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SATURN ASCENDING PHASE GUIDANCE AND CONTROL TECHNIQUES

by
F. Brooks Moore and Melvin Brooks
George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Huntsville, Ala.

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(C-1 + C-5 Vehicles)

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ABSTRACT 2458-62

The Saturn guidance and control concept must be sufficiently broad to accommodate a variety of vehicle configurations and engine specifications coupled with a large assortment of mission objectives and flight paths. The guidance concept under development at Marshall Space Flight Center meets these requirements and is termed the Adaptive Guidance Mode. This mode functions by accepting the present vehicle flight variables and engine parameters as initial conditions and defining the optimum path ahead which meets the mission requirements. This information is supplied in the form of attitude and cutoff commands. The optimum path is generally that requiring minimum fuel consumption. The control mode orients and stabilizes the vehicle along this optimum path even under perturbing influences.

The guidance and control system for implementing the Adaptive Guidance Mode is also designed for use in any Saturn vehicle and mission complex. The guidance and control system consists principally of the following major sub-systems:

(a) A four gimbal inertial platform system with necessary items for maximum flexibility and unlimited attitude freedom.

(b) A general purpose digital computing system capable of meeting the requirements for all foreseeable missions.

(c) An analog control system which orients and stabilizes the vehicle.

LIST OF SYMBOLS

(X_s, Y_s, Z_s)	=	Spacefixed coordinate system for guidance computations
(X, Y, Z)	=	Spacefixed coordinate system for gravitation computation
(ξ, η, ζ)	=	Inertial measuring coordinate system
$\dot{X}_s, \dot{Y}_s, \dot{Z}_s$	=	Components of velocity in guidance coordinate system
X_s, Y_s, Z_s	=	Components of displacement in guidance coordinate system
$\dot{X}_{so}, \dot{Y}_{so}, \dot{Z}_{so}$	=	Initial launch components of velocity in guidance coordinate system
X_{so}, Y_{so}, Z_{so}	=	Initial launch components of displacement in guidance coordinate system
X, Y, Z	=	Components of displacement in gravitation coordinate system
$\dot{\xi}, \dot{\eta}, \dot{\zeta}$	=	Measured components of velocity in inertial coordinate system
$\delta\dot{\xi}, \delta\dot{\eta}, \delta\dot{\zeta}$	=	Incremental components of velocity from accelerometers
$\ddot{\xi}, \ddot{\eta}, \ddot{\zeta}$	=	Components of acceleration in inertial coordinates
$\bar{r}_o, \dot{\bar{r}}_o$	=	Instantaneous position and velocity respectively in the guidance coordinate system
$\bar{r}_c, \dot{\bar{r}}_c$	=	Position and velocity respectively at cutoff in the guidance coordinate system
t_c	=	Time of cutoff reckoned from launch
t_o or t	=	Flight time reckoned from launch and obtained from guidance computer clock to solve guidance equations

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t_f = Time-to-go before cutoff, reckoned from launch

$\left(\frac{F}{M}\right)$ = Vehicle longitudinal acceleration

$\left(\frac{F}{M}\right)_o$ = Instantaneous value of vehicle longitudinal acceleration

g_x, g_y, g_z = Components of acceleration due to gravity in gravitation coordinate system

χ_y χ_x χ_z = Eulerian angles defining desired orientation of vehicle-fixed coordinate system in spacefixed guidance coordinate system (X_S, Y_S, Z_S) when executed in the sequence χ_y, χ_x and χ_z about the vehicle-fixed axis indicated with subscripts. The vehicle-fixed coordinate system is parallel with the guidance coordinate at launch

χ_{yo}, χ_{xo}, χ_{zo} = Instantaneous steering commands

φ_r = Inner gimbal angle (roll)

φ_{pl} = Inner-middle gimbal angle (pitch limited)

φ_y = Outer-middle gimbal angle (yaw)

φ_{op} = Outer gimbal angle (outer pitch)

$(\chi_p - \varphi_p)$ = Pitch error signal from platform resolver chain

$(\chi_y - \varphi_y)$ = Yaw error signal from platform resolver chain

$(\chi_r' - \varphi_r)$ = Roll error signal from platform resolver chain

$\dot{\varphi}_p$ $\dot{\varphi}_y$ $\dot{\varphi}_r$ = Angular velocity about the vehicle pitch, yaw, and roll axes respectively

$\dot{\varphi}$ = Rate of change of angle between the vehicle longitudinal axis and vertical inertial reference

φ = Instantaneous angle between the vehicle longitudinal axis and the vertical inertial reference

α_p, α_y = Angle-of-attack in the pitch plane and yaw plane respectively

β_p, β_y, β_r = Gimbaled engine angular position for pitch, yaw, and roll respectively

$\ddot{\gamma}$ = Acceleration perpendicular to vehicle longitudinal axis and obtained from control accelerometer

$\bar{\chi}$ = Desired yaw heading for delta minimum crossrange guidance

$\left(\frac{\dot{M}}{M}\right)_o$ = Instantaneous value of the ratio of mass flow rate to total mass

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I. INTRODUCTION

Since the development of a large multi-stage launch vehicle requires such a vast commitment of manpower and resources, the economy of the country cannot support the development of a particular vehicle for each specialized mission. Scheduling is an equally important consideration, since an extended period of time is required for development of the launch vehicle. If the development of a launch vehicle could commence only after a particular mission and payload were well defined, years of development time would be lost. The launch vehicle must therefore be capable of accommodating a variety of payloads which are under parallel development, as well as payloads which will be an outgrowth of missions yet to be formulated.

In order to be compatible with the overall flexibility requirements of the vehicle, the guidance concept under development for the Saturn is sufficiently broad in scope to encompass all foreseeable vehicle configurations and mission objectives. Moreover, the implementation of the concept is not dependent upon the type of navigation sensors employed. The mathematical structure of this concept which is termed the Adaptive Guidance Mode, is sufficiently comprehensive to handle satisfactorily foreseeable specific guidance problems. A particular solution to a guidance problem is derived from the general mathematical structure when the conditions and requirements of the particular flight are specified. The form of the guidance and cutoff equations is invariant with changing missions and vehicles.

The guidance and control system employed in the implementation of the Adaptive Guidance Mode is also being designed for the greatest flexibility in application, so that a minimum of constraints are imposed on the utility of the vehicle. The goal of maximum flexibility is considered subservient to only one other prime requirement; namely, maximum possible reliability.

The tremendous expense of each vehicle, in addition to the fact that the vehicle will eventually transport human passengers, compels the designers of the guidance and control system to set maximum possible reliability as the primary goal.

The guidance and control methods and concepts described in this paper are applicable, in general, to both the Saturn C1 and C5 vehicles. The detailed description of hardware is limited to that employed in the C1. In addition to fulfilling the primary requirements of a variety of space missions, the C1 will also be used as a test vehicle for the C5. The design of hardware employed in the C5 system will be influenced by experience gained on the C1, as well as generally representing a more advanced state-of-the-art. In the C5 system particular emphasis will be placed on redundancy as a means of improving reliability of the overall system.

II. THE CONCEPT OF THE ADAPTIVE GUIDANCE MODE

The guidance concept for the Saturn vehicles must be applicable to a large variety of vehicle configurations, vehicle performance capabilities, and mission requirements. The powered flight time of the Saturn vehicle is very long by ballistic missile standards ranging from approximately 600 seconds for earth orbits to approximately 1000 seconds for lunar probes. With some multi-engine, multi-stage vehicles, the potential for completing the intended mission exists even after the failure of an engine during flight. Under these conditions the necessary high injection accuracy may be difficult to obtain with a guidance scheme employing conventional time programs or other programs predicated upon a single reference trajectory.

Any general guidance concept should provide a volume or class of reference trajectories which satisfy the mission criteria. Guidance signals must then be derived during flight from the reference trajectory which is appropriate for the flight conditions and employed to direct the control system to cause the vehicle to follow a satisfactory path. In addition, some means must be provided to initiate cutoff when the mission criteria have been satisfied.

The Adaptive Guidance Mode which fulfills the Saturn configuration and mission flexibility requirements is described in detail in reference 1; however, a brief summary is given here of the concept so that the system integration can be outlined.

A spacefixed coordinate system, as shown in Figure 1, is convenient for describing the Adaptive Guidance Mode. In the Saturn the thrust direction is controlled in order to keep the vehicle on the desired trajectory. A conventional way of defining the thrust direction when the direction is established by the orientation of a rigid body such as a rocket vehicle is by specifying a set of three Eulerian angles, X_y, X_x, and X_z. These angles define three successive rotations taken in a specified sequence about three body-fixed coordinate axes. The body-fixed axes are initially parallel to the spacefixed axes; hence, the angles define the orientation of the vehicle in the spacefixed coordinate system. For a time varying thrust direction, these three angles are functions of time. The control system turns the vehicle into the orientation defined by these angles and stabilizes the vehicle during flight.

Steering commands which are stored functions of time and which provide only a single reference trajectory are satisfactory only if no perturbing influences occur during flight. A change in thrust level or other perturbations at any time during flight requires a new reference trajectory for use between the perturbed point and cutoff. In addition, a new set of time variable steering commands and a new cutoff time must be selected which is compatible with the mission objectives and new trajectory. When perturbations occur the Adaptive Guidance Mode generates new steering and cutoff commands based on measurements and calculations made on-board the vehicle. These steering commands generate a new trajectory which meets the mission objectives in an optimum manner. The criteria for an optimum is generally minimum fuel consumption.

Sufficient information must be obtained during flight to determine the steering and cutoff commands for generating a trajectory connecting the initial point and the desired cutoff point. The information required is the initial flight coordinates ($\bar{r}_o$, $\dot{\bar{r}}_o$, t_o) and the initial values of suitable parameters for determining the vehicle and engine characteristics.

If the thrust and mass flow rate are assumed to remain constant for the remainder of the burning time of the stage at the values existing when on-board measurements are made, and standard values for subsequent stages are assumed, then the knowledge of $\left(\frac{F}{M}\right)_o$ and $\left(\frac{\dot{M}}{M}\right)_o$ is sufficient to determine the vehicle and engine characteristics. The parameter $\left(\frac{\dot{M}}{M}\right)_o$ is not required to be measured directly; it can be computed as a function of time where the mass flow rate is changed during flight to represent the number of engines operating. Lesser changes in the mass flow rate can be adequately represented by a nominal value.

