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SATURN APOLLO GUIDANCE ERROR ESTIMATION

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Abstract

A brief sketch of the development of the equations for a weighted least squares estimator is given, the equations for both collective and recursive estimators being included. Four possible problem sources that may be encountered in the application of the estimator are identified. Various "success" parameters are defined in an attempt to predict the success with which the method has been applied. The application of the estimation technique to the problem of computing various error parameters associated with the ST-124M guidance platform is described and the numerical results obtained using a manufactured data case are presented. These results are used to form conclusions about the effectiveness of the "success" parameters and preferred approaches to the problem of system evaluation using techniques of estimation theory.

I. Introduction

Statistical treatment of data dates back to the least squares techniques devised by Gauss. From this beginning, very sophisticated estimation techniques have evolved. Since each of these techniques requires a large number of numerical computations, the development of high speed computers has encouraged work in this area and in recent years a great number of investigators have reported their work in the technical literature. References 1-6 are representative of such reports.* The basic idea in all these techniques is to take a large number of measurements, with attendant measurement inaccuracies, of a particular set of values and from this overdetermined system to compute the set of values that best represents the result of the measurement process. The measurements involved can be either direct measurements, where the parameters of interest are measured or indirect measurements where known functions of the parameters of interest are measured. There are various ways of defining the criteria that leads to the "best" estimate of the parameters. This brief statement gives a background for the definition of the problem to be treated.

*The list of these works included in the present work represent ones that have become familiar to the authors. The omission of any specific work from this list is not intended to reflect the author's opinion of its usefulness.

The Saturn Apollo IB and V vehicles use the ST-124M platform as the heart of the guidance system. This platform provides an inertial reference frame and accelerometers from which intelligence for the guidance computer is obtained. Certain errors are associated with the platform that lead to slightly inaccurate measurements with the result that the vehicle will fly a trajectory that differs from the design or nominal trajectory. The measured deviations of the trajectory from nominal are used as indirect measurements of the error parameters that cause the deviation and a "best" estimate of these parameters is computed, "best" in this case being defined as the weighted least squares estimate. The method permits the use of direct preflight measurements of the error parameters so that all available information is used in obtaining the estimate. However, problems arise in this procedure that have not been fully resolved. These problems can usually be traced to one or more of the following sources:

(1) The flight measurements and the preflight measurements are significantly different.

(2) The effects of two or more parameters on the trajectory are so closely correlated that a near singular situation develops in their simultaneous computation.

(3) The set of error parameters considered do not include certain parameters that have a significant effect on the trajectory.

(4) The assumption of linear relationships between trajectory deviations and the error parameters is not valid.

The development of the equations used in the computations will be outlined and methods that attempt to eliminate some of the above problems will be given. Finally, numerical results obtained in a test case will be reviewed.

II. Statistical Formulation[7,8]

The weighted least-squares adjustments are made on the basis of observed velocity deviations between the nominal trajectory and the trajectory as determined by the tracking measurements. A basic assumption in the derivation which follows is that the tracking data contains no systematic errors, although the equations developed could easily be modified to include the effects of such errors. Based on this assumption the observed velocity deviations are expressed mathematically by a first order Taylor expansion about a zero nominal value:

$$\overline{\Delta V}_i = B_{G_i} \overline{K}_G + \overline{\eta}_i \qquad (1)$$

where $\overline{\Delta V}_i$ is the vector velocity deviation for time t_i, B_{G_i} is a $(3 X_k)$ matrix of partial derivatives of the velocity deviation with respect to the error parameters (k is the number of error parameters being considered), $\overline{K}_G$ is a $(kX1)$ column matrix of error parameters and η_i is a $(3X1)$ column matrix representing noise contributions to the measurements. The $\overline{\Delta V}_i$ are obtained by differencing telemetry and tracking measures of velocity for each time point. $\overline{K}_G$ is defined by the error model equations being used. The partial derivative matrix B_{G_i} is computed using these equations. The noise vectors over a given flight are assumed to have zero mean and a known standard deviation. One further assumption simplifies the procedure greatly. This assumption is that the three noise components associated with a given measurement are uncorrelated and that there is no correlation from one measurement to the next.

Measurements of the error parameters are made prior to each flight and these measurements are included in the total amount of information to be used in obtaining the best estimate of these error parameters. These "a priori" measurments are denoted by $\overline{K}_{G_o}$ which is a $(kX1)$ column matrix.

The weighted least squares estimate is the particular set of values for the components of $\overline{K}_G$ that minimizes the function

$$F = \sum_{i=1}^{N} \overline{\eta}_i^T W_i \overline{\eta}_i + (\overline{K}_G - \overline{K}_{G_o})^T W_{K_{G_o}} (\overline{K}_G - \overline{K}_{G_o}) \quad (2)$$

where W_i and $W_{K_{G_o}}$ are the weighting matrices for the flight measurements and the preflight measurements respectively. Further interpretation and discussion of the weighting matrices are given in a later section. Close examination of the function F shows that it is the sum of the weighted squares of the noise components of the flight measurements plus a term whose effect is to constrain the solution to lie in a region determined by the specific values of $\overline{K}_{G_o}$ and $W_{K_{G_o}}$

The desired weighted least-squares solution is obtained by determining the partial derivatives $\partial F / \partial K_{G_i}$ and setting these expressions equal to zero. Such an operation is effected by solving for $\overline{\eta}_i$ from Equation (1), substituting this expression into Equation (2), and finally performing the required differentiations. The resulting equations, when solved for the error parameters, become

$$\overline{K}_G = C_{K_G}\left[\left(\sum_{i=1}^{N} B_{G_i}^T W_i \overline{\Delta V}_i\right) + W_{K_{G_o}} \overline{K}_{G_o}\right] \quad (3)$$

where

$$C_{K_G} = \left[\left(\sum_{i=1}^{N} B_{G_i}^T W_i B_{G_i}\right) + W_{K_{G_o}}\right]^{-1}$$

defines the covariance matrix of the guidance error parameters.

