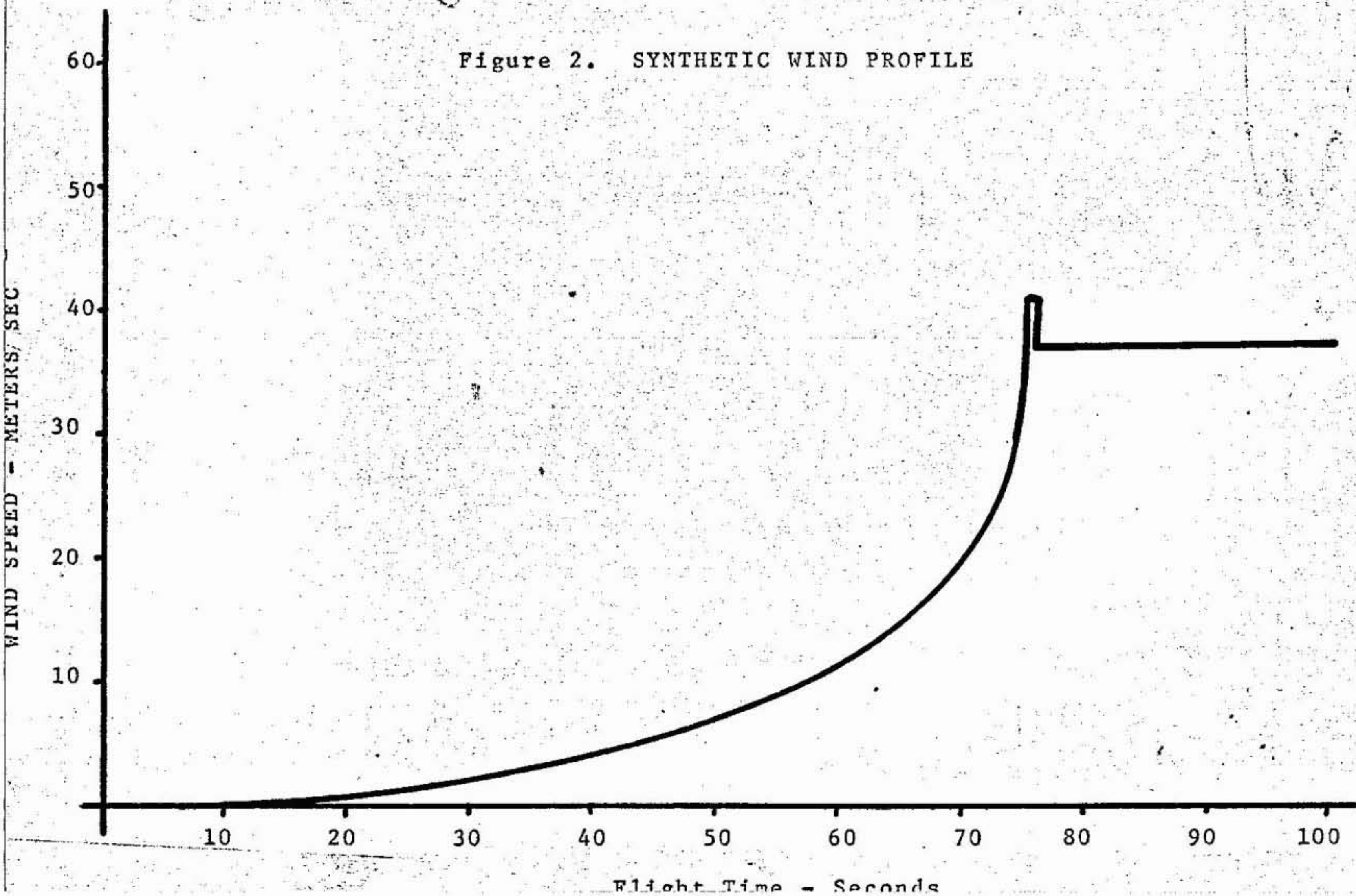

Figure 3. VEHICLE PITCH ANGLE