Hence, the information which must be obtained during flight from measurements and calculations consists of the initial conditions, $\left[\bar{r}_o, \dot{\bar{r}}_o, \left(\frac{F}{M}\right)_o, t_o\right]$. Since the gravitational acceleration is a known function of position, $\bar{r}_o$, $\dot{\bar{r}}_o$, and $\left(\frac{F}{M}\right)_o$ may be calculated from the outputs of the integrating accelerometers. Time may be supplied by the clock mechanism in the guidance computer.

The values of $\bar{r}_o$, $\dot{\bar{r}}_o$, $\left(\frac{F}{M}\right)_o$ and t_o are updated continuously during flight. Thus, the present conditions calculated from instantaneous measurements are taken as the initial conditions for the remainder of the flight. If a perturbation occurs at some point on the trajectory, the new values are determined from measurements and calculations and employed as new initial conditions. New steering and cutoff commands are then generated which shape the trajectory for minimum fuel consumption consistent with the required end conditions.

To demonstrate the concept, select a set of initial conditions, $\left[\bar{r}_o, \dot{\bar{r}}_o, \left(\frac{F}{M}\right)_o, t_o\right]$ at some point on the trajectory which is determined by calculations on-board the vehicle (See Figure 1). This point constitutes the initial conditions for a powered trajectory which terminates at the cutoff point, $\left[\bar{r}_c, \dot{\bar{r}}_c, t_c\right]$. The cutoff point constitutes the solution to a set of mission equations which are assumed to have been formulated. In general, there exists an infinite number of possible paths which will pass through the initial point and satisfy the mission equations. However, that particular path which optimizes some particular parameter is generally more desirable than the others. The criteria for an optimum is generally minimum fuel consumption. Since the mass flow rate is assumed constant, minimum flight time will give minimum fuel consumption.

Associated with each trajectory is a set of time variable Eulerian angles, $\chi_y(t)$, $\chi_x(t)$, and $\chi_z(t)$ which define the thrust direction.

If these angles define the thrust direction for the optimum trajectory described by the functions $\bar{r}(t)$ and $\dot{\bar{r}}(t)$, then regardless of the location of the point on the trajectory where the initial conditions, $\left[\bar{r}_o, \dot{\bar{r}}_o, \left(\frac{F}{M}\right)_o, t_o\right]$, are obtained, the same set of functions $\chi_y(t)$, $\chi_z(t)$, and $\chi_z(t)$ will give the optimum from this point forward. These functions will have the initial values χ_{yo}, χ_{xo}, and χ_{zo} at the initial point. The trajectory will also be described by the same functions $\bar{r}(t)$ and $\dot{\bar{r}}(t)$. However, if a perturbation occurs at some point, the initial conditions at that point will not be the same as those obtained for the original optimum path. Hence, a new set of functions, $\bar{r}'(t)$ and $\dot{\bar{r}}'(t)$, will be required to describe the optimum trajectory from the perturbed point to the cutoff point. Moreover, the steering functions will have new initial values χ'_{yo}, χ'_{xo}, and χ'_{zo} at this point. In addition, the time required for the vehicle to reach cutoff under the perturbed conditions will be different from the time required for the unperturbed conditions. Consequently, for each set of initial conditions obtained during flight there corresponds a set of initial values, χ_{yo}, χ_{xo}, and χ_{zo}, for the three Eulerian angles and a certain flight time t_f, remaining before the mission equations are solved and cutoff is initiated. Thus,

$$\chi_{yo} = \chi_{yo}\left[\bar{r}_o, \dot{\bar{r}}_o, \left(\frac{F}{M}\right)_o, t_o\right]$$

$$\chi_{xo} = \chi_{xo}\left[\bar{r}_o, \dot{\bar{r}}_o, \left(\frac{F}{M}\right)_o, t_o\right]$$

$$\chi_{zo} = \chi_{zo}\left[\bar{r}_o, \dot{\bar{r}}_o, \left(\frac{F}{M}\right)_o, t_o\right]$$

$$t_f = t_f\left[\bar{r}_o, \dot{\bar{r}}_o, \left(\frac{F}{M}\right)_o, t_o\right]$$

If the functions χ_{yo}, χ_{xo}, χ_{zo}, and t_f can be found and solved at each point on the trajectory during flight, the steering signals and cutoff signal will be obtained for the optimum trajectory.

The techniques of the Calculus of Variation are employed to generate a volume of expected trajectories for specific vehicles and missions. Numerical curve fitting methods are employed to obtain satisfactory series solutions for the guidance and cutoff commands.

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III. CONTROL CONSTRAINTS

The Saturn Control System is designed to receive the steering commands from the guidance system and to apply moments to the vehicle in such a manner that it is properly directed to fulfill the assigned mission. In addition, the control system must provide vehicle stability in the presence of upsetting moments, such as those caused by aerodynamic disturbances and thrust misalignments. The control system must fulfill these requirements in the presence of a number of constraints attributable to the vehicle structural and aerodynamic characteristics, as well as basic characteristic limitations of the control system itself.

Although the derivation of control moments has been accomplished in a variety of ways on the earlier generation of missiles leading up to the large launch vehicle, the only scheme which is practical for employment on the Saturn C1 and C5 vehicles presently under development is limited to the gimbaling of the main propulsion engine. The angle through which a large propulsion unit can be practically gimbaled is limited, however, by the gimbal bearing and propellant flex-lines, as well as the vehicle structural considerations. The constraints which are thereby placed on the control system must be considered in the system design. Transducers which sense the wind loads must be employed to direct the vehicle in such a manner that the resultant lateral loads and the required engine deflections are retained within the allowable limits. The associated control loop gains required to accomplish the necessary structural load relief and control torque minimization are changed as a precalculated function of time.

The fact that a multi-stage vehicle is relatively long and flexible places another constraint on the control system. Permissable location of the control sensors must be determined and required signal shaping must be derived from accurate knowledge of the structural bending characteristics. The affect of local structural compliances within the gimbaled engine loop as well as possible excitation from propellant sloshing must also be considered in the determination and verification of the control system stability.

The nominal rigid body control frequency must be held above a certain minimum in order to provide adequate response to wind disturbances and to allow for some uncertainty of aerodynamic and structural data. The rigid body control frequency therefore lies within the crowded spectrum of sloshing frequencies of the multiple propellant tanks and the structural bending and local compliance frequencies.

Assuming that all characteristics of the complex system could be accurately determined and assuming no conflicting stabilization requirements of the various modes, the system could obviously be stabilized by electrically shaping at proper points within the various loops. On the Saturn, however, a more conservative approach has been followed. Any sloshing mode which appears to be a potential problem is damped by proper baffling in the tanks throughout the critical regions. Active damping is applied to the engine gimbal compliance loop through the

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utilization of a load-sensitive feedback within the engine actuation system.

Alleviation of the sloshing and gimbal compliance problems through the above methods allows more flexibility in coping with the structural bending and other local compliance problems through the proper location of control sensors, supplemented by electrical compensation.

IV. GUIDANCE AND CONTROL SYSTEM MECHANIZATION

The mechanization of the guidance and control modes in the Saturn vehicles results in an integration of the two functions through the sharing of hardware sub-systems. In particular, the stabilized platform supplies the inertial reference frame for the guidance mode and provides the attitude reference for the control mode. The system mechanization results in a completely self-contained inertial system which integrates the functions of these two modes to effect guidance and stability of the vehicle during the ascending phase without assistance from ground stations.

A simplified diagram of the Saturn Guidance and Control System is shown in Figure 2. The system consists principally of three major sub-systems: A four-gimbal inertial platform having three orthogonal accelerometers mounted on the stable element and attitude resolvers on each gimbal, a general purpose digital guidance computer, and an analog control computer with associated sensors and actuators. Interface requirements between these sub-systems are provided in the guidance signal processor. Physical descriptions of some of the major components employed in the system are included in a later section of this paper. The following is a functional description of the operation of the system, indicating the method of derivation of the required guidance intelligence, the conversion of this information into the proper co-ordinate system, and the execution through the control system.

IVa. GUID IMPLEMENTATION

The coordinate systems used for describing the implementation of the guidance system are shown in Figure 3. The spacefixed system, (X, Y, Z) is used for computing the acceleration of gravity. The spacefixed system (X_s, Y_s, Z_s) is the one in which the guidance functions are computed, and the system (ξ, η, ζ) defines the inertial measuring directions of the accelerometers on the stabilized platform.

A functional diagram of the basic computations required in the guidance computer is shown in Figure 4. Incremental outputs from the converters on the integrating accelerometers are summed to give the components of inertial velocity. The sum of these velocity components and the corresponding components of initial spacefixed velocity are integrated to give components of displacement.

Components of the acceleration due to gravity are computed as functions of displacement at intervals of approximately one-half second in a feedback loop. The first integral of each component of gravitational acceleration is summed with the corresponding velocity components so that the required spacefixed velocity and position components become available for solving the guidance and cutoff equations. These computations are performed throughout the entire ascent phase.

The thrust per unit mass, $\left(\frac{F}{M}\right)_o$, is computed approximately once each second by differentiating each total accelerometer velocity output and taking the square root of the sum of the squares of the resulting derivatives. The computations described are those required to solve the equations given in Figure 5.

Guidance and cutoff commands are computed as functions of the flight coordinates and vehicle parameters at intervals of approximately one second. Time is supplied by the clock mechanism in the guidance computer. In general, there are three separate steering polynomials of the form given in Figure 6. However, the length of each equation and the specific terms included vary with the mission. The polynomials for χ_{yo}, χ_{xo}, and χ_{zo} are solved during the second and subsequent stages. The cutoff polynomial, also shown in Figure 6, is solved only during the latter phase of the final stage; all other stages are burned until fuel depletion occurs.

The first stage tilt program is obtained as an expansion in time only because of structural and control problems. This is sufficient, however, since the Adaptive Guidance Mode is capable of handling the expected deviations at first stage burnout.

Most missions require the vehicle to be constrained to fly in a spacefixed plane during the ascending phase. Under this condition, "Delta Minimum" guidance may be used in the crossrange direction and Adaptive Guidance in the pitch plane. The polynomial for χ_{zo} provides the tilt angle in the flight plane while simple equations for χ_{yo} and χ_{xo} provide "Delta Minimum" guidance in the crossrange direction.