The equations above will be referred to as the collective least squares solution. By considering these expressions using N data points and N+1 data points and making liberal use of various matrix indentities, a recursive estimator can be obtained. The equations for the recursive estimator take the form

$$\overline{K}_{G_i} = \overline{K}_{G_{i-1}} + F_i\left[\overline{\Delta V}_i - B_{G_i}\overline{K}_{G_{i-1}}\right], \quad i = 1, 2, --- \quad (4)$$

and

$$C_{K_{G_i}} = C_{K_{G_{i-1}}} - F_i B_{G_i} C_{K_{G_{i-1}}}, \quad i = 1, 2, --- \quad (5)$$

$$F_i = C_{K_{G_{i-1}}} B_{G_i}^T \left[W_i^{-1} + B_{G_i} C_{K_{G_{i-1}}} B_{G_i}^T\right]^{-1} \quad (6)$$

The subscript i used with K_G or C_{K_G} indicates that data up to and including the i^{th} data point have been used in the computations. For the other quantities the subscript i indicates data for a particular data point. It is noted here without proof that equations 4, 5, and 6 can be obtained by either the Kalman method or by the maximum likelihood method if the appropriate assumptions are made.

III. Required Data[8]

The equations of the previous section are used to compute the weighted least squares estimates of the error parame The data used in the computations are briefly described below.

Preflight Estimates and their Variances

The preflight estimates are obtained from laboratory measurements and prelaunch pad measurements. They are represented by a (k x 1) column matrix where k is the number of parameters to be considered in the computation. $\overline{K}_{G_o}$ is the symbol used for the vector of preflight estimates.

The variances of the preflight estimates are the squares of the standard deviations, σ_i associated with the measurements. The inverse of the variance is used to obtain the weight matrix assigned to the preflight estimates. This weight matrix is given by

$$W_{K_{G_o}} = \begin{bmatrix} \left(\frac{1}{C_1\sigma_1}\right)^2 & & & \bigcirc \\ & \left(\frac{1}{C_2\sigma_2}\right)^2 & & \\ & & \ddots & \\ \bigcirc & & & \left(\frac{1}{C_k\sigma_k}\right)^2 \end{bmatrix} \quad (7)$$

where C_i are parameters used to change $W_{K_{G_o}}$ in a convenient way.

Delta Velocities and their Variances

The delta velocities, $\overline{\Delta V}_i$ are three component vectors obtained by differencing the velocity measurements obtained from telemetry data and from error free tracking data. Thus, the entire velocity discrepancy is assumed to be due to systematic errors in the guidance system with random noise errors superimposed. There is one such vector for each time point for which data is available.

The variances associated with the $\overline{\Delta V}$'s are obtained by estimating the width of the envelope of the curve obtained when the $\overline{\Delta V}$'s are plotted against time. The variances are the squares of one-half the envelope width. Variances are assigned to regions of the curve. The weight matrix assigned to a particular $\overline{\Delta V}$ is the inverse of the variance assigned the region of the $\overline{\Delta V}$ versus time curve from which that ΔV is taken. The weights obtained in this way are diagonal (3 x 3) matrices and are denoted by W_i.

Partial Derivative Matrices

The matrices of partial derivatives B_{G_i} are computed using the error model equations of the following section. The nominal acceleration profile is used for this purpose so the partial derivative matrices are valid only as long as the true trajectory does not differ greatly from the nominal. However, for post flight evaluation the partial derivatives are usually computed using inflight measurements of acceleration so this problem does not exist.

Each B_G matrix has dimensions 3 x k and there is one such matrix for each time point considered.

IV. Error Model Equations[9]

The error model equations relate errors in acceleration to the various errors associated with the guidance platform. The most complete error model consists of 30 error terms. However, the trajectories flown by vehicles using the ST-124M platform during research and development testing do not have significant cross range accelerations so that terms proportional to the cross-range acceleration are not considered. Other error sources have been determined to be relatively insignificant. The result is that the number of error terms considered has been reduced to 18. The error terms considered are related to errors in the inertial acceleration by the equations

$$\Delta a_x = B_x + S_x a_x + a_y(\delta_z + \dot{\delta}_z t + \dot{\delta}_{z/\ddot{x}} V_x + \dot{\delta}_{z/\ddot{y}} V_y)$$

$$\Delta a_y = B_y + S_y a_y - a_x(\delta_z + M_{yz} + \dot{\delta}_z t + \dot{\delta}_{z/\ddot{x}} V_x + \dot{\delta}_{z/\ddot{y}} V_y)$$

$$\Delta a_z = B_z + a_x(\delta_y + \dot{\delta}_y t + \dot{\delta}_{y/\ddot{x}} V_x + \dot{\delta}_{y/\ddot{y}} V_y) - a_y(\delta_x + \dot{\delta}_x t + \dot{\delta}_{\dot{x}/\ddot{x}} t + \dot{\delta}_{x/\ddot{x}} V_x + \dot{\delta}_{x/\ddot{y}} V_y)$$

(8)

where Δa_x, Δa_y, Δa_z are the errors in the inertial acceleration components, and the system error parameters (which correspond to the elements of the K_G matrix) are defined as follows:

B_i = bias error of the i-accelerometer

S_i = scale factor error of the i-accelerometer

$\delta_{i.}$ = misalignment error associated with rotation about the i-axis

$\dot{\delta}_i$ = platform constant drift rate about the i-axis

M_{ij} = non-orthogonality between i and j axes (positive if axes form an angle greater than 90 degrees)

$\dot{\delta}_{i/\ddot{j}}$ = platform "g-sensitive" drift about the i-axis due to acceleration parallel to the j-axis

The partial derivative elements are the time integrals of the coefficients of the error terms in equations (8).

