109

X1.8

1 March 1960 Report No. DT-TM-1-60

STANDARD PROCEDURE FOR USING UNITS OF MASS, WEIGHT, FORCE, PRESSURE AND ACCELERATION

by Schuler, A.E.
NASA-MSFC

ARMY BALLISTIC MISSILE AGENCY

REDSTONE ARSENAL, ALABAMA

ABMA Form 485, Rev 1 May 59 Previous edition is obsolete.

1 March 1960 Report No. DT-TM-1-60

STANDARD PROCEDURE FOR USING UNITS OF MASS, WEIGHT, FORCE, PRESSURE AND ACCELERATION

by

A. E. Schuler

TEST LABORATORY

Arranged and Processed

by

Reports and Publications Section
Weapon System Information Office

FOREWORD

The field of missiles and rockets deals with quantities of matter at various locations with different accelerations of gravity. The weight of these masses changes with gravity and the measurements of liftoff weight, fuel weight, etc., result in different values, depending on whether mass or weight units are used.

Pressure and thrust are independent of the acceleration of gravity, but the instruments for measuring these values are calibrated with standard masses, producing different weight-forces and calibration curves at different locations.

Most sections of ABMA and other agencies or companies use pounds or kilograms as units of mass, weight or force, and the influences of different accelerations of gravity are often disregarded or treated incorrectly. These discrepancies become increasingly unacceptable with larger missiles and greater distances between operation sites.

Therefore, the following Standard Procedure has been prepared to insure consistent and uniform terms and units of mass, weight, force, pressure and acceleration. All sections and individuals concerned are urged to use these units and procedures.

Wernher von Braun

WERNHER VON BRAUN
Director
Development Operations Division

TABLE OF CONTENTS

SECTION I. INTRODUCTION

The acceleration of gravity varies up to .55% over the earth and the weight of the same body changes accordingly, while the mass remains the same. In most fields of engineering the difference of gravity is negligible and it is common practice to use terms and quantities of mass and weight interchangeably. The same is often the case in the field of rocket propulsion. Masses are being called weight and sometimes the mass data are obtained correctly from measurements, but are converted into weight units in an effort to correct for a different acceleration of gravity. In all, there is widespread confusion and inconsistency in using terms and units of mass, weight, force, pressure, and acceleration. Therefore, a standard procedure for using these units is necessary.

SECTION II. SYSTEMS OF UNITS

The absolute and the gravitational systems of units use 1 m/sec^2 or 1 ft/sec^2 as unit of acceleration; the units of mass and force are such that 1 force unit = 1 mass unit x 1 acceleration unit. Therefore, the weight of a body is numerically about 9.8 or 32.2 times its mass. There are some mixed systems of units in use, employing the mass unit in kilograms or pounds mass from the absolute system and the force unit in kiloponds or pounds force from the gravitational system of units. In these systems the numerical values of mass and weight are equal at standard acceleration of gravity (g_s). This is very desirable. However, the current mixed systems still use 1 m/sec^2 or 1 ft/sec^2 as unit of acceleration, resulting in various confusing or incorrect forms of Newton's equation. When kilograms and kiloponds are used as units of mass and weight, the proper unit of gravity is calculated by the following equation of quantities (Groessengleichung):

$$a = \frac{1\ \text{kp}}{1\ \text{kg}} \quad \frac{9.80665\ \text{kg m/sec}^2}{1\ \text{kg}} = 9.80665\ \text{m/sec}^2 = 1\ g_s$$

This Standard Procedure is based on the following units:

TABLE I

Property	Metric Units		English Units	
Mass	Kilogram	kg	Pound Mass	lbm
Force	Kilopond	kp	Pound Force	lbf
Acceleration	Standard Gravity (9.80665 m/sec^2)	g_{s_m}	Standard Gravity (32.17398 ft/sec^2)	g_{s_e}