The typical "Delta Minimum" guidance equation for this application will take the form

$$\overline{X} = e_o Z_s + e_1 \dot{Z}_s$$

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that means we do it twice?

where $\bar{X}$ is the heading required in yaw to correct the crossrange deviations; it is generally one degree or less. Because of the interchange of the roll and yaw axes during flight and since $\bar{X}$ is small, the two equations needed for implementation take the form:

$$\chi_{xo} = (e_o Z_s + e_1 \dot{Z}_s) \cos \chi_{zo}$$

$$\chi_{yo} = (e_o Z_s + e_1 \dot{Z}_s) \sin \chi_{zo}$$

The coefficients, e_o and e_1, may be constants or linear functions of time. When the crossrange position and velocity, Z_s and $\dot{Z}_s$, are zero χ_{xo} and χ_{yo} become zero.

The Eulerian angles, χ_{yo}, χ_{xo}, and χ_{zo} computed in the digital computer are converted to analog command signals to be executed in the control system by the positioning of three command resolvers located in the Guidance Signal Processor.

A block diagram of one channel indicating the particular scheme of implementation employed in the Saturn C1 is shown in Figure 7. The presently commanded attitude direction and the newly calculated direction are compared in the digital computer. The difference is changed to desired rate of attitude correction by a scaling constant. The required rate is then placed on the ladder network. The dc output voltage of the ladder network indicates the desired corrective rate. After modulation and amplification, this signal energizes an ac motor which in turn drives a command resolver and an incremental encoder. After proper shaping, the output of the encoder is fed back to the computer where the information is used for comparison with the desired angle computed on the next computer cycle.

The spacefixed components obtained from the command resolvers in the signal processor are transformed by the four resolvers on the gimbals of the stabilized platform into components in the actual vehicle coordinate system.

A flow diagram of the resolver chain through which the attitude commands derived in the digital computer are processed is shown in Figure 8. This chain is unique in that each resolver is made to transform two axes simultaneously instead of one as is generally done. This double duty is made possible by exciting the chain with two inharmonically related frequencies. Filters separate the signals obtained for demodulation. The signals are then demodulated to provide attitude error signals for use by the control system. When these error signals are driven to zero, the actual vehicle coordinate system is aligned with the desired vehicle coordinate system and the thrust acts in the optimum direction.

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IVb. CONTROL IMPLEMENTATION

The control system can be broken down into three main categories; sensing, computing, and engine actuation. The required information from the sensors includes the vehicle attitude, with respect to a reference, the attitude rate and a means of sensing lateral loads.

In the Saturn system the desired attitude information is derived by the digital computer and referenced to the vehicle through the platform resolver chain, as previously described.

The attitude rate information is derived either from rate gyros or by differentiation in the control computer of the attitude signals. The latter is considered a simpler and more reliable method and is, therefore, utilized when the location of the platform, with respect to the structural bending mode shapes, permits. In the C1 configuration, for instance, the Block I vehicles are stabilized by differentiation of the attitude signal, since the platform location atop the S-I stage is a permissable location with respect to bending. In the Block II vehicles, however, the platform is located in the Instrument Unit forward of the S-IV stage. The shape of the bending is greater at this location and may force the use of rate gyros located at a structurally optimum position for the overall configuration. After separation of the S-I stage, the vehicle can again be stabilized during S-IV burning by simple lead networks. Final determination of the method of stabilization of each stage of a particular configuration can only be determined after reasonably accurate data on the structural characteristics are known. Although the general philosophy will be the same for the C5 as for the C1 vehicles, sufficiently accurate structural data on the C5 is not yet available to the extent that definite implementation can be determined.

A means of accomplishing lateral load relief is necessary while the vehicle is within the atmosphere. A method, generally termed angle-of-attack control, is employed wherein the instantaneous angle-of-attack is sensed and reduced by directing the vehicle attitude in the required direction to reduce the angle-of-attack. The lateral load torques caused by the wind, and consequently the stabilizing torques which must be derived from the gimbaled engines, are thereby reduced. On earlier Saturn vehicles, the wind information is derived from angle-of-attack transducers which sense the direction of air flow directly. Body-fixed accelerometers which are mounted with their sensitive axes perpendicular to the longitudinal axis may also be used. Stabilization of the vehicle in the presence of structural bending can be more troublesome, however, since the accelerometer senses a higher derivative of the bending oscillation. The problems associated with the use of accelerometers in the later C1 and the C5 vehicles are being actively pursued, however, since the structural and aerodynamic characteristics of the Apollo, and possibly other payloads, make the direct sensing of the angle-of-attack difficult. The required ratio between the attitude and angle-of-attack or control accelerometer gain at any particular time of flight is a function of the particular configuration and trajectory. Figure 9 shows the particular gain program employed in conjunction with the angle-of-attack transducers on Saturn C1, Block I. Figure 10 is a program which may be similarly employed on the Block II vehicles, assuming the use of control accelerometers.

The signals from the various sensors are shaped, mixed in the proper gain and phase relationship, and routed to the appropriate engine actuators by the analog control computer.

The control equations which are implemented in the control computer for the pitch, yaw, and roll axes are, respectively:

$$\beta_p = a_{op} \; (\chi_p - \varphi_p) + a_{1p} \dot{\varphi}_p + b_{op} \alpha_p$$

$$\beta_y = a_{oy} \; (\chi_y - \varphi_y) + a_{1y} \dot{\varphi}_y + b_{oy} \alpha_y$$

$$\beta_r = a_{or} \; (\chi_r - \varphi_r) + a_{1r} \dot{\varphi}_r$$

The above equations are idealized and representative only at low frequencies. At higher frequencies, lags in the control system and the necessary shaping for bending stabilization add additional terms.

The nature and extent of the required sensor signal shaping is determined from an overall analysis considering rigid body control requirements, structural bending, local compliances at the sensor locations, local compliances within the engine gimbal loop, and sloshing modes of the various tanks. As previously mentioned, however, an attempt is made to relieve the constraints by damping the gimbal and sloshing modes by means other than shaping in the control computer. The shaping network provides the phase characteristics required to stabilize the rigid body and first bending mode, whereas signals generated by the higher modes are sufficiently attenuated so that deleterious feedback through the control system is prevented.

The Saturn Control System is integrated to the greatest extent possible. Unnecessary duplication of sensing and computing is avoided. The control intelligence signals are routed from the control computer to the appropriate gimbaled engine actuators by a main trunk line which runs through the various stages. The signals are switched to each subsequent stage as the burned-out stage is separated. Figure 11 shows schematically how this is accomplished on the C5 vehicle. A more detailed diagram of the control computer-actuation system integration arrangement for the C1 vehicles can be seen in Figure 12.

On the C1 vehicle, hydraulic actuators are utilized to position the four gimbaled engines of the S-I stage and six gimbaled engines of the S-IV stage. The hydraulic power for each gimbaled engine is obtained from a pump attached to the engine turbopump shaft. As an example, a schematic of an H-1 engine hydraulic system as used on the S-I stage is shown in Figure 13. A pressure-feedback hydraulic valve is used on each actuator to provide active damping within the H-1 gimbal loop.

Hydraulic actuators are also being developed for the S-Ic, S-II, and S-IVb stages of the C5 configuration. Some means of active damping within the hydraulic gimbal loop will probably, likewise, be employed on all C5 stages. Pneumatic actuators which are more compatible with

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the environment of the hydrogen-oxygen stages may eventually be employed in the upper C5 stages, depending on the length of the orbital coast times which are imposed by the particular missions which evolve.

All stages of the Saturn C1 and C5 configurations, with the exception of the S-IVb stage, have multi-engine propulsion systems. Control about the three axes, pitch, yaw, and roll, can be accomplished by gimbaling the outer engines of the multi-engine stages. On the S-IVb stage, however, only pitch and yaw are controlled by the single main engine. Control about the roll axis will be accomplished using hydrogen gas ejected through auxiliary nozzles or by a multiple arrangement of small hypogolic nozzles. If the latter arrangement is used, the nozzles will be sized so that the vehicle can be stabilized about all three axes, utilizing the same nozzles, during coasting phases.

V. DESCRIPTION OF MAJOR G&C SYSTEM COMPONENTS:

Va. STABILIZED PLATFORM

The stabilized platform developed for use with the Saturn vehicles is designated the ST-124 and is shown in Figure 14. The ST-124 has unlimited gimbal freedom about three axes provided by a four gimbal configuration. This feature is important when complicated maneuvers are required as may occur in rendezvous operations. Gimbal lock is prevented by a redundant gimbal. This redundancy exists about the pitch axis when the vehicle is in the launch position.

A diagram showing the gimbal order and the essential elements mounted on the stable element is shown in Figure 15. The nomenclature of the gimbal angles, corresponding to the vehicle orientation at launch, is from the inside, φ_r (roll), φ_{pl} (pitch limited), φ_y (yaw), and φ_{op} (outer pitch). The outer pitch gimbal angle is continuously driven into correspondence with the Eulerian angle X_{zo}, the steering command. This forces the pitch limited gimbal and the yaw gimbal to be orthogonal during steady-state conditions. The pitch limited gimbal angle is limited to ±20° and departs from zero only during transient conditions.

The stable inner element carries three single-degree-of-freedom gyroscopes for stabilization, three pendulous integrating gyro accelerometers mounted in an orthogonal triad, three pendulums for preflight leveling, and one porro-prism and digital encoder for azimuth alignment. The gyroscopes, accelerometers, and pendulums have gas bearings.

One pancake resolver is mounted on each gimbal for guidance and vehicle stabilization. Additional resolvers are mounted on the gimbals as a part of the platform stabilization network.

Vb. GUIDANCE COMPUTER

The Saturn Guidance Computer is a general purpose digital machine. The Saturn C1 will use the existing IBM ASC-15 computer which is a whole number machine with incremental portions. The machine contains a rotating drum memory and is constructed with replaceable welded modules. Detailed characteristics of this machine are omitted, since it is quite similar to one employed in a military system.

The computer planned for the C5 configuration will employ advanced techniques in packaging, such as deposited circuits, and solid state non-destructive memory with random access. Lower power consumption, expandable memory, improved environmental capability, and versatility will be considerations in the design. Redundancy of various types will be employed to enhance the computer reliability.