The mathematical error model described in the preceding paragraph was derived for the ST-124 platform currently in use on Saturn vehicles. A detailed mathematical development may be found in Reference 9, in which small-angle assumptions were not made. Equations (7) represent the result of assuming small-angle errors and simplifying the results obtained in Reference 9.

V. Success Parameters[8]

It has been stated that certain problems occur in the estimation procedure that have not been fully resolved. This section presents definitions of several "success" parameters designed to determine the success with which the platform error parameters have been estimated.

Correlation Coefficients

Three different correlation coefficients are computed. These are:

ρ_{ij} = ordinary correlation coefficient,

ρ'_{ij} = partial correlation coefficient,

ρ''_i = multiple correlation coefficient.

ρ_{ij} is designed to show to what extent the effects of the i^{th} parameter can be represented by the effects of the j^{th} parameter or vice versa. However, the ρ_{ij} are computed using the covariance matrix, C_{K_G}, and as a result have the effect of the other K-2 parameters indirectly included.

ρ'_{ij} is similar to ρ_{ij}, but it is computed from the W_{K_G} $(=C_{K_G}{}^{-1})$ matrix and is a measure of the dependence of the i^{th} parameter on the j^{th} parameter when the other K-2 parameters remain fixed.

ρ''_i is designed to show to what extent the effects of the i^{th} parameter can be represented by a combination of all other parameters.

The equations for the computation of the correlation coefficients are:

$$\rho_{ij} = \frac{(C_{K_G})_{ij}}{\left[(C_{K_G})_{ii}\,(C_{K_G})_{jj}\right]^{\frac{1}{2}}},$$

$$\rho'_{ij} = -\frac{(W_{K_G})_{ij}}{\left[(W_{K_G})_{ii}\,(W_{K_G})_{jj}\right]^{\frac{1}{2}}},$$

and

$$\rho''_i = 1 - \frac{1}{(C_{K_G})_{ii}\,(W_{K_G})_{ii}}$$

The correlation coefficients should indicate cases in which trouble is expected. In the case where either of the correlation coefficients is unity, no solution to the problem exists since some of the matrices to be inverted will be singular. In cases where the correlation coefficients are large there is a possibility of compensating errors being introduced into the solution.

Figure-of-Merit Parameter

The figure-of-merit parameter is designed to show the relative effect of the preflight data and the in-flight data on the solution for the error parameters. This parameter is denoted by $(FM)_i$ and is computed according to

$$(FM)_i = \frac{(W^{-1}_{K_{G_o}})^{\frac{1}{2}}_{ii} - (C_{K_G})^{\frac{1}{2}}_{ii}}{(W^{-1}_{K_{G_o}})^{\frac{1}{2}}_{ii}} \times 100 \qquad (10)$$

There will be one such parameter for each of the error model parameters. The values range from 0 when the flight data have no effect on the solution to 100 when the pre-flight estimates have no effect. A value of 29.3 indicates that the flight data and the preflight data contribute equally. A value of 10.6 indicates that the preflight estimates contribute twice as much as the flight data so that parameters with a figure-of-merit less than 10.6 are not considered to be determined by the flight data. A value of 53.2 indicates that the flight data contributes twice as much as the preflight estimates so that parameters with a figure-of-merit larger than 53.2 will affect the solution even though high correlation with other parameters exists.

These numerical values are obtained by noting the variance relationship

$$\frac{1}{\sigma_i^2 \text{ total}} = \frac{1}{\sigma_i^2 \text{ flight}} + \frac{1}{\sigma_i^2 \text{ preflight}} \qquad (11)$$

which holds when a single parameter is estimated or when there is no correlation between parameters. The factor $(W^{-1}_{K_{G_o}})^{\frac{1}{2}}_{ii}$ is the preflight estimate of the standard deviation of the i^{th} error parameter and $(C_{K_G})^{\frac{1}{2}}_{ii}$ is the standard deviation resulting from the estimation process.

VI. Numerical Results

In this section a numerical test case is examined to show the approach followed in trying to determine a method for detecting and circumventing the previously listed problem areas. For this test case a set of typical numerical values for the error parameters was used with the partial derivatives generated for a particular flight to generate a set of manufactured $\overline{\Delta V}$'s. The nominal test case was then computed using these $\overline{\Delta V}$'s and B_G's with typical values for W and $W_{K_{G_o}}$. The "a priori" estimates for the error parameters were chosen to be one-tenth the values used in manufacturing the $\overline{\Delta V}$'s. The nominal case data was then varied to determine the particular combination of test data that yielded the closest agreement with the known solution. The success parameters were also computed in an attempt to determine their effectiveness.

Figures 1-4 show the behavior of the estimates of four of the error parameters as the relative weight of the flight data is decreased (this decrease in weight of the flight data is accomplished by decreasing the value of the C parameters of equation 7). Figures 5-8 show the recursive development of the same parameters. One would suspect that for large relative weighting of the flight data the estimate would approach the true solution while for small relative weighting of the flight data the estimate would approach the "a priori" value assigned to the parameter. This is the observed behavior.

However, for intermediate values of the relative weights the estimate is not a weighted average of the extreme values as one would suspect but some of the estimates deviate sharply from this type of behavior. A careful study of the values of the various success parameters gives no indication that they can predict this odd behavior. Similar results are shown for the recursive development of the parameters. These curves essentially show the effect of varying the relative weights of the flight and preflight data since including more flight data in the computation reduces the relative effect of the "a priori" data. This behavior is probably due to a combination of the first two problem areas given in the introduction, but the success parameters apparently do not provide a means for predicting the behavior. Figures 9-12 show the behavior of the multiple correlation, ρ_i'', and the Figure of Merit, FM_i, for a typical parameter. The behavior was the same for all platform error parameters so the results for only one case is needed.