These units are derived from the basic units of mass, length, and time. The kilogram (kg) is the International Standard of mass as represented by the platinum-iridium cylinder in Sèvres, France. The symbol kgm may be used in personal work to avoid confusion with the old habit of using kg as unit of weight or force. The kilopond (kp) is the new International Unit of force, defined as the weight of 1 kg mass at the standard acceleration of gravity. 1 kp = 980665 dyne. The unit of acceleration is one standard acceleration of gravity, accepted by international agreement as $g_s = 9.80665$ m/sec^2. This gravity prevails at about 45° latitude and sea level. The English units in the above system of units are analogous. With these units, indicated in parentheses, the Newton equation is:

$$F\ (\mathrm{kp}) = m\ (\mathrm{kg}) \cdot a\ (g_{s_m})\ \text{or}\ F\ (\mathrm{lbf}) = m\ (\mathrm{lbm}) \cdot a\ (g_{s_e}).$$

When using these units, the same body has the same numerical values of mass and weight under the standard acceleration of gravity. The acceleration is already being expressed in g quite frequently, and the above units will be very practical in most cases. These units should be used in all measurements and field reports. For aeroballistic or some other calculations and reports, the acceleration unit of 1 m/sec^2 or 1 ft/sec^2 and the mass unit of 1 kp m^{-1} sec^2 or 1 lbf ft^{-1} sec^2 are being used. The gravitational system with these units and the conversion factors between the different units are listed in Section VII.

This Standard Procedure uses the following symbols for the different accelerations of gravity:

g_s = Standard acceleration of gravity in metric or English units.

g_{s_m} = Standard acceleration of gravity in metric units.

g_{s_e} = Standard acceleration of gravity in English units.

g_1 = Local acceleration of gravity

g_c = Acceleration of gravity at the location where the instrument was calibrated with calibration masses.

SECTION III. MASS

Mass is the quantity of matter or a measure of the inertia of a body. The unit of mass is kilogram (kg) or pound mass (lbm).

1 kg = 2.204 622 lbm.

Mass standards are available at the National Bureau of Standards in Washington, D. C. (NBS). Secondary standards, or so-called "dead weights", are available at Redstone Arsenal, Patrick Air Force Base, and other agencies and companies concerned with missile activities. These dead weights are calibrated at the NBS or scale companies by comparison with primary standards of mass. At different locations, the dead weights have the same mass, but different weights. Therefore, the term "dead weight" is misleading. They are actually dead masses. A missile also has the same mass but different weights at different locations. For example, there is a .11% difference between Detroit and Patrick. The same applies to fuel weight, liftoff weight, and all other data pertaining to the weighing of objects. They are actually masses and must be treated as such. They should also be called "dead masses" or "calibration masses", "mass of fuel", etc., as it is already done for "mass flowrate", "mass characteristics of missiles", or "mass moment of inertia". The NBS uses the word "weight" for pieces of metal, representing standards of mass, but there the word "weight" is equivalent to the name of a body used in the process of weighing. The force of habit will uphold for some time the word "weight" in that sense, especially when the masses are determined by weighing. This should be avoided, but is acceptable as long as the word "weight" is used only as a name for quantities. All numerical values must be expressed in units of mass and must be clearly specified as such.

Most weighing systems are calibrated by calibration masses and the results are in units of mass, provided the calibration is made at the same location. Therefore, no correction shall be applied to these results. When load cells or proving rings have been calibrated at another location with the gravity g_c, the mass measurements with such instruments, at the local gravity g_1, must be corrected by the factor g_c/g_1. For example, the load cells or proving rings calibrated at NBS are compared with standards of masses at an acceleration of 9.8008 m/sec^2. At Redstone Arsenal, the acceleration of gravity is 9.7964 m/sec^2 and more mass is required to produce the same weight-force obtained for a certain mass in Washington. Therefore, the mass measurements with an NBS calibrated proving ring must be corrected by $9.8008/9.7964 = 1.0004$. Using the percent deviations from Table II, the correction is $\Delta g_c - \Delta g_1 = -0.06\% - (-0.10\%) = +0.04\%$.