Vc. GUIDANCE SIGNAL PROCESSOR

There are a variety of functions related to the guidance computer but which are not made a part of the computer for several reasons: These functions may change for various missions; they may change if other vehicle systems change; and they may be added after the original design has been accomplished. A Guidance Signal Processor is included in the Saturn C1 to provide the desired flexibility. Although it is a sizeable unit, its component count is not high and most of its sections are non-interacting. A block diagram of the GSP is shown in Figure 16. The major modules included are:

(1) The attitude command resolver system (including frequency sources and demodulators)

(2) Telemetry shift register and counter

(3) Accelerometer signal shaper

(4) Computer turn on sequencer

(5) Radio command system switching

(6) Miscellaneous telemetry shapers

Items (1) and (2), which comprise about 80% of the unit, are described below.

The operation of the attitude command resolver system has been partially described in a previous section. The unit includes a rate servo system which turns the command resolver. The actual mechanical position of the shaft is fed back to the computer through an incremental encoder (.05° resolution) for comparison, so that long term rate errors

M0064-14

do not accumulate. Large surges of command values to the vehicle control system are minimized by a speed limitation of 2°/second.

The majority of the auxiliary equipment for the resolver chain system is located in the processor. Two frequency sources (1500 cps and 1800 cps) are included. These are derived from the basic 400 cps voltage. The voltage is controlled to 1 or 2%, because any error becomes a direct gain error in the overall vehicle control loop.

The application of the resolvers has also been discussed earlier. Each resolver has a non-rotating dummy mounted in the same housing. The dummies are used instead of networks for more exact compensation in the presence of temperature and other variations.

The demodulators are phase and frequency sensitive, using the 1500 cps and 1800 cps sources as references. Thus, in one case a resolver output is demodulated in two demodulators, one demodulator using a 1500 cps reference and giving an output to the roll axis, and the other demodulator using an 1800 cps reference and giving a yaw output. A third demodulator using a 1500 cps reference demodulates the output of another resolver to give the pitch signal. All demodulator outputs have a 3 volt/degree output, accurate to 1 or 2%, over a range of ±15°.

Another resolver is mounted on the shaft of the pitch module to furnish a direct order to position the outer pitch gimbal of the platform.

Guidance telemetry functions are performed with the telemetry shift register and counter. Most of the computer words that are desirable for telemetering pass through the computer accumulator and those that do not can be written on it simply. In the Saturn C-1 vehicle these are buffered and continuously fed to the processor. An order to telemeter is given each time one of the desired words passes through the register. When the telemeter order reaches the processor, the telemeter gate opens and the next word from the accumulator enters the shift register. The data is then available in parallel form for inspection by the PCM telemeter system during flight and the GSE before launch.

The processor is packaged in a box 15" x 18" x 9", weighing 65 pounds. The frame is of cast aluminum.

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Vd. C-1 CONTROL COMPUTER

The control computer is an analog device which accepts attitude error information from the stabilized platform, control accelerometers and rate gyros and after performing certain computations, provides proper steering signals to the gimbaled engines.

As an example, an attitude error signal to the computer is routed to a gain programmer which provides continuous gain programming in accordance with a preselected pattern through shaping networks which modify the signal as required for proper dynamic response and to the servo amplifiers which provide proper scaling and polarity to drive the servo valves on engine actuators. A feedback signal from each engine actuator is also fed into the proper servo amplifier.

A block diagram of the Saturn C1 Control Computer, with its associated inputs and outputs, is shown in Figure 12. The major modules of the computer are the servo amplifiers, the shaping networks, and the gain programmer.

The servo amplifier employs a magnetic amplifier section for signal mixing and scaling. Polarity selection is also made at the amplifier input. The individual windings on the magnetic core provides isolation of the incoming signals from each other and from ground. The magnetic amplifier section is followed by two stages of transistor amplification. Complete signal isolation from ground and from other circuits is provided by a small dc power supply which is packaged with the amplifier. Each servo amplifier is a separate plug-in module.

The shaping network circuits are also packaged as plug-in modules. The networks for individual channels are confined to individual modules which can be removed or replaced separately. In this manner rapid changes in shaping networks can be made using data acquired from the previous flight.

Passive elements only are used in the shaping networks. Typical networks of the type planned for C1, Block II are shown in Figure 17. While the values of reactive elements required arequite large, careful component selection and design keeps the overall size reasonable. For instance, the largest inductor used (2000 henries) weighs less than one pound.

The gain programmer is a simple cam mechanism driven by a synchronous motor. The cam positions a potentiometer to set the gain in each channel. The plug-in concept has been employed in the design of this module also.

The control computer weighs approximately 35 pounds and is packaged in a 21" x 14" x 6" box.

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Ve. CONTROL RATE GYRO

The control rate gyro is a single degree of freedom fluid damped type, with an angular variable differential transformer pick-off. The full scale range of the gyro is plus and minus 10 degrees per second, with an ac output of 0.2 volts per degree.

The gyros have two self-checking features which allow operational checks of the instruments remotely. A signal generator is built into the spin motor to indicate the speed at which it is turning. An electro-magnetic torquer is attached to the gimbal so that it may be torqued, and thus give an output signal simulating an input rate to the gyro. With these features it is possible to determine if the motor is turning at the proper speed, if the gyro gimbal is free to move, and if the pick-off and associated circuitry are operating properly.

Three gyros are used in a three-axis package on Saturn to measure the angular rate of movement of the vehicle in the pitch, yaw and roll planes. A picture of the three-axis package is shown in Figure 18.

Vf. CONTROL ACCELEROMETER

The control accelerometer is a linear, spring-mass, fluid damped type, with an inductive pick-off. The range of the instrument is plus and minus 10 meters per second square ~~per second~~ with a basic ac output of 0.5 volts per meter per second^2 ~~per second~~. This ac signal output is amplified and demodulated, all within the accelerometer package, with a resultant dc signal output of 4.5 volts per meter per second^2 ~~per second~~.

This instrument also contains a feature which permits remote checking after installation in the vehicle. Through the use of an internal electro-magnetic coil, it is possible to deflect the seismic mass a calibrated amount, and thus check the electrical output of the instrument without subjecting it to acceleration.

Two control accelerometers are used on the Saturn vehicle. They will measure the lateral acceleration perpendicular to the longitudinal axis in the pitch and yaw planes. The output of the instruments is used in the control system to reduce lateral structural loading. A picture of the control accelerometer is shown in Figure 19.

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Vg. ACTUATOR

As an example of the type of actuators employed on the Saturn, a picture of the S-1 actuator which gimbals the H-1 engine is shown in Figure 20. This hydraulic actuator is of the linear, double acting, equal piston area type. The source pressure is 3000 psi. The piston area is 5 square inches, and the actuator is capable of producing a maximum gimbal velocity of approximately 25 degrees/second. A pressure feedback valve is used to control the actuator. The pressure feedback valve has a hydraulic feedback loop in which the pressure on each side of the actuator piston is fed back to the servo-valve spool. This type valve has a strong stabilizing effect on the controlled load. The actuator also includes an internally enclosed linear transducer which is used to indicate the actuator position to the control computer and to telemetry.

VI. APPLICATION OF ADAPTIVE GUIDANCE TO SATURN C-1

A great amount of effort is being expended in studying techniques for determining empirical Adaptive Guidance equations for various missions and vehicle combinations. Numerous techniques for numerical curve fitting operations are being applied to determine the best method of representing the volume of expected trajectories which provide minimum fuel consumption. The data given here is the result of only one such study, but is indicative of the type of information to be expected from other studies with different vehicles and mission constraints.

The vehicle employed was a Saturn C1, Block II. This is a two stage vehicle with eight engines in the first stage and six in the second stage. The thrust of each engine in the first stage was assumed to be 188K lbs; the thrust of each engine in the second stage was assumed to be 15K lbs. The first stage trajectory was shaped to favor seven engines burning in the first stage from liftoff to fuel depletion. This tilt program has been found to give satisfactory results with either seven or eight engines burning during the first stage.

Three similar orbital missions were established for this study. These orbits were required to be circular at altitudes of 100, 200, and 300 nautical miles, but no restrictions were placed upon the plane of the orbits, or the position of the vehicle in the orbits. This type of mission is referred to as being range-independent.

A volume of optimum trajectories were computed which satisfied the mission requirements and which represented the probable volume of space through which the vehicle might be expected to fly. The volume included those trajectories resulting from an engine failure in the first stage. All flights were constrained to the same plane.

An arbitrary sampling of "points" was made from the volume of trajectories for each mission. These "points" consisted of the set of parameters which affect the thrust angle, χ_{zo}, for the optimum trajectory. Although the sampling was arbitrary, it was consistent for the three orbits.

Polynomials were fitted to these points to obtain expressions for the tilt angle, χ_{zo}. The polynomials were selected arbitrarily and, therefore, were not the best; however, the same polynomials were used to fit the points for each mission and are typical. The polynomials selected contained 8, 36, 47, and 57 terms.

The criteria for determining the best fit for the polynomials is difficult to determine. Numerous indicators are used, such as relative mission achievement, relative amount of propellants consumed, and the root-sum-square error. The root-sum-square is used in this report for comparison purposes. Although a small root-sum-square error is necessary for a good steering function, it is not a guarantee of a good steering function.

The cutoff equations used in the study forced cutoff to occur when the correct velocity was obtained so that the errors would result in path angle and altitude errors only. Moreover, this type of cutoff criteria enabled a comparison to be made between the various steering polynomials without being influenced by the cutoff polynomial.

Table I compares the root-sum-square errors resulting from the various polynomials for the three orbits.

Trajectory simulations were made for the limits of the volume of trajectories used to obtain the steering polynomials. Table II gives the results of the simulation for the 100 nautical mile orbit. The errors in path angle and altitude are those obtained when a simulation of guidance was made using the empirically determined steering functions. The reference for the errors was the exact path angle and altitude required to satisfy the mission.

The techniques used in these somewhat arbitrary simulations and the results obtained are intended only to give an indication of the usefulness of the Adaptive Guidance Mode in the Saturn vehicle and the length of the guidance polynomials to be expected. A great number of detailed simulations have been made using the Adaptive Guidance Mode with various missions. In addition, simulated guidance studies have been made in which preliminary hardware error specifications were included. The results of these combined studies indicate that the guidance concept and the implementing system are completely satisfactory for all foreseeable Saturn vehicles and missions even under the conditions of severe disturbance.