VII. Conclusion

The failure of the success parameters to predict the behavior of the estimates of the platform error parameters for this ideal manufactured data case indicates that they will be of little value in analyzing the results of a real data case. However, the results obtained do indicate that the estimation technique employed is valid under certain conditions. These are (1) that an adequate error model be used in the analysis and (2) that the flight data and the preflight data be used as two separate determinations of the error parameters to be combined using engineering judgment as to their validity. The problem of obtaining an adequate error model can be solved for a given system by constant reevaluation in light of the results obtained. The separation of the flight data estimate from the preflight estimates can be solved by assigning high relative weight to the flight data as was done in this test case.

The problem of correlation will usually exist and probably can be solved by some iterative method of fixing one parameter and solving for another using the values of the correlation coefficients as a guide. Further investigation of this problem was beyond the scope of the present work as was the problem of non-linearity.

VIII. References

1. Deutsch, R., Estimation Theory, Prentice-Hall, 1965.

2. Rauch, H. E., Tung, F., and Striebel, C. T., Maximum Likelihood Estimates of Linear Dynamic Systems, AIAA Journal, Vol. 3, No. 8, 1965.

3. Kalman, R. E., A New Approach to Linear Filtering and Prediction Problems, Journal of Basic Engineering 82, 1960, pages 35-45.

4. Kalman, R. E., and Bucy, R. S, New Results in Linear Filtering and Prediction Theory, Journal of Basic Engineering, March, 1961, pages 95-108.

5. Smith, G. L., Schmidt, S. F., and McGee, L. A., Application of Statistical Filter Theory to the Optimal Estimation of Position and Velocity on Board a Circumlunar Vehicle, NASA Technical Report TR-R-135, 1962

6. Brown, D. C., A General Solution to the Problem of the Adjustment of Trajectory Observations, RCA Data Processing Technical Memo No. 54, March 6, 1959.

7. Beal, T. R. and J. A. Stanton, Procedure and Program for Guidance Error Model Coefficient Recovery; Volume I, Development of Procedure, LMSC/HREC A711877, Lockheed Missiles & Space Company, Huntsville Research & Engineering Center, January 1966.

8. Crenshaw, H. T., Analysis of the Recursive Solution of the Guidance Error Model Coefficient Recovery (GEMCR) Computer Program, Lockheed Missiles and Space Co., LMSC/HREC A783339, October 1966.

9. Beal, T. R., Error Analysis of ST-124 Platform, Final Report Flight Evaluation Support Studies, Lockheed Missiles & Space Co., HREC/LMSC A033332, Vol. III, January 1964.