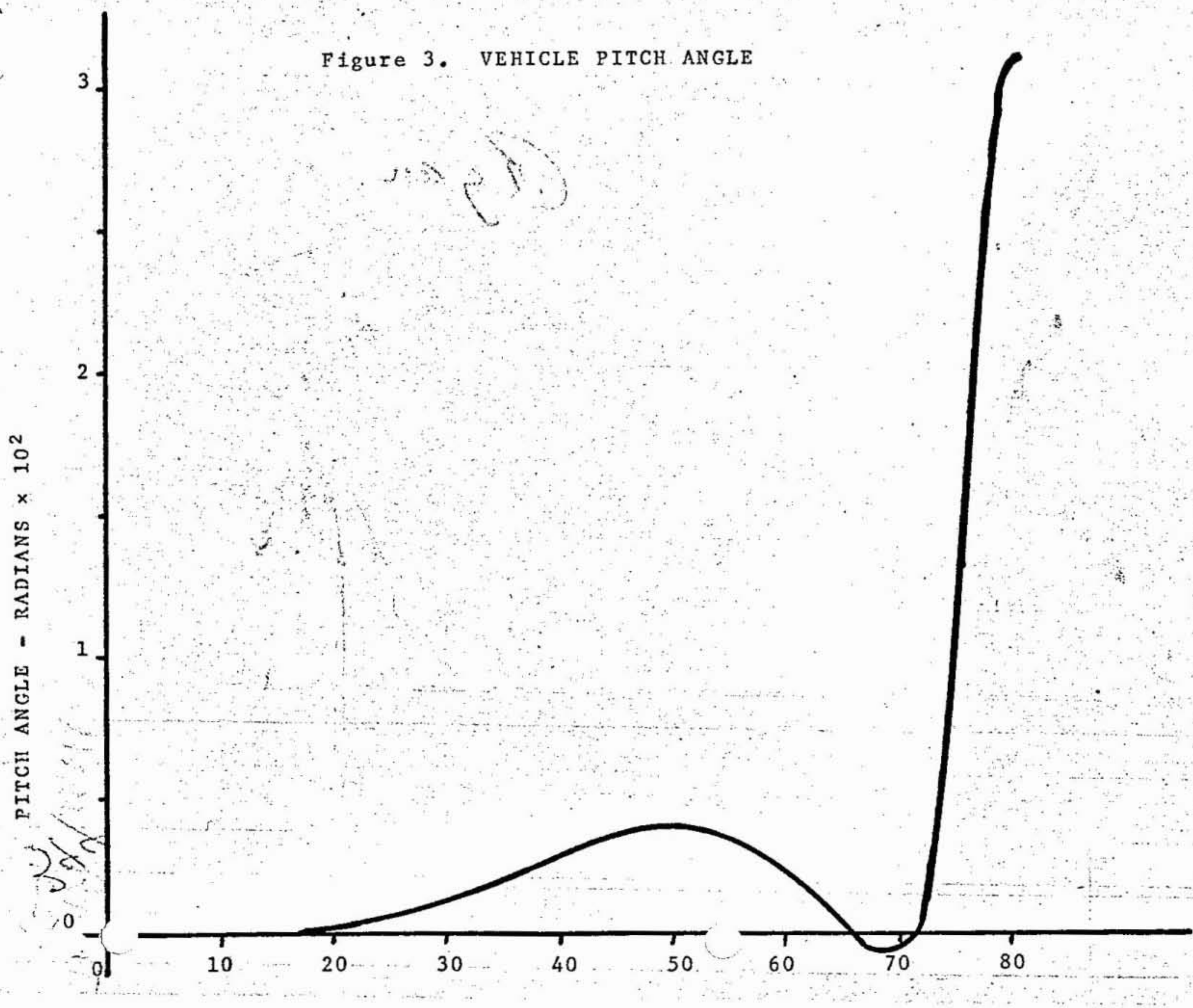

Figure 4. VEHICLE PITCH RATE