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REFERENCES

1. Miner, W. E. and Schmieder, D. H., "The Path-Adaptive Mode for Guiding Space-Flight Vehicles", ARS Reprint 1944-61, August 1961

2. Schmieder, D. H. and Brand, N. J., "Implimentation of the Path-Adaptive Guidance Mode in Steering Techniques for Saturn Multi-stage Vehicles", ARS Reprint 1945-61, August 1961

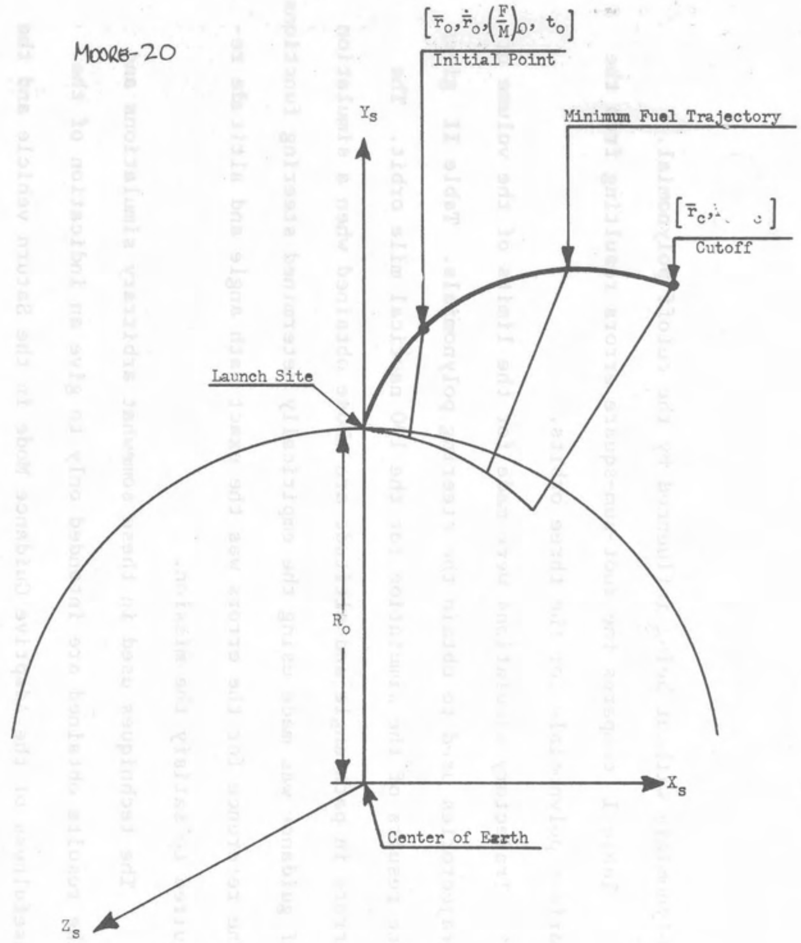
3. Winch, John B., "Present Theory and Techniques Applied to Space Vehicle Problems", in Status Report No. 1 on Theory of Space Flight and Adaptive Guidance, MTP-AERO-62-61, Aeroballistics Division, George C. Marshall Space Flight Center, Huntsville, Alabama, March 1, 1962

4. Hoelker, R. F. and Miner, W. E., "Introduction Into the Concept of the Adaptive Guidance Mode", Aeroballistics Internal Note 21-60, Aeroballistics Division, George C. Marshall Space Flight Center, Huntsville, Alabama, December 28, 1960

5. Geisler, Ernst D. and Haeussermann, Walter, "Saturn Guidance and Control", Astronatuics, ARS, February 1961

6. Gassaway, G. G., Haeussermann, W., and Moore, F. B., "Guidance and Control Systems for Space Carrier Vehicles", in From Peenemunde to Outer Space, George C. Marshall Space Flight Center, Huntsville, Alabama, March 23, 1962

7. Moore, Richard L. and Thomason, Herman E., "Gimbal Geometry and Attitude Sensing of the ST-124 Stabilized Platform", MTP-G&C-G-61-38, Guidance and Control Division, George C. Marshall Space Flight Center, Huntsville, Alabama, September 27, 1961



SATURN POWERED TRAJECTORY

Figure 1

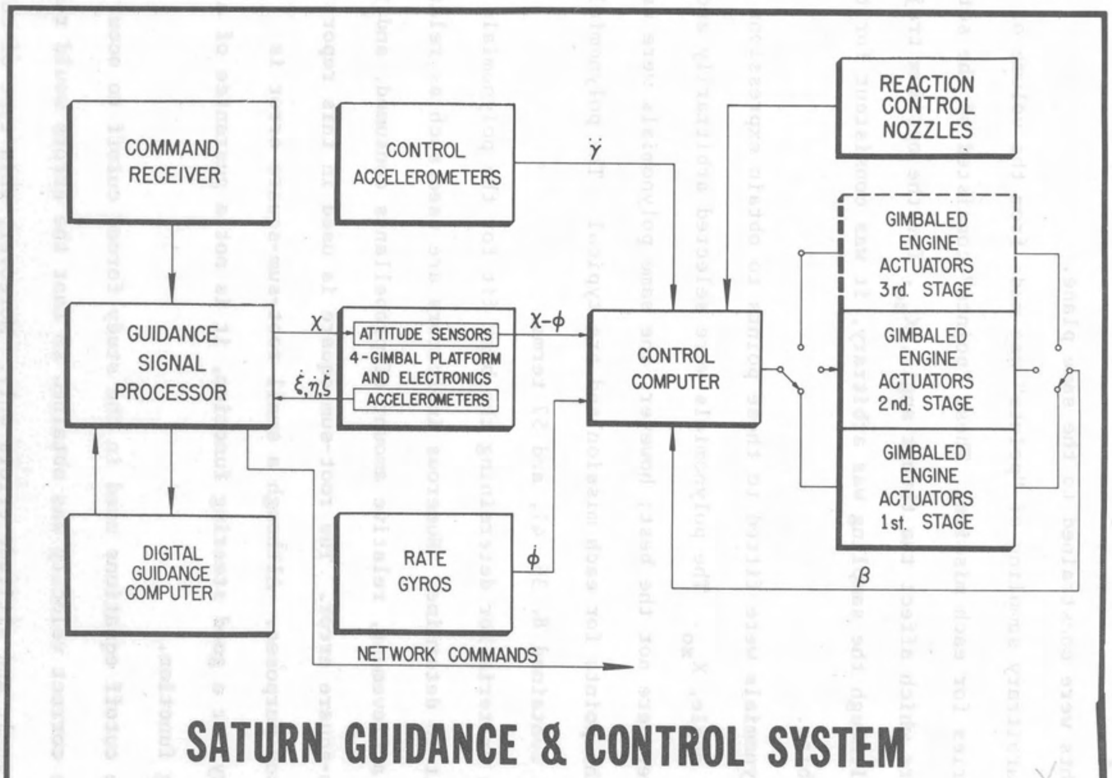


Figure 2

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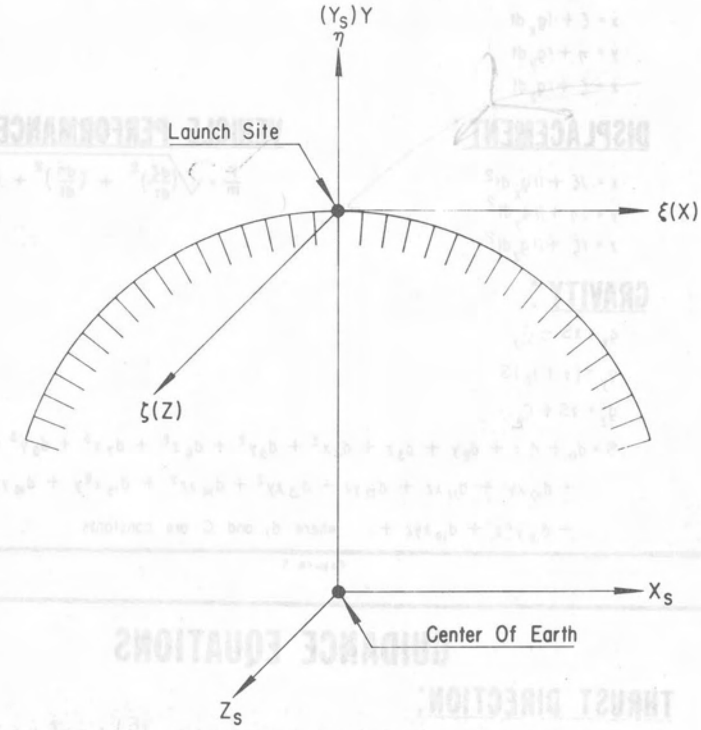
SATURN GUIDANCE COORDINATE SYSTEM



X_S = Down Range Direction Along Launch Horizontal.
Y_S = Direction Of Launch Verticle.
Z_S = Direction Perpendicular To Flight Plane.
ξ, η, ζ = Inertial Measuing Directions.
X,Y,Z = Space Fixed Coordinate System For Gravity Computation.