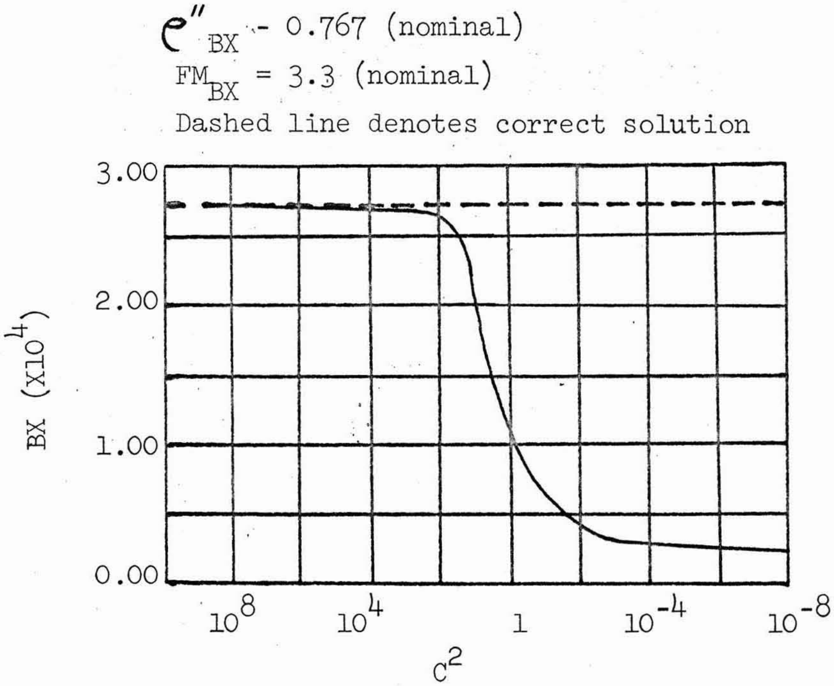

Figure 1 - Estimate of BX as a Function of Relative Weights

Figure 1 - Estimate of BX as a Function of Relative Weights

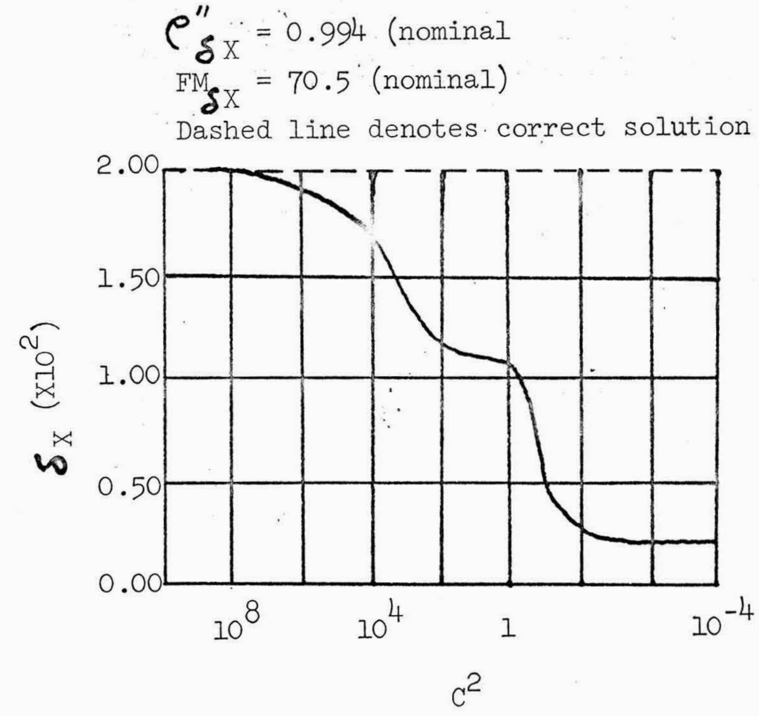

Figure 2 - Estimate of δ_X as a Function of Relative Weights

Figure 2 - Estimate of δ_X as a Function of Relative Weights

$\rho''_{\dot{\delta}_Y} = 0.999$ (nominal)

$FM_{\dot{\delta}_Y} = 31.5$ (nominal)

Dashed line denotes correct solution

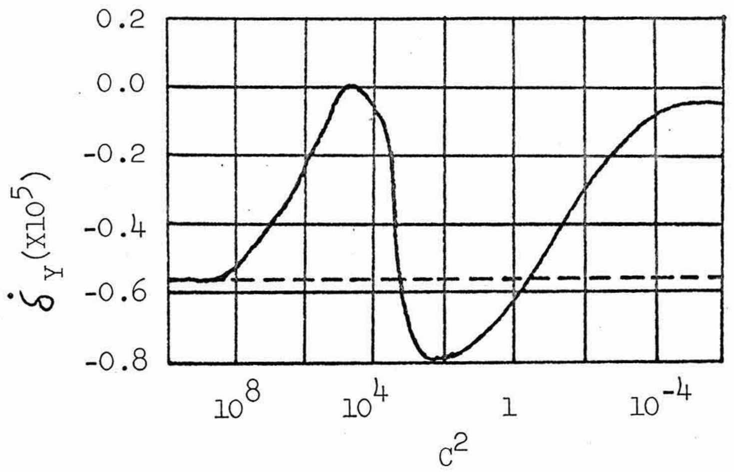

Figure 3 - Estimate of $\dot{\delta}_Y$ as a Function of Relative Weights

Figure 3 - Estimate of $\dot{\delta}_Y$ as a Function of Relative Weights

$\rho''_{MYZ} = 0.963$ (nominal)

$FM_{MYZ} = 50.5$ (nominal)