When the mass of lox or fuel filling is determined with differential pressure gages, the result is in units of mass, provided the Δp gage is calibrated at the same location by liquid columns or by pressure balances using calibration masses. It is important that these data must not be multiplied by g_1/g_s to obtain correct pressure units as prescribed for pressure measurements in Section V of this procedure or in the instructions of some calibration standards. The Δp values for filling control or density measurements are not in units of force per unit area = pressure, but they are measures of mass. The units must be called psi (m), in Hg (m), etc. to differentiate them from standard pressure units.

When the mass of lox, fuel or other liquids is determined by weighing, a correction must be made for buoyancy of air. The buoyancy effect on a body "X" equals the mass (m_x) times the specific gravity of air ($\gamma_a \approx 0.0012$) divided by the specific gravity of the body (γ_x). The weighing systems are calibrated with masses, having a specific gravity between 6.8 to 8.4, and the buoyancy effect is about

$$\frac{0.0012}{8} \cdot m_x = 0.00015 \cdot m_x = 0.015\%.$$

The results from the weighing systems have a buoyancy error of

$$\left(\frac{0.0012}{\gamma_x} - \frac{0.0012}{8}\right) .100\%$$

This is about:

0.11% for water

0.14% for hydrocarbon fuels

0.10% for lox

These average figures for buoyancy corrections are accurate enough for different densities of the respective liquids above. Liquid hydrogen has a larger variation in density, and its buoyancy effect may vary from 1.5% to 4%. Therefore, the buoyancy must be calculated for each particular specific gravity of LH_2. The same requirements pertain to the buoyancy correction when weighing complete missiles; e.g., the empty JUPITER missile has a specific gravity of about 4.0 and its buoyancy effect is only 0.015%. When the missile is fueled and the pressurized compartments are closed, the specific gravity of the JUPITER is about 0.8 and the buoyancy effect is 0.13%. This correction must be applied when the liftoff mass is determined by weighing. In turn, the buoyant force of 158 pounds must be added to the thrust at liftoff, and decreasing buoyant forces must be added with decreasing air densities during flight as outlined in DSLB Memo No. 176.

Derived from mass units are:

a. Mass flowrates in kg/sec or lbm/sec.

b. Density in kg/dm^3, kg/ltr, g/cc, lbm/ft^3, lbm/in^3 and lbm/gal.

Calibration masses are used to measure these values or to calibrate flowmeters and densitometers; therefore, no corrections must be made.

The specific gravity is the ratio of the density of a medium to the density of water at 4°C. This dimensionless figure is to be preferred to density values, especially the English values. In the metric system, the density is numerically the same as the specific gravity values, referring to water at 4°C. The specific gravity of fuel is often referred to water at 60°F (API gravity). This causes confusion and must be discontinued.

SECTION IV. WEIGHT AND FORCE

The force is the cause of deformations or change in motion of a body and the weight of a body is the force with which the earth pulls that body toward its center. The unit of weight and force is 1 kilopond (kp) or 1 pound force (lbf). This is the weight of a unit mass under the standard acceleration of gravity.

At other accelerations of gravity g_1, a mass m has the weight of $m \cdot g_1/g_s$. Therefore, the calibration masses must be multiplied by g_1/g_s or corrected by $\Delta g\%$ when calibrating load cells, proving rings, or other force measuring devices. Thrust is one of our most important force values and the above correction is essential, although the gravity error at Redstone is only -0.1%, while the inaccuracy of the thrust measurement is about ±0.5%. The error of the thrust measurement is composed of several small errors and at the present state of accuracy, a systematic error of 0.1% is not negligible. The calibration curves must present the corrected values and details as to how they were obtained.

Presently, the NBS calibrations of load cells or proving rings give the number of pounds mass and the output caused by the weight of these masses at local acceleration of gravity. Therefore, the NBS data must be corrected by 9.8008/9.80665 = 0.994 or -0.06% when NBS calibrated instruments are used for force measurements or as standards for calibration of force measuring devices. After this correction, the load cells or proving rings measure true force at any location.