Figure 3

SATURN GUIDANCE MECHANIZATION

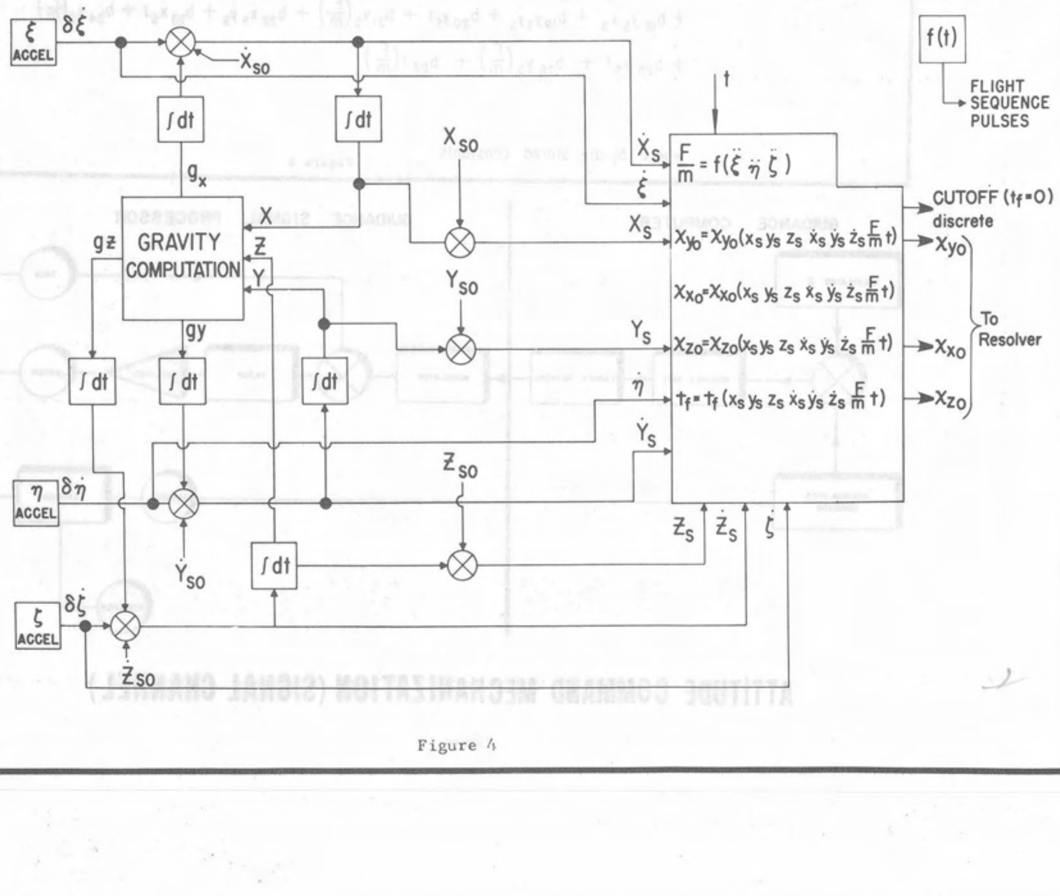


Figure 4

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BASIC EQUATIONS

VELOCITY:

$$\dot{x} = \dot{\xi} + \int g_x\,dt$$
$$\dot{y} = \dot{\eta} + \int g_y\,dt$$
$$\dot{z} = \dot{\zeta} + \int g_z\,dt$$

DISPLACEMENT:

$$x = \int \dot{\xi} + \iint g_x\,dt^2$$
$$y = \int \dot{\eta} + \iint g_y\,dt^2$$
$$z = \int \dot{\zeta} + \iint g_z\,dt^2$$

VEHICLE PERFORMANCE:

$$\frac{F}{m} = \sqrt{\left(\frac{d\dot{\xi}}{dt}\right)^2 + \left(\frac{d\dot{\eta}}{dt}\right)^2 + \left(\frac{d\dot{\zeta}}{dt}\right)^2}$$

GRAVITY:

$$g_x = xS + C_1$$
$$g_y = (y + r_0)S$$
$$g_z = zS + C_2$$

$$S = d_0 + d_1 x + d_2 y + d_3 z + d_4 x^2 + d_5 y^2 + d_6 z^2 + d_7 x^3 + d_8 y^3 + d_9 z^3$$
$$+ d_{10} xy + d_{11} xz + d_{12} yz + d_{13} xy^2 + d_{14} xz^2 + d_{15} x^2 y + d_{16} yz^2 + d_{17} x^2 z$$
$$+ d_{18} y^2 z + d_{19} xyz + \ldots$$ where d_i and C are constants

Figure 5

GUIDANCE EQUATIONS

THRUST DIRECTION:

$$\chi_{YO}, \chi_{XO}, \chi_{ZO} = a_0 + a_1 x_S + a_2 y_S + a_3 \dot{x}_S + a_4 \dot{y}_S + a_5 t + a_6\left(\frac{F}{m}\right) + a_7 x_S^2 + a_8 y_S^2 + a_9 \dot{x}_S^2$$
$$+ a_{10} \dot{y}_S^2 + a_{11} t^2 + a_{12}\left(\frac{F}{m}\right)^2 + a_{13} x_S y_S + a_{14} x_S \dot{x}_S + a_{15} x_S \dot{y}_S + a_{16} x_S t$$
$$+ a_{17} x_S\left(\frac{F}{m}\right) + a_{18} y_S \dot{x}_S + a_{19} y_S \dot{y}_S + a_{20} y_S t + a_{21} y_S\left(\frac{F}{m}\right) + a_{22} \dot{x}_S \dot{y}_S$$
$$+ a_{23} \dot{x}_S t + a_{24} \dot{x}_S\left(\frac{F}{m}\right) + a_{25} \dot{y}_S t + a_{26} \dot{y}_S\left(\frac{F}{m}\right) + a_{27} t\left(\frac{F}{m}\right)$$

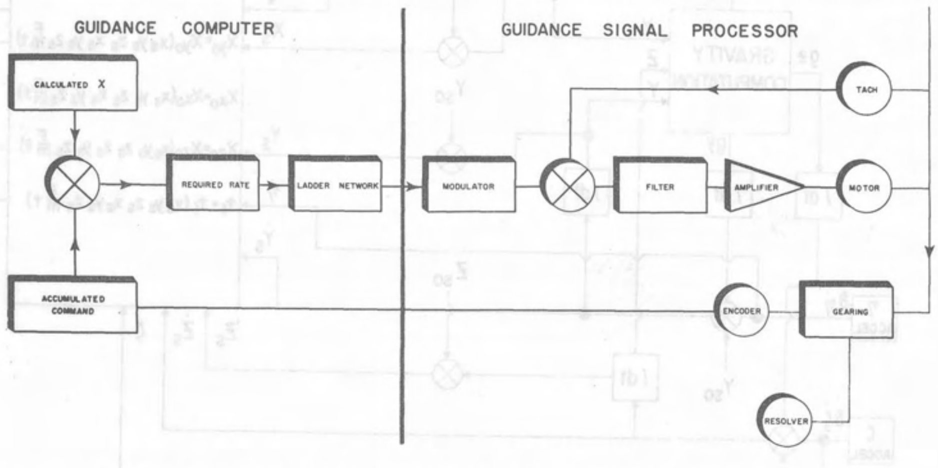
where a_i are stored constants different for each stage.

CUTOFF:

$$t_f = b_0 + b_1 x_S + b_2 y_S + b_3 \dot{x}_S + b_4 \dot{y}_S + b_5 t + b_6\left(\frac{F}{m}\right) + b_7 x_S^2 + b_8 y_S^2 + b_9 \dot{x}_S^2 +$$
$$b_{10} \dot{y}_S^2 + b_{11} t^2 + b_{12}\left(\frac{F}{m}\right)^2 + b_{13} x_S y_S + b_{14} x_S \dot{x}_S + b_{15} x_S \dot{y}_S + b_{16} x_S t + b_{17} x_S\left(\frac{F}{m}\right)$$
$$+ b_{18} y_S \dot{x}_S + b_{19} y_S \dot{y}_S + b_{20} y_S t + b_{21} y_S\left(\frac{F}{m}\right) + b_{22} \dot{x}_S \dot{y}_S + b_{23} \dot{x}_S t + b_{24} \dot{x}_S\left(\frac{F}{m}\right)$$
$$+ b_{25} \dot{y}_S t + b_{26} \dot{y}_S\left(\frac{F}{m}\right) + b_{27} t\left(\frac{F}{m}\right)$$

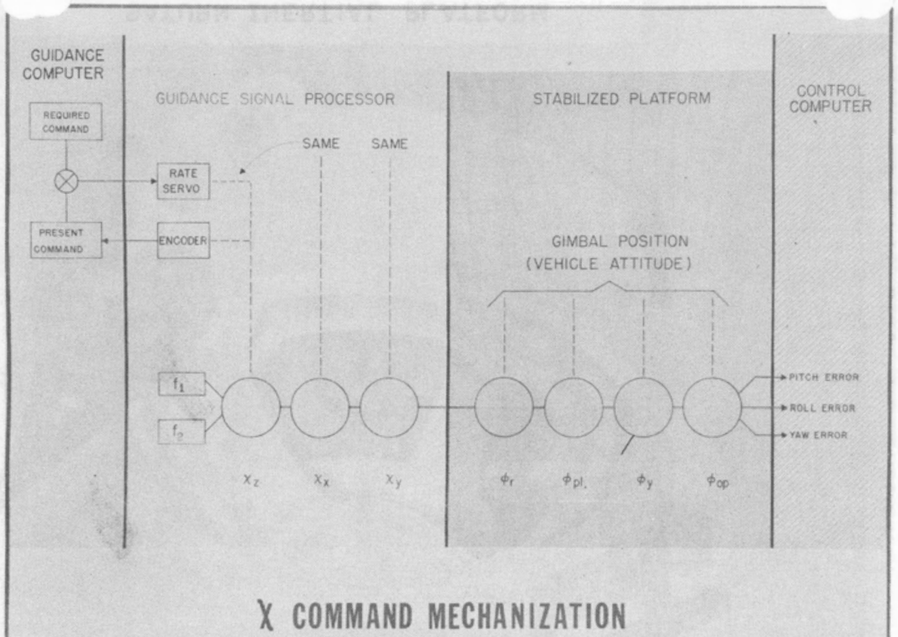
where b_i are stored constants.