Dashed line denotes correct solution

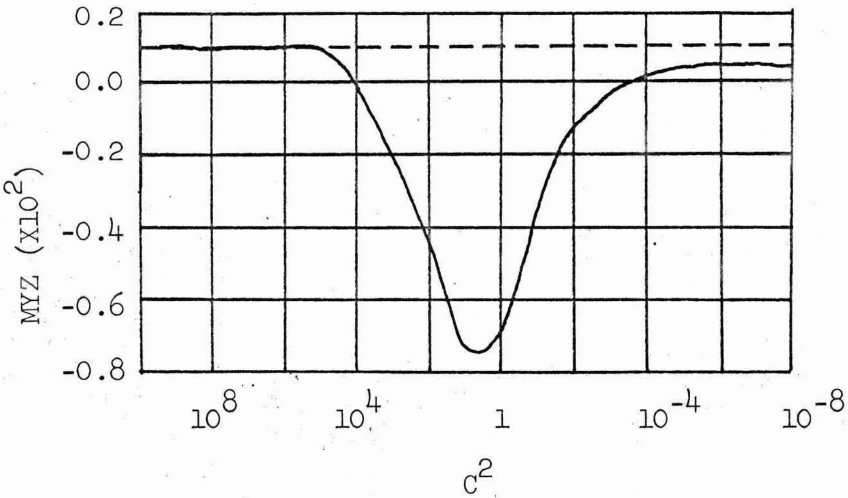

Figure 4 - Estimate of MYZ as a Function of Relative Weights

Figure 4 - Estimate of MYZ as a Function of Relative Weights

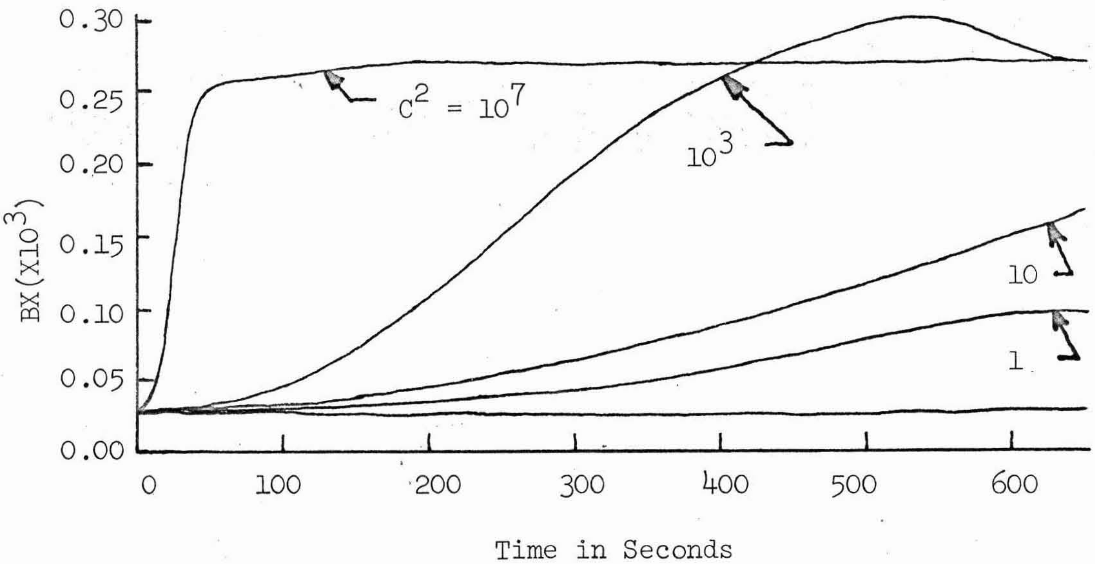

Figure 5 - Recursive Development of B_X

Figure 5 - Recursive Development of B_X

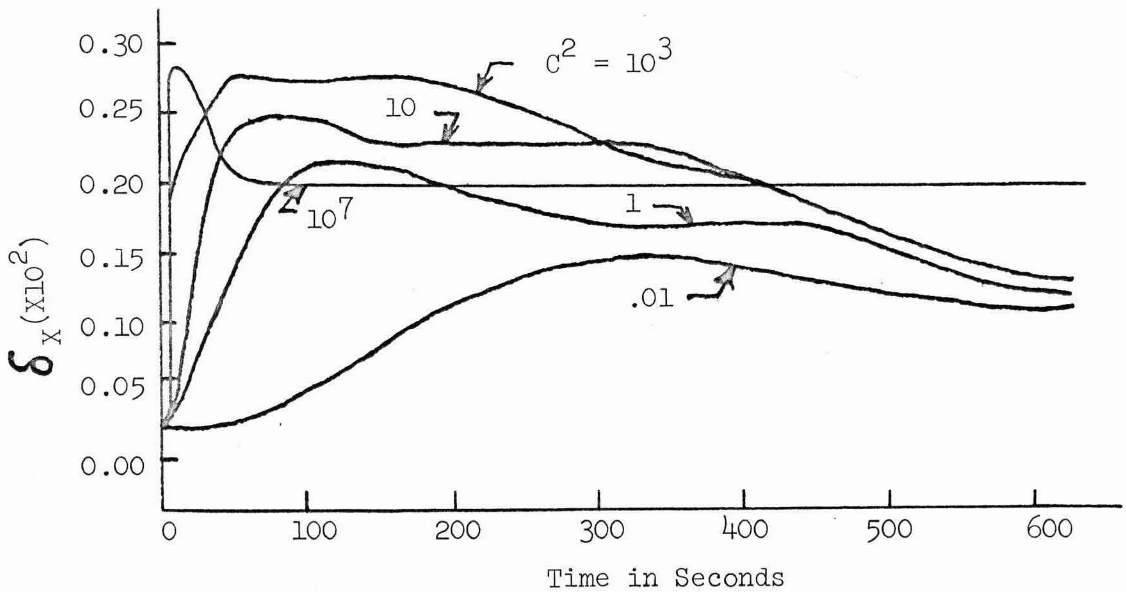

Figure 6 - Recursive Development of δ_X

Figure 6 - Recursive Development of δ_X

