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SUMMARY

This report presents the logic leading to a mathematical expression for mission availability. Mission availability is treated as the probability that the cumulative downtime occurring during a mission of given length will be less than the time constraint. This is opposed to more general approaches such as steady state or instantaneous availability or operating time versus real time.

We intend to present a practical and usable mathematical model by deduction and demonstration. The development is based on exponentially distributed downtimes. Experience shows that certain systems follow exponential downtime distributions except near zero. This error is often so small that it may be neglected. A future report will present a downtime distribution which will account for this small error.

KEY WORDS

Availability, Reliability, Probability, Launch Probability, Repair Time, Downtime

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CONTENTS

	PAGE
Summary	1
Introduction	2
Discussion	3
Appendix	15

SUMMARY

Mission availability is the probability that the cumulative repair time occurring during a mission of operating time, T, will be within a given time constraint t_c.

Although a much more sophisticated definition could be derived, the above meets all ESE requirements with no sacrifice in clarity.

The mathematical expression for mission availability A_M is

$$A_M = 1 - e^{-\lambda T} \sum_{j=1}^{\infty} \frac{(\lambda T)^j}{j!} e^{-\frac{1}{j}\mu t_c}$$

where e is the naperian base

λ is the failure rate

T is the mission time

μ is the repair rate (or maintenance action rate)

t_c is the total repair time constraint for all repairs

$j!$ is factorial j

INTRODUCTION

The development is based on exponentially distributed times between failures as well as exponentially distributed times to repair. With respect to failures, this means that failure events are randomly occurring events (in time), or that any equal time increment has an equal probability of incurring a failure. With respect to the repair periods, it means that the length of any repair period which follows a failure is of random length (or that any equal repair time increment, Δt_r , has an equal probability of containing a repair action completion).

DISCUSSION

Figure I(a) shows how random failures might appear along a time axis with no repair time shown. This is the usual concept upon which the exponential failure model is based. The reliability is based on mission times such that the probability of a time increment without a failure, occurring is the reliability. Intuitively then the mission times are restricted to the order of magnitude of failure free intervals.

If the mission is such that brief periods of inoperability can be tolerated so that failures can be repaired (unscheduled maintenance) the mission time can be greatly extended, as in Figure I(b).

The graph in I(b) is the graph from I(a) with the "repair times", τ , inserted after each failure. The assumption for this study is that if a sufficient number of the τ's were plotted according to magnitude, the resulting histogram would appear as in Figure II.

Several analogies can now be established. In Figure I(b) the mean time between failures is the sum of the t's divided by the number of t's. Analogously to this, the repair times could be laid end to end as in Figure I(c) and the mean repair time then is the sum of all repair times, τ , divided by the number of τ 's. The reciprocal of this mean which is analagous to the failure rate is the repair rate μ .

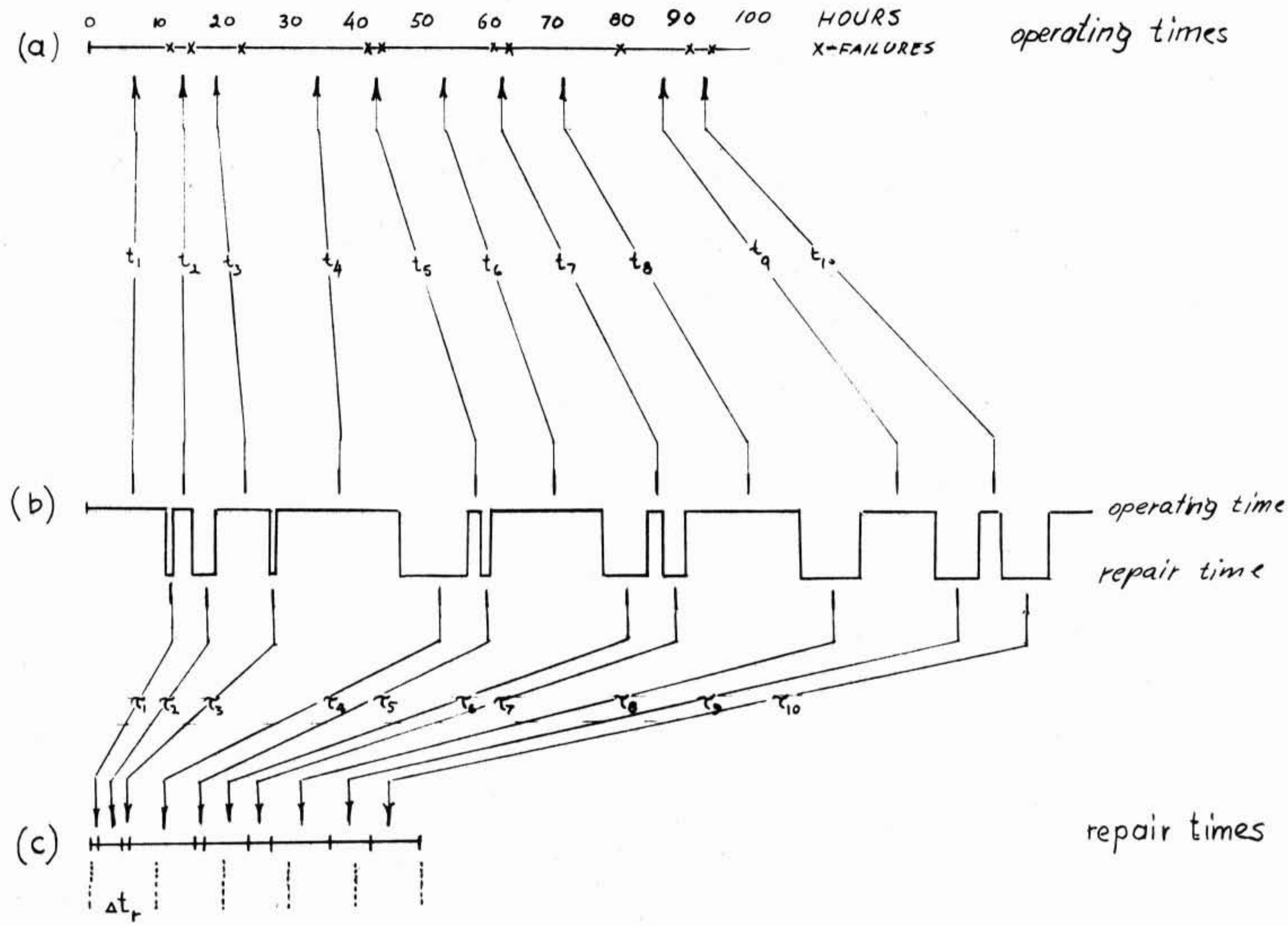
(a)
0 10 20 30 40 50 60 70 80 90 100 HOURS
X-FAILURES
operating times
t_1 t_2 t_3 t_4 t_5 t_6 t_7 t_8 t_9 t_{10}
(b)
operating time
repair time
τ_1 τ_2 τ_3 τ_4 τ_5 τ_6 τ_7 τ_8 τ_9 τ_{10}
(c)
Δt_r
repair times

Fig I

If the frequency distribution of the times to repair is exponential (as in Fig. II.), then the probability that a repair time of less than t_c will occur is

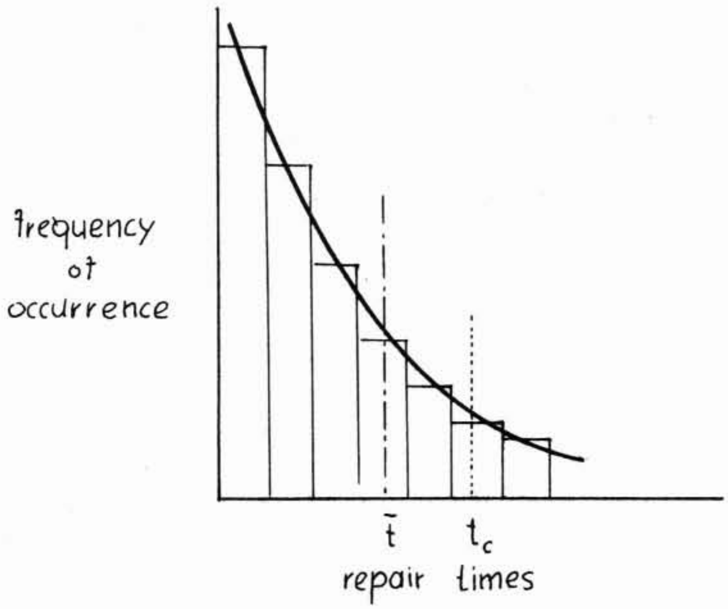

Fig. II

$$\frac{\int_0^{t_c} \mu e^{-\mu t}\, dt}{\int_0^{\infty} \mu e^{-\mu t}\, dt} = \left[1 - e^{-\mu t_c}\right] = P_r$$

$P_r = (1 - e^{-\mu t_c})$ is the probability that a repair time will occur that is less than t_c.

The probability that 1 repair is completed in time less than t_c is $(1 - e^{-\mu t_c})$; the probability that each of two repair occurrences are of time duration less than t_c, each is the event that the first is of time duration less than t_c <u>and</u> that the second is of time duration less than t_c. This implies the product of the two probabilities

$$(1 - e^{-\mu t_c})^2$$

Similarly for i repair occurrences each of time duration less than t_c the probability is

$$(1 - e^{-\mu t_c})^i$$

The probability that exactly f failures will occur during the mission time T is given by

$$P_f = \frac{(\lambda T)^f e^{-\lambda T}}{\underline{|f}}$$

The probability P_i of the event that exactly i failures occur and that each is repaird in time less than t_c is the product of $P_f \cdot (P_r)^i$

$$P_i = \frac{(\lambda T)^i e^{-\lambda T}}{\underline{|i}} (1 - e^{-\mu t_c})^i$$

The probability that exactly 1, <u>or</u> exactly 2, <u>or</u> exactly n failures occur and that they are repaired in times less than t_c is the sum of these probabilities.

$$P_1 = \frac{(\lambda T)^1 e^{-\lambda T}}{\underline{\lvert 1}} (1 - e^{-\mu t_c})^1$$

$$P_2 = \frac{(\lambda T)^2 e^{-\lambda T}}{\underline{\lvert 2}} (1 - e^{\mu t_c})^2$$

$$P_3 = \frac{(\lambda T)^3 e^{-\lambda T}}{\underline{\lvert 3}} (1 - e^{-\mu t_c})^3$$

$$\begin{matrix} \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \end{matrix}$$

$$P_n = \frac{(\lambda T)^n e^{-\lambda T}}{\underline{\lvert n}} (1 - e^{-\mu t_c})^n$$

$$\sum_{n=1}^{i} P_n = e^{-\lambda T} \sum_{n=1}^{i} \frac{(\lambda T)^n}{\underline{\lvert n}} (1 - e^{-\mu t_c})^n$$

$$= e^{-\lambda T} \sum_{n=1}^{i} \frac{[(\lambda T)(1 - e^{-\mu t_c})]^n}{\underline{\lvert n}}$$

For <u>all</u> possibilities i goes to ∞, and the equation becomes:

$$\sum_{n=1}^{\infty} P_n = e^{-\lambda T} \sum_{n=1}^{\infty} \frac{(\lambda T)^n}{\underline{\lvert n}} (1 - e^{-\mu t_c})^n$$

The summation is now in the form

$$\frac{x^1}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} \cdots \frac{x^n}{\lfloor n} \cdots = e^{x} - 1$$

where x is $(\lambda T)\cdot(1-e^{-\mu t_c})$ and so

$$\sum_{n=1}^{\infty} P_n = e^{-\lambda T}\left\{e^{[(\lambda T)(1-e^{-\mu t_c})]} - 1\right\}$$

$$= \left(e^{-\lambda T}\right)\left(e^{\lambda T[1-e^{-\mu t_c}]}\right) - \left(e^{-\lambda T}\right)$$

$$= \left(e^{-\lambda T}\right)\left(e^{\lambda T} e^{-\lambda T e^{-\mu t_c}}\right) - \left(e^{-\lambda T}\right)$$

Note that $e^{-\lambda T} \cdot e^{\lambda T} = e^{0} = 1$ therefore the above becomes

$$\sum_{n=1}^{\infty} P_n = e^{-\lambda T e^{-\mu t_c}} - e^{-\lambda T}$$

This equation now represents the probability of a number of failures occurring and each individual failure being repaired in a time period less than t_c.

However the equipment has also a probability of being failure free for a time between the above repaired failures. (Note that the above calculation refers only to failures occurring and their being repaired). The probability of no failure is the reliability of the equipment, $e^{-\lambda T}$. This must now be added to the "failure repair" equation. So the resulting equation, the probability of availability for the entire period T, the mission time, for the equipment, becomes

$$A_E = e^{-\lambda T e^{-\mu t_c}} - e^{-\lambda T} + e^{-\lambda T}$$

or

$$A_E = e^{-\lambda T e^{-\mu t_c}}$$

This equation could of course have been obtainable immediately by summing

$$P_n = \frac{(\lambda T)^n e^{-\lambda T}}{\underline{|n}} (1 - e^{-\mu t_c})^n \qquad \text{from } n = 0 \text{ to } n \to \infty$$

That is,

$$\sum_{n=0}^{\infty} P_n = e^{-\lambda T e^{-\mu t_c}} = A_E$$

but this logic becomes rather abstruse when the attempt is made to interpret the equation practically.

If the repair times are now taken 2 or 3 or j at a time as in Fig. III, it can be intuitively seen that the ensuing distribution must also be exponential.

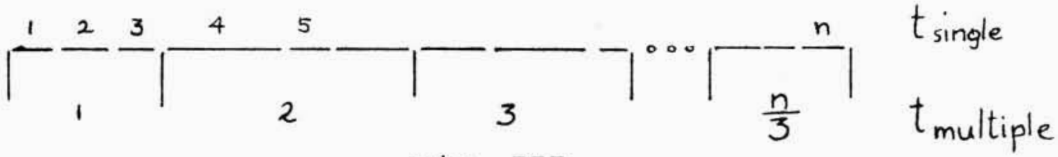

Fig. III

The mean of the distribution taking repair times one at a time would be the "Single" mean or $\frac{\sum t_{single}}{n} = \frac{\text{total time}}{n}$

The mean of the distribution taking, for example, three repair times at a time would be the "Multiple Mean"

$$\frac{\sum t_{multiple}}{\frac{n}{3}} = \frac{\text{total time}}{\frac{n}{3}}$$

since there are now only 1/3 as many time periods to be added.

Therefore, with respect to the single repair times the "new" mean repair time is $\frac{3\sum t}{n}$. Hence the new repair rate $\mu_r = \frac{1}{\frac{3\sum t}{n}} = \frac{1}{3}\mu$ or in general $\mu_r = \frac{1}{3}\mu$.

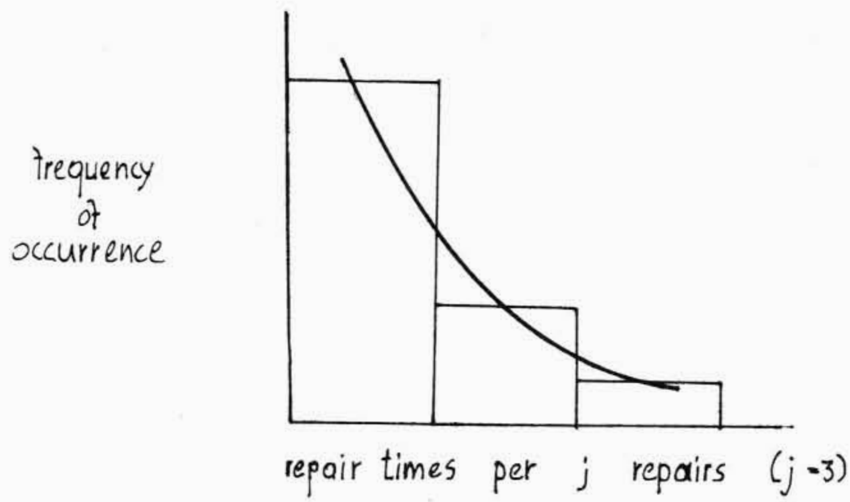

Fig. IV

The relation between μ_τ and μ can be seen intuitively; that is, if the average time per one repair is $\hat{t}$ then the average time per 3 repairs might well be $3\hat{t}$.

The probability then that j randomly occurring repair times are of duration less than t_c is

$$\frac{\int_0^{t_c} e^{-\frac{1}{j}\mu t}\,dt}{\int_0^{\infty} e^{-\frac{1}{j}\mu t}\,dt}$$

similarly to the earlier case, $P = (1 - e^{-\frac{\mu}{j}t_c})$

This is the probability that the sum of j repair times is less than t_c.

The probability then that exactly j failures occur, <u>and</u> that exactly j repair times are of total duration less than t_c is the product of the two probabilities

$$P_j = e^{-\lambda T}\frac{(\lambda T)^j}{\lfloor j}(1 - e^{-\frac{1}{j}\mu t_c})$$

The probability then that exactly one failure occurs and that it is repaired within the time constraint t_c <u>or</u> that two failures occur and both are repaired within the time constraint t_c <u>or</u> that ----i failures occur and that these i are all repaired within the time constraint t_c is the sum of these probabilities or,

$$\sum_{j=1}^{i} P_i = P_1 + P_2 + P_3 + \cdots + P_i$$

Summing these terms

$$P_1 = e^{-\lambda T}\frac{(\lambda T)^1}{\lfloor 1}\left(1 - e^{-\frac{1}{j}\mu t_c}\right)$$

$$P_2 = e^{-\lambda T}\frac{(\lambda T)^2}{\lfloor 2}\left(1 - e^{-\frac{1}{j}\mu t_c}\right)$$

$$P_3 = e^{-\lambda T}\frac{(\lambda T)^3}{\lfloor 3}\left(1 - e^{-\frac{1}{j}\mu t_c}\right)$$

$$\vdots \qquad \vdots$$

$$P_i = e^{-\lambda T}\frac{(\lambda T)^i}{\lfloor i}\left(1 - e^{-\frac{1}{j}\mu t_c}\right)$$

$$\sum_{j=1}^{i} P_j = e^{-\lambda T}\sum_{j=1}^{i}\frac{(\lambda T)^j}{\lfloor j}\left(1 - e^{-\frac{1}{j}\mu t_c}\right)$$

For all possible events; that is from one failure to all (∞) failures the equation becomes

$$\sum_{j=1}^{\infty} P_j = e^{-\lambda T}\sum_{j=1}^{\infty}\frac{(\lambda T)^j}{\lfloor j}\left(1 - e^{-\frac{1}{j}\mu t_c}\right)$$

$$= e^{-\lambda T} \sum_{j=1}^{\infty} \left[\frac{(\lambda T)^j}{\underline{|j}} - \frac{(\lambda T)^j}{\underline{|j}} \, e^{-\frac{1}{j}\mu t_c} \right]$$

$$= e^{-\lambda T} \left\{ \sum_{j=1}^{\infty} \frac{(\lambda T)^j}{\underline{|j}} - \sum_{j=1}^{\infty} \frac{(\lambda T)^j}{\underline{|j}} \, e^{-\frac{1}{j}\mu t_c} \right\}$$

$$= e^{-\lambda T} \left\{ (e^{\lambda T} - 1) - \sum_{j=1}^{\infty} \frac{(\lambda T)^j}{\underline{|j}} \, e^{-\frac{1}{j}\mu t_c} \right\}$$

$$\sum_{j=1}^{\infty} P_j = 1 - e^{-\lambda T} - e^{-\lambda T} \sum_{j=1}^{\infty} \frac{(\lambda T)^j}{\underline{|j}} \, e^{-\frac{1}{j}\mu t_c}$$

This is the probability that the repair actions are successful. In addition to this must be considered the probability, $P_o = e^{-\lambda T}$, that no failure will occur. This is mission availability $A_M = \sum_{j=1}^{\infty} P_j + R$ where R is the probability of no failure, $P_i = P_o = R$, the reliability. Then

$$A_M = 1 - e^{-\lambda T} - e^{-\lambda T} \sum_{j=1}^{\infty} \frac{(\lambda T)^j}{\underline{|j}} \, e^{-\frac{1}{j}\mu t_c} + e^{-\lambda T}$$

$$A_M = 1 - e^{-\lambda T} \sum_{j=1}^{\infty} \frac{(\lambda T)^j}{\underline{|j}} \, e^{-\frac{1}{j}\mu t_c} \qquad \text{Q.E.I.}$$

This last addition would not have been necessary if

$$P_j = e^{-\lambda T} \frac{(\lambda T)^j}{\underline{|j}} \left(1 - e^{-\frac{1}{j}\mu t_c}\right)$$

were summed from j = 0 (for no failure) to j = ∞ , but this leads to more abstruse reasoning in the solution.

It is also interesting to note that A_M becomes 1 for $\lambda = 0$ and A_M becomes 0 for $T = \infty$. Also when no repair is allowed A_M becomes R (the reliability).

These are the limiting conditions which would be expected.

APPENDIX

In the case where the equipment is non-maintainable, that is $t_c = 0$,

$$A_M = 1 - e^{-\lambda T} \sum_{j=1}^{\infty} \frac{(\lambda T)^j}{\underline{|j}} \cdot 1 \qquad \text{since } e^{-\frac{1}{j}\mu t_c} = 1 \text{ for } t_c = 0$$

$$= 1 - e^{-\lambda T}(e^{\lambda T} - 1)$$

$$= 1 - 1 + e^{-\lambda T}$$

$$A_M = e^{-\lambda T} = R_T$$

the reliability for time T.

Some other interesting limiting conditions supporting the correctness of the equation are

$$\lim_{T\to\infty} A_M = \lim_{T\to\infty} \left\{ 1 - e^{-\lambda T} \left[\lim_{j\to\infty} \frac{(\lambda T)^j}{\underline{|j}} \right] \cdot \left[\lim_{j\to\infty} e^{-\frac{1}{j}\mu t_c} \right] \right\}$$

$$= \lim_{T\to\infty} \left\{ 1 - e^{-\lambda T} \left[e^{\lambda T} - 1 \right] \cdot \left[1 \right] \right\}$$

$$\lim_{T\to\infty} A_M = \lim_{T\to\infty} \left\{ e^{-\lambda T} \right\} = 0$$

similarly

$$\lim_{\lambda\to 0} A_M = 1$$

Examples

Availability for LUT and LCC

Given

LCC Availability, A_M = .995

MTTR $\frac{1}{\mu}$ = .5 hrs; μ = 2

Mission Time, T = 15 hrs.

Total repair time, tc = 1 hour

$\therefore$ Reliability required for 15 hrs is $.9639 + \epsilon$

for 7 hrs is $.9830 + \epsilon$

Solving A_M for λ by computer yields

$$\lambda = .00245 \times T$$

$$\therefore R_T = e^{-.00245 \times T}$$

$$\therefore R_{15} = e^{-.00245 \times 15} = .9639 + \epsilon$$

$$R_7 = e^{-.00245 \times 7} = .9830 + \epsilon$$

(ϵ designates error)

LUT to operate for last seven hours without maintenance with reliability of .99. This implies a failure rate of no more than .001436.

Given

Reliability R = .99 for 7 hrs.

$$\therefore \quad e^{-.010051} = e^{-7\lambda}$$

$$.010051 = 7\lambda$$

$$.001436 = \lambda$$

So for 8 hrs $R = e^{-.001436 \times 8} = .98858 + \epsilon$

$\therefore A_M = .99844$ (by computer)

LCC	$A_M = .995$		$\lambda = .00245$
LUT	$A_M = .99844$	$A = R_7 = .99$*	$\lambda = .001436$
	8 Hrs.	7 Hrs.	

* LUT is non-maintainable during last 7 hrs.

Computer Program

Manual computations of the availability equation proved to be very laborious and cumbersome. Calculations become particularly time consuming in the case where $\frac{(\lambda T)^j}{j!}$ converges very slowly. Experience has shown that the number of terms to be calculated depends on the value of λT. If $\lambda T \leq 1$. 3 to 10 terms are sufficient. If $\lambda T > 1$. the number of terms needed is about $3\lambda T$. These difficulties were overcome by a computer program written by Mr. C. Metelmann.

To do this, the sum:

$$\sum_{j=1}^{\infty} \frac{(\lambda T)^j}{j!} e^{-\frac{\mu t_c}{j}}$$

was treated as follows:

if $$\frac{(\lambda T)^j}{j!} = S$$

then $$S \cdot \frac{\lambda T}{j+1} = \frac{(\lambda T)^{j+1}}{(j+1)!}$$

Each of the terms generated must be multiplied by

$$e^{-\frac{\mu t_c}{j}}$$

and then added to the sum.

It was decided to stop generating terms on two conditions:

(a) If the term $S \cdot e^{-\frac{\mu t_c}{j}}$ becomes smaller than 1×10^{-10}

(b) If 100 terms had been calculated without condition (a) being fulfilled.

The program written in FORTRAN II is available upon request.

Graphic Presentations

The following charts show some typical results. The charts are all based on the same repair rates $\mu = .333$ and the same time constraint $t_c = 2.5$ hrs.

Fig. V shows the general form of the availability function for various failure rates λ.

Fig. VI is an extraction from figure V of the area $.95 \leq A_M \leq 1$ and $T < 600$ hrs. In figure VI the reliability scale has also been added. For example: given $T = 470$ and $\lambda = .0002$; $R = .91$ and $A_M = .96$ can be read from the graph.

Fig. VII shows an example of the relation between reliability and availability for identical failure rates and the same mission time.

MISSION AVAILABILITY
for Repair Rate μ = 0.3333
and Time Constraint t_c = 2.5 Hrs.