Figure 6



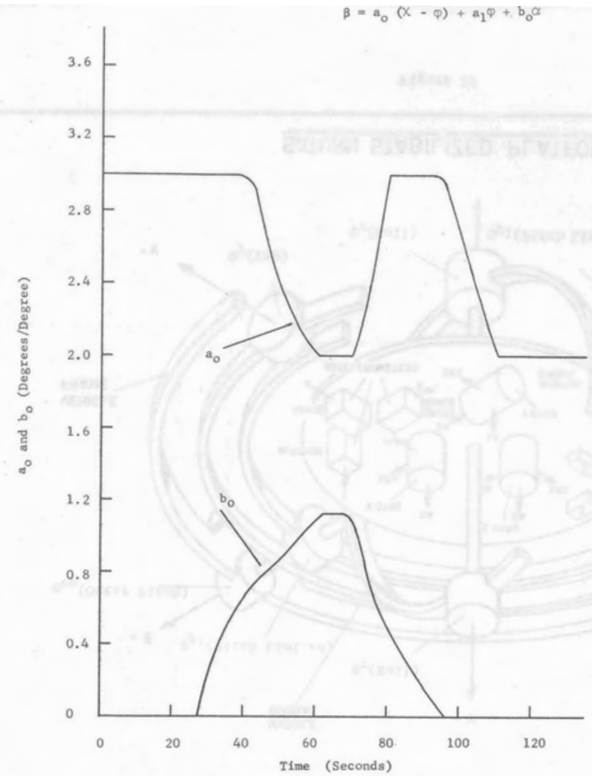
ATTITUDE COMMAND MECHANIZATION (SIGNAL CHANNEL)

Figure 7



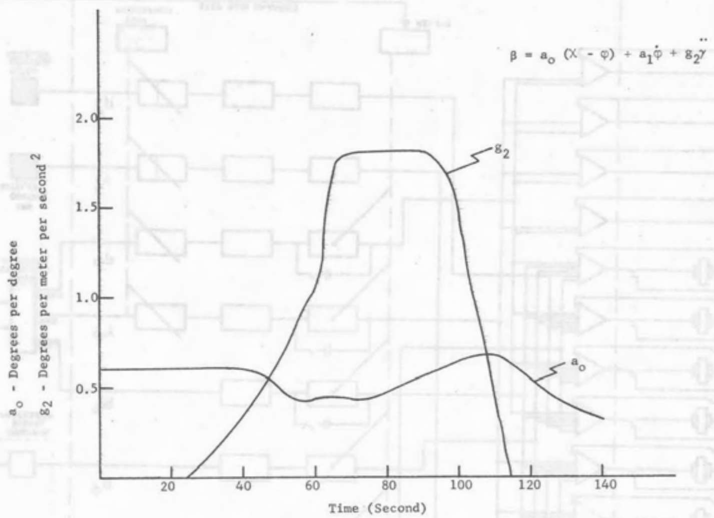
χ COMMAND MECHANIZATION

Figure 8



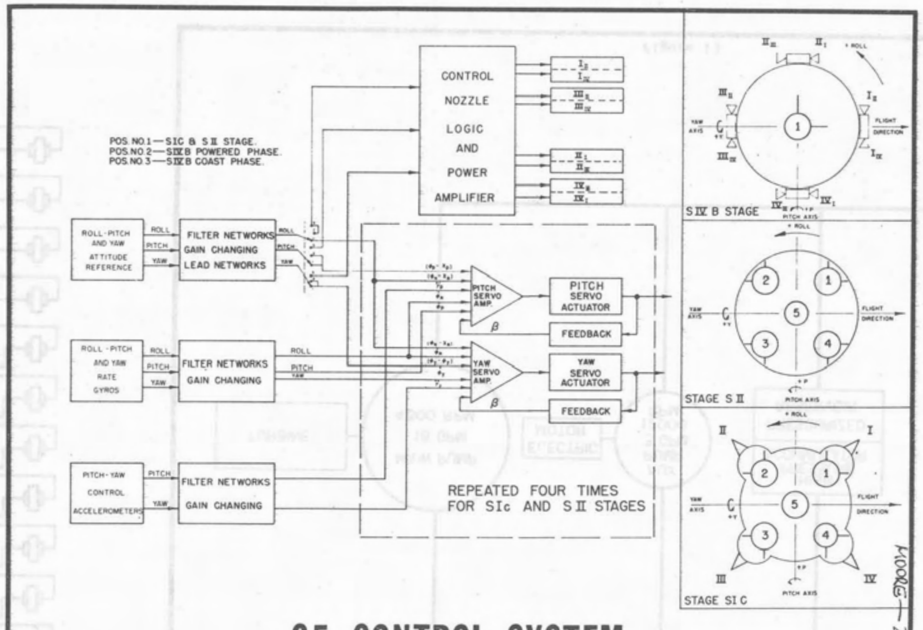
Gain Program, Saturn C-1, Block II, Angle-of-Attack Transducer

Figure 9



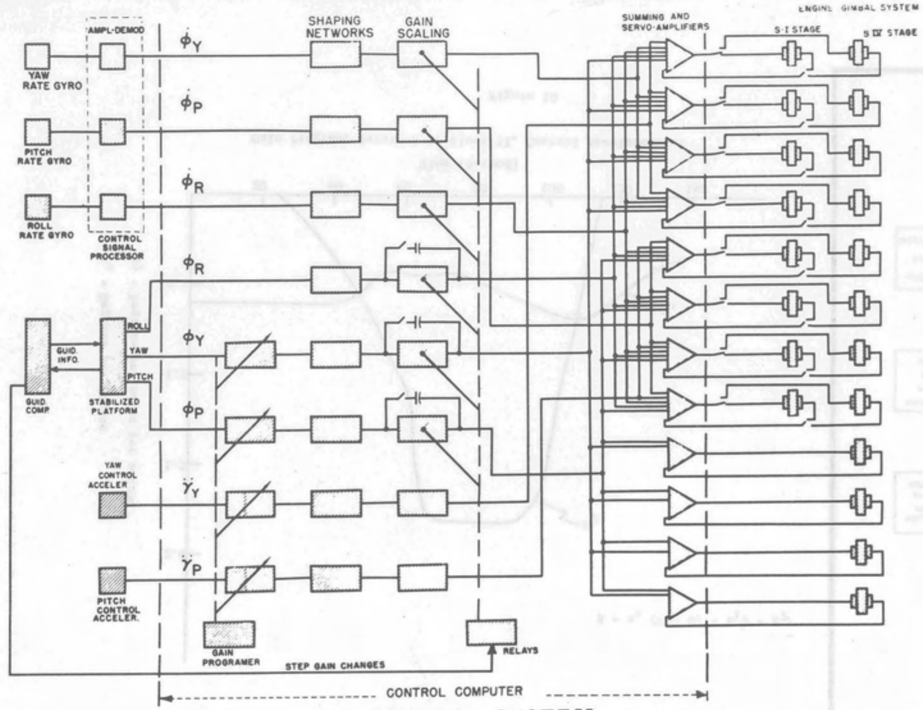
Gain Program, Saturn C-1, Block II, Control Accelerometer

Figure 10



C5 CONTROL SYSTEM

Figure 11



C-1 CONTROL SYSTEM

Figure 12

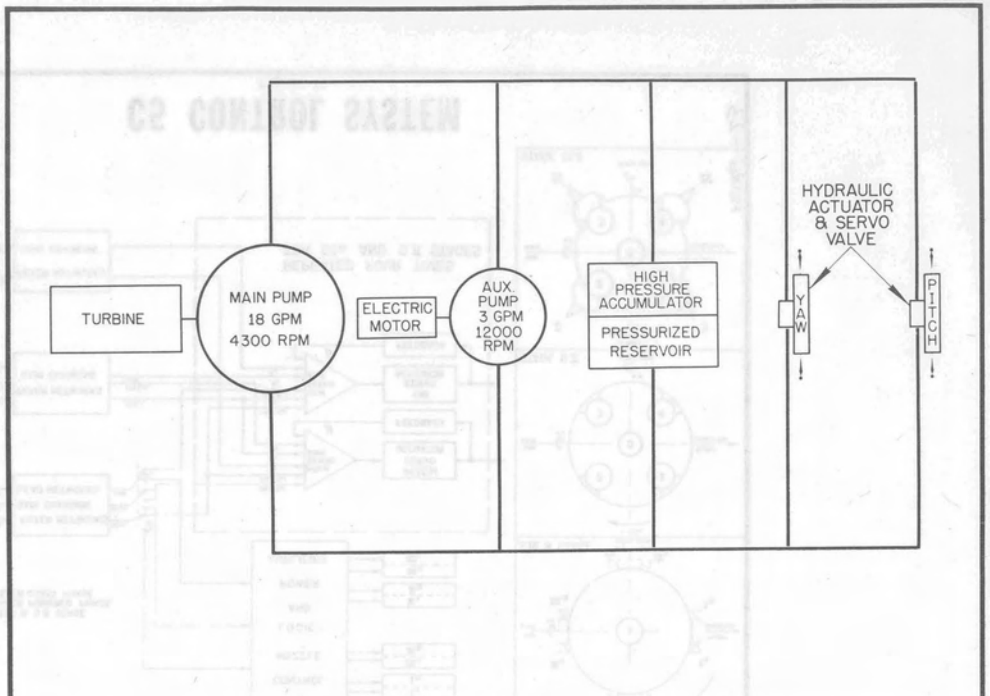
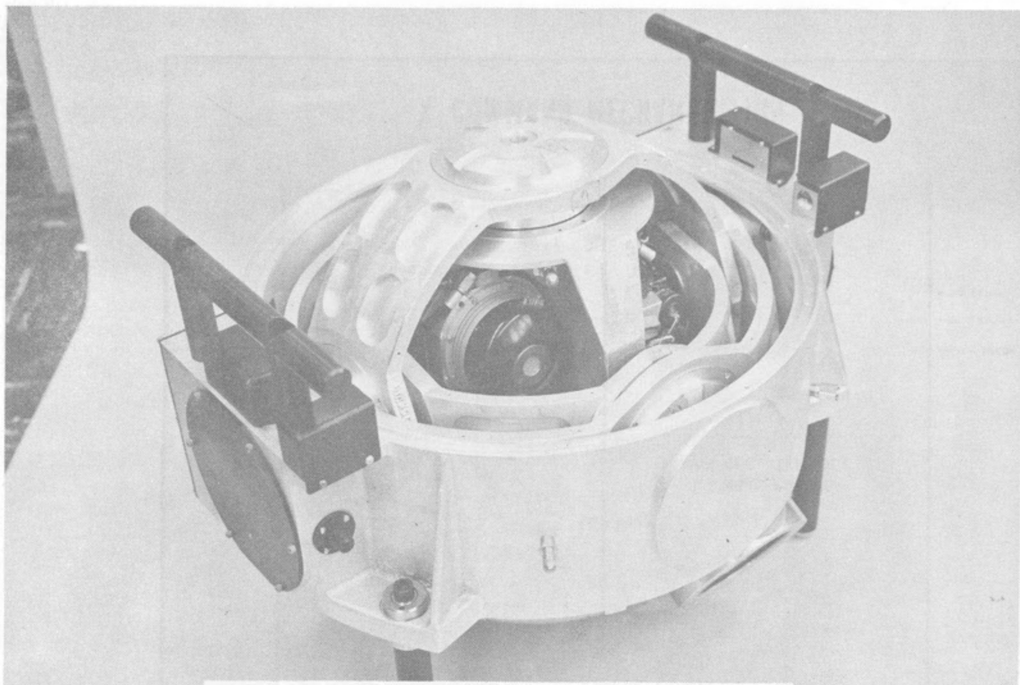
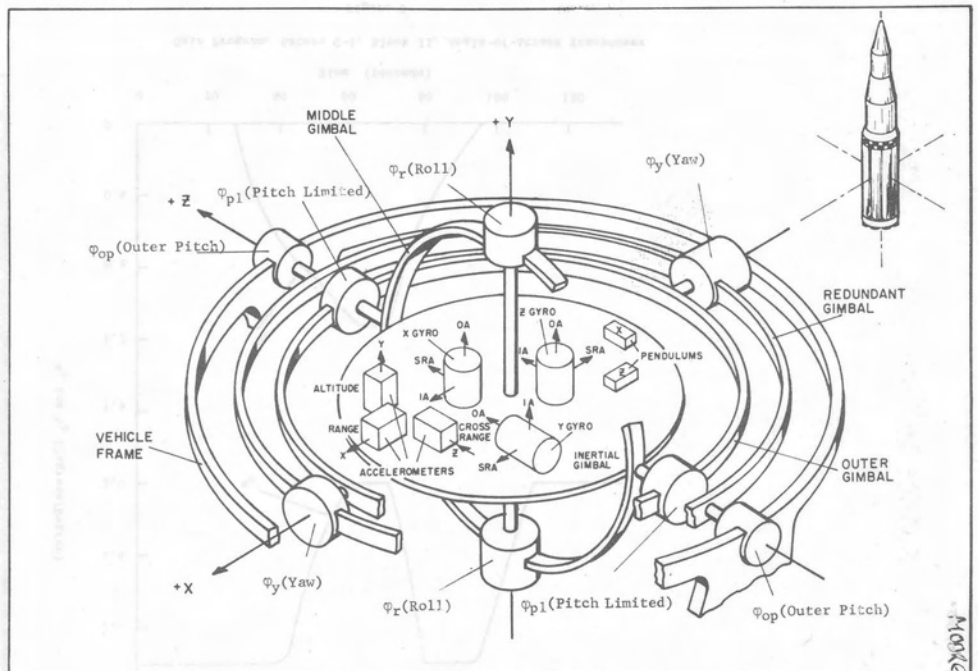