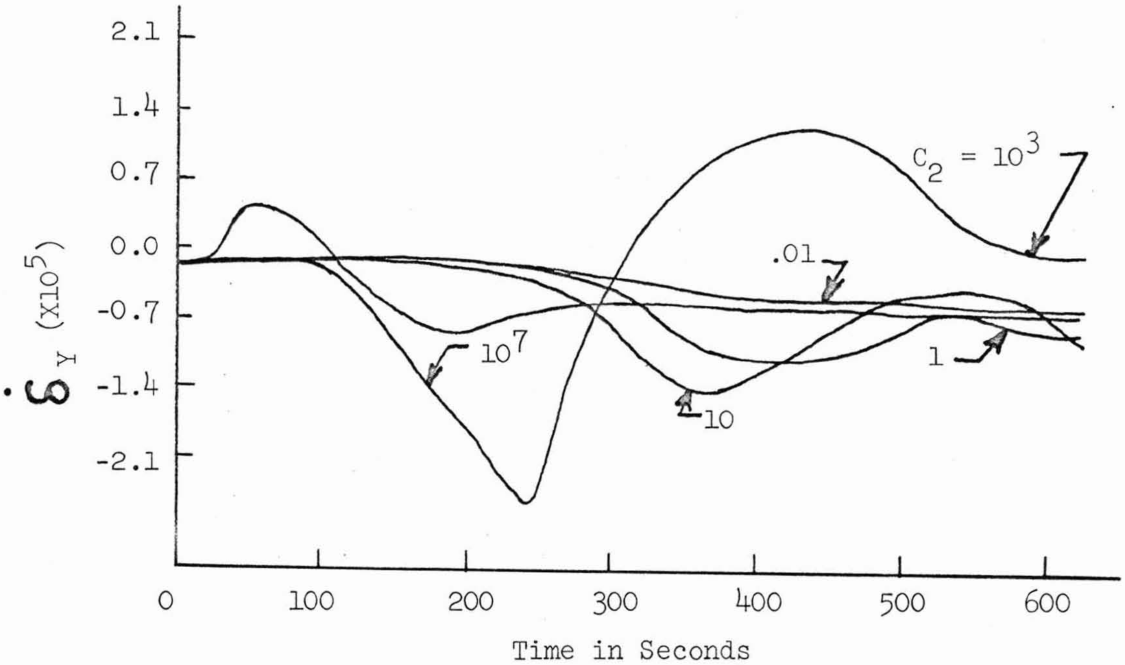

Figure 7 - Recursive Development of $\dot{\delta}_Y$

Figure 7 - Recursive Development of $\dot{\delta}_Y$

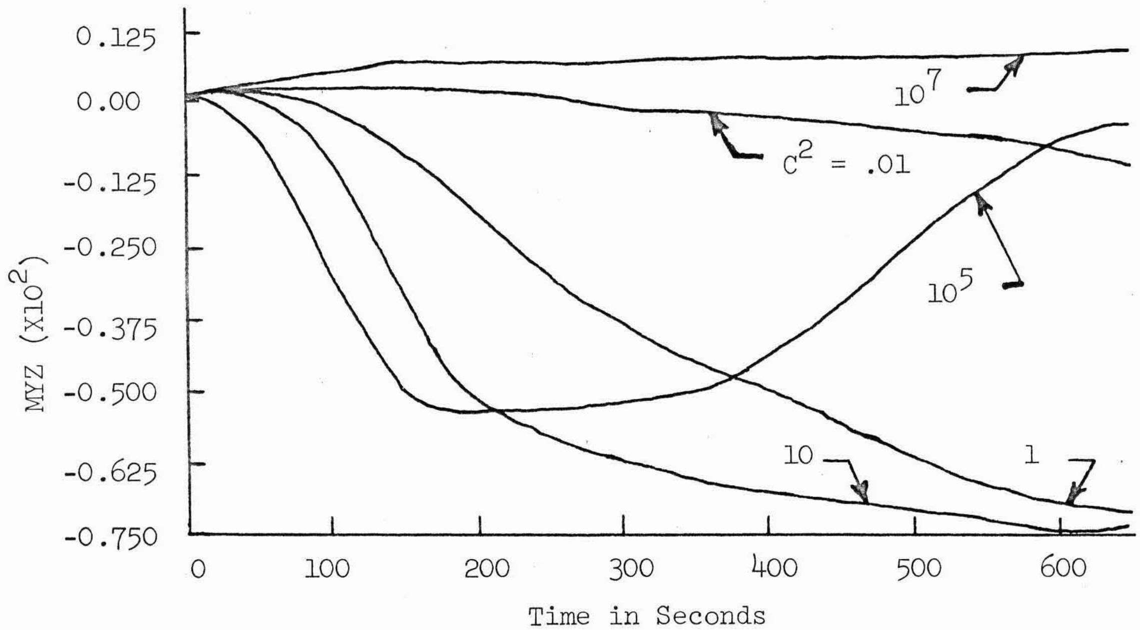

Figure 8 - Recursive Development of MYZ

Figure 8 - Recursive Development of MYZ

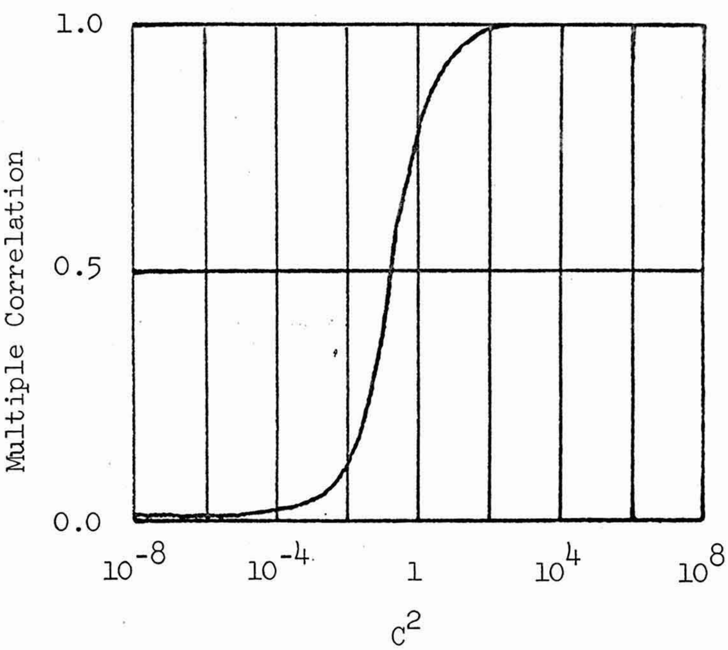

Figure 9 - Typical Value of Multiple Correlation as a Function of Relative Weights

Figure 9 - Typical Value of Multiple
Correlation as a Function
of Relative Weights