SECTION V. PRESSURE

Pressure is the force per unit area. The basic unit of pressure is 1 kp/cm^2 (at) or 1 lbf/sq in (psi). There are many other derived units which are well known and covered by respective tables.

Basic standards for calibration of pressure gages are: dead weight pressure balance, dead weight tester and manometers with liquid columns. All these instruments use certain quantities of mass to provide forces per unit area or pressure. Therefore, the calibration values have to be corrected by g_1/g_s or $\Delta g\%$ as described for force in Section III. The calibration curves must present corrected values and details as to how they were obtained.

The NBS applies this correction when it calibrates pressure gages, yet it does not correct to true force values when calibrating load cells or proving rings as outlined in Section IV.

SECTION VI. ACCELERATION OF GRAVITY

The following table lists the accelerations of gravity for different locations connected with missile work and the deviations of these values from the International Standard.

TABLE II

Location	m/sec^2	ft/sec^2	$\Delta g\%$
1. International Standard about 45° latitude, sea level	9.80665	32.17398	
2. Chrysler Plant, Detroit	9.8031	32.1623	-0.04
3. Bureau of Standards, Washington	9.8008	32.1548	-0.06
4. Redstone Arsenal	9.7964	32.1404	-0.10
5. Rocketdyne, Santa Susana	9.7949	32.127	-0.11
6. Patrick Air Force Base	9.7924	32.1266	-0.14
7. Equator	9.7803	32.0875	-0.27
8. Pole	9.8322	32.2578	+0.26

The International Standard acceleration is a value adopted by agreement. The other values are measured or calculated and the same has to be done for other locations not listed. The numerous tables of gravity values, based on the Potsdam System or other measurements, do not cover all locations of plants, laboratories, test stands, and launching sites connected with missile activities. There are several equations for g_1 resulting in values which differ up to $1.6 \cdot 10^{-4}$ m/sec^2. All formulas use the geographical latitude θ and altitude a, but different considerations are made for internal density layers of the earth, ellipticity, etc. Quite common is the equation by Helmert:

$$g_1\ (m/sec^2) = 9.80616 - 0.025928 \cos 2\theta - 0.000069 \cos^2 2\theta - 0.0003086\ a.$$

θ is the geographic latitude, a is the elevation in meters, and the result is in m/sec^2. In English units, the gravity is:

$$1\ g_1\ (ft/sec^2) = 1\ g_1\ (m/sec^2) \cdot 3.280833.$$

The result is rounded off to the nearest 10^{-4} decimal because of the above uncertainties and the needlessness for more decimals.

At different locations within the same area, the change of g_1 is negligible since the maximum change is 0.0005% at Patrick Air Force Base, 0.002% at Redstone, and 0.003% at Santa Susana. At different altitudes on tall missiles, the change is less than 0.003% for 100 m or 328 ft, which is also negligible in most cases.

SECTION VII. GRAVITATIONAL SYSTEM OF UNITS

The gravitational or technical system uses the following units:

TABLE III

Property	Metric Units	English Units
Force	Kilopond (kp)	Pound Force (lbf)
Mass	kp m^{-1} sec^2	lbf ft^{-1} sec^2
Acceleration	m/sec^2	ft/sec^2

The systems of units in Section II and Section VII use the same units of force. The units of acceleration differ by the standard acceleration of gravity $g_s = 9.80665\ m/sec^2 = 32.17398\ ft/sec^2$. The conversion factors between the different units of mass are listed in Table IV.

TABLE IV

kg	kp $m^{-1}sec^2$	lbm	lbf $ft^{-1}sec^2$ (slug)
1	0.1019716	2.204622	0.0685219
9.80665	1	21.61996	0.671970
0.453592	0.0462536	1	0.03108102
14.59388	1.488162	32.17398	1