Figure 13



SATURN INERTIAL PLATFORM
S-124
Figure 14



SATURN STABILIZED PLATFORM

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Figure 15

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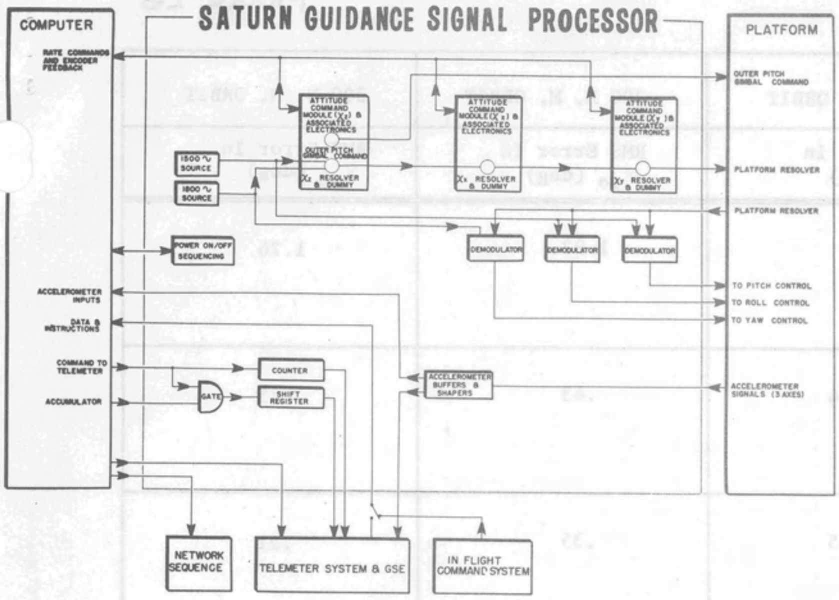
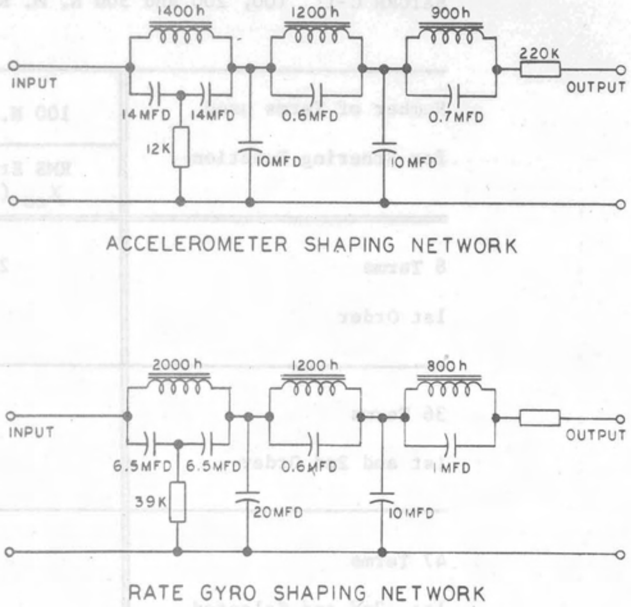
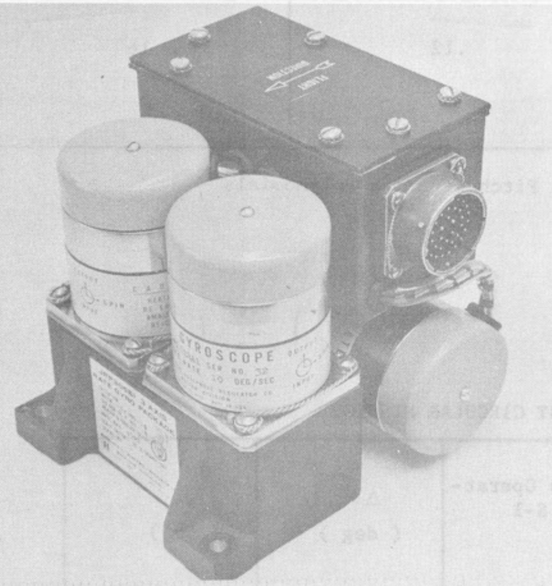


Figure 16



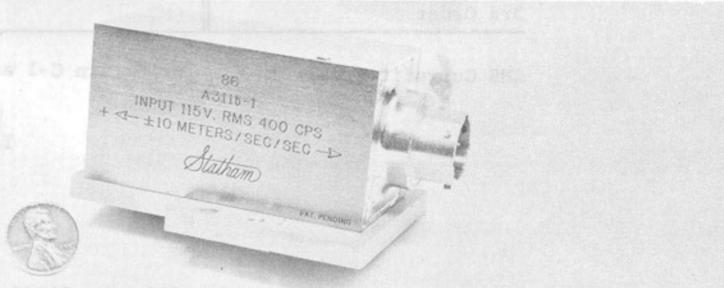
TYPICAL SATURN C-I CONTROL SHAPING NETWORKS

Figure 17



THREE-AXIS CONTROL RATE GYRO PACKAGE

Figure 18



CONTROL ACCELEROMETER

Figure 19

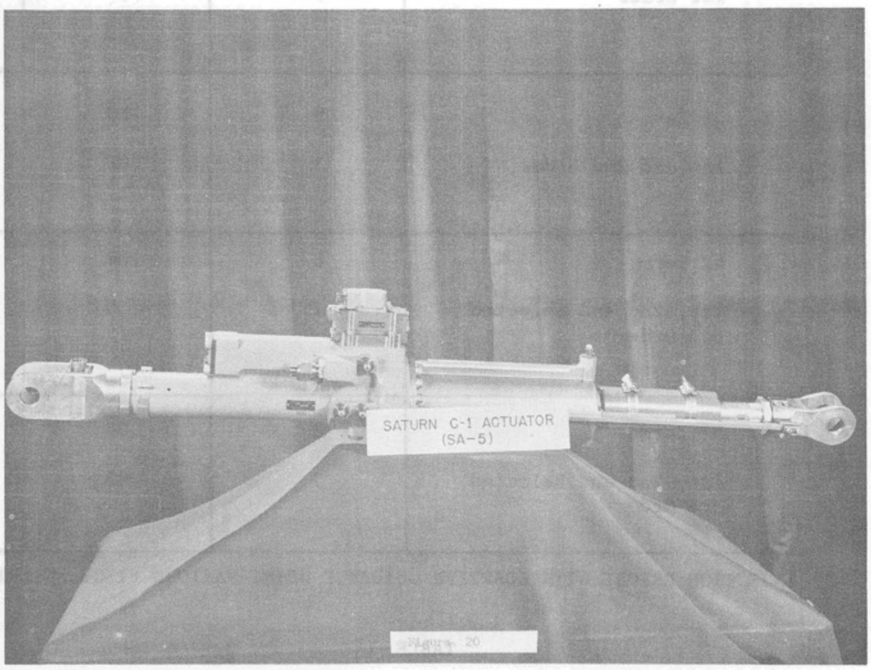


Figure 20

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SATURN C-1: 100, 200 and 300 N. M. RANGE INDEPENDENT MISSION

Number of Terms used for Steering Function	100 N. M. ORBIT	200 N. M. ORBIT	300 N. M. ORBIT
	RMS Error in X_{zo} (deg)	RMS Error in X_{zo} (deg)	RMS Error In X_{zo} (deg)
8 Terms 1st Order	2.31	1.92	1.76
36 Terms 1st and 2nd Order	.64	.43	.25
47 Terms 1st, 2nd and Selected 3rd Order	.45	.35	.21
57 Terms 1st, 2nd and Selected 3rd Order	.22	.12	.16

RMS Curvefit Errors in X_{zo} for Saturn C-1 and Various Pitch Steering Polynomials

TABLE I

SATURN C-1: 100 N.M. RANGE-INDEPENDENT CIRCULAR MISSION

Number of Terms Used For Steering Functions	Engines Operating in S-1 Stage	$\Delta\theta$ (deg)	Δh (km)
1. 8 Terms	8	.39	4.9
1st Order	7	-.67	-3.7
2. 36 Terms	8	.10	.2
1st and 2nd Order	7	.01	-.3
3. 47 Terms	8	.04	.4
1st, 2nd and Selected 3rd Order	7	0	.4
4. 57 Terms	8	.06	-.2
1st, 2nd and Selected 3rd Order	7	05	-.1

INJECTION ERRORS WITH ADAPTIVE GUIDANCE USING VARIOUS PITCH STEERING POLYNOMINOLS

TABLE II