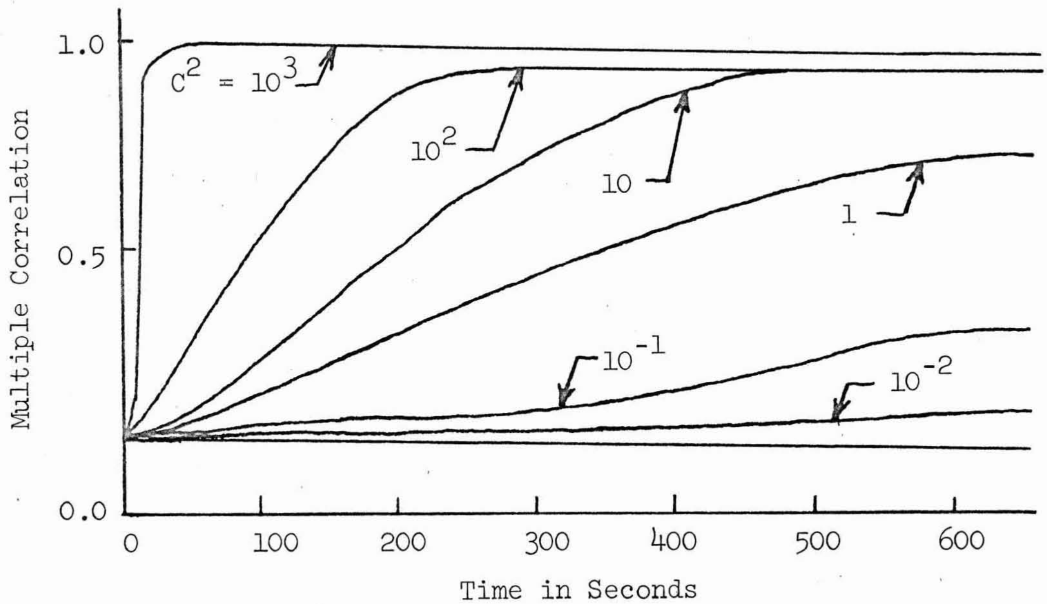

Figure 10 - Typical Recursive Development of Multiple Correlation

Figure 10 - Typical Recursive Development of Multiple Correlation

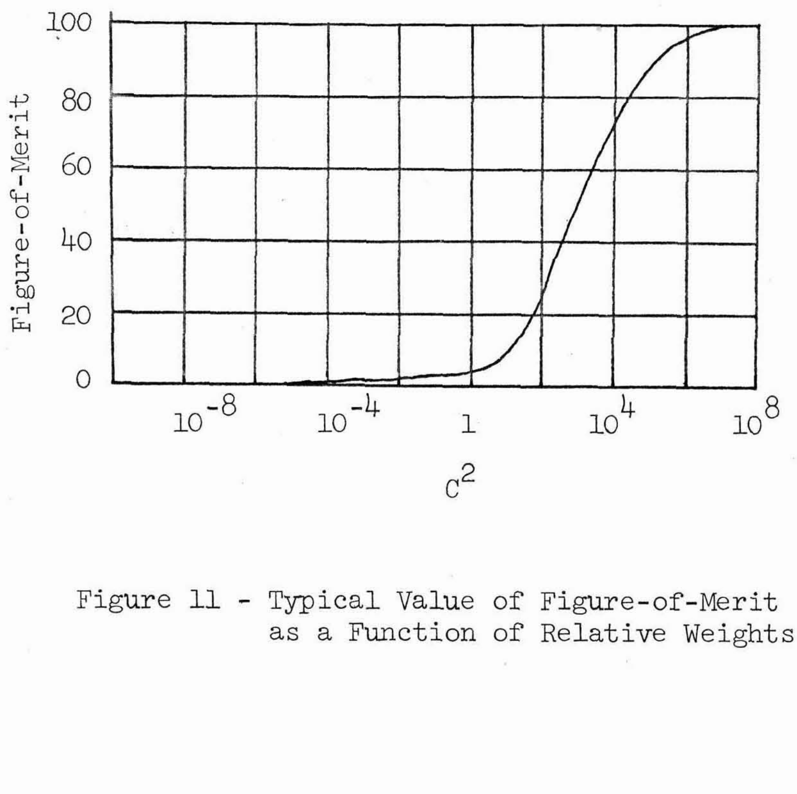

Figure 11 - Typical Value of Figure-of-Merit as a Function of Relative Weights

Figure 11 - Typical Value of Figure-of-Merit
as a Function of Relative Weights

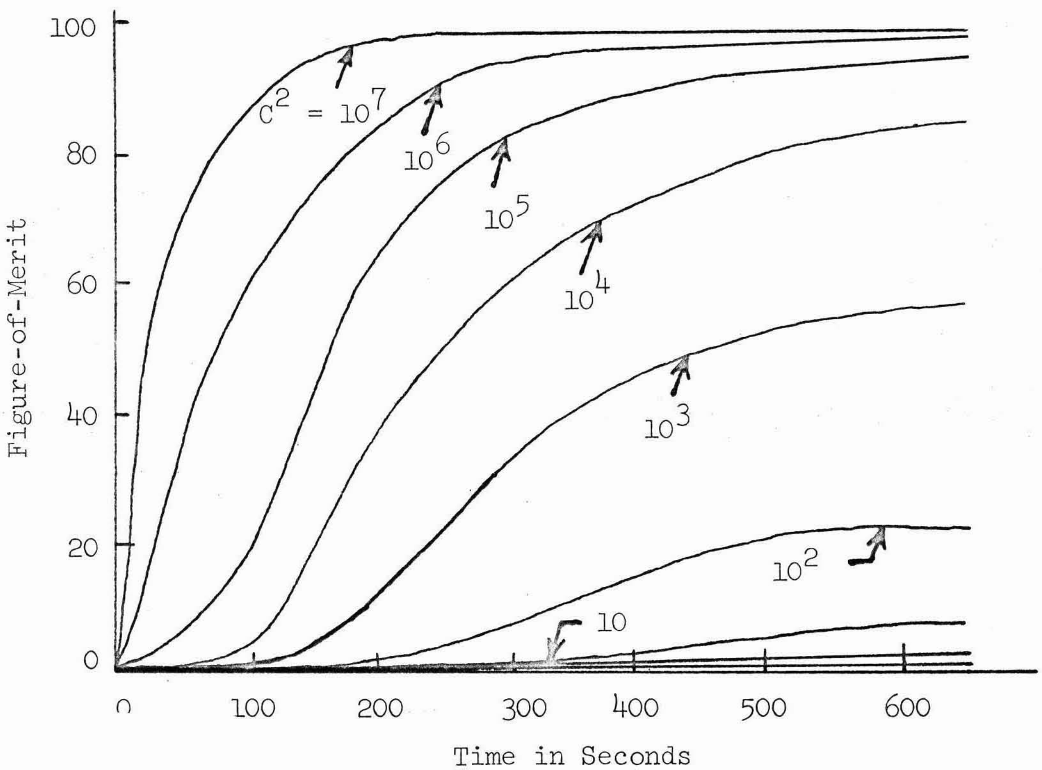

Figure 12 - Typical Recursive Development of Figure-of-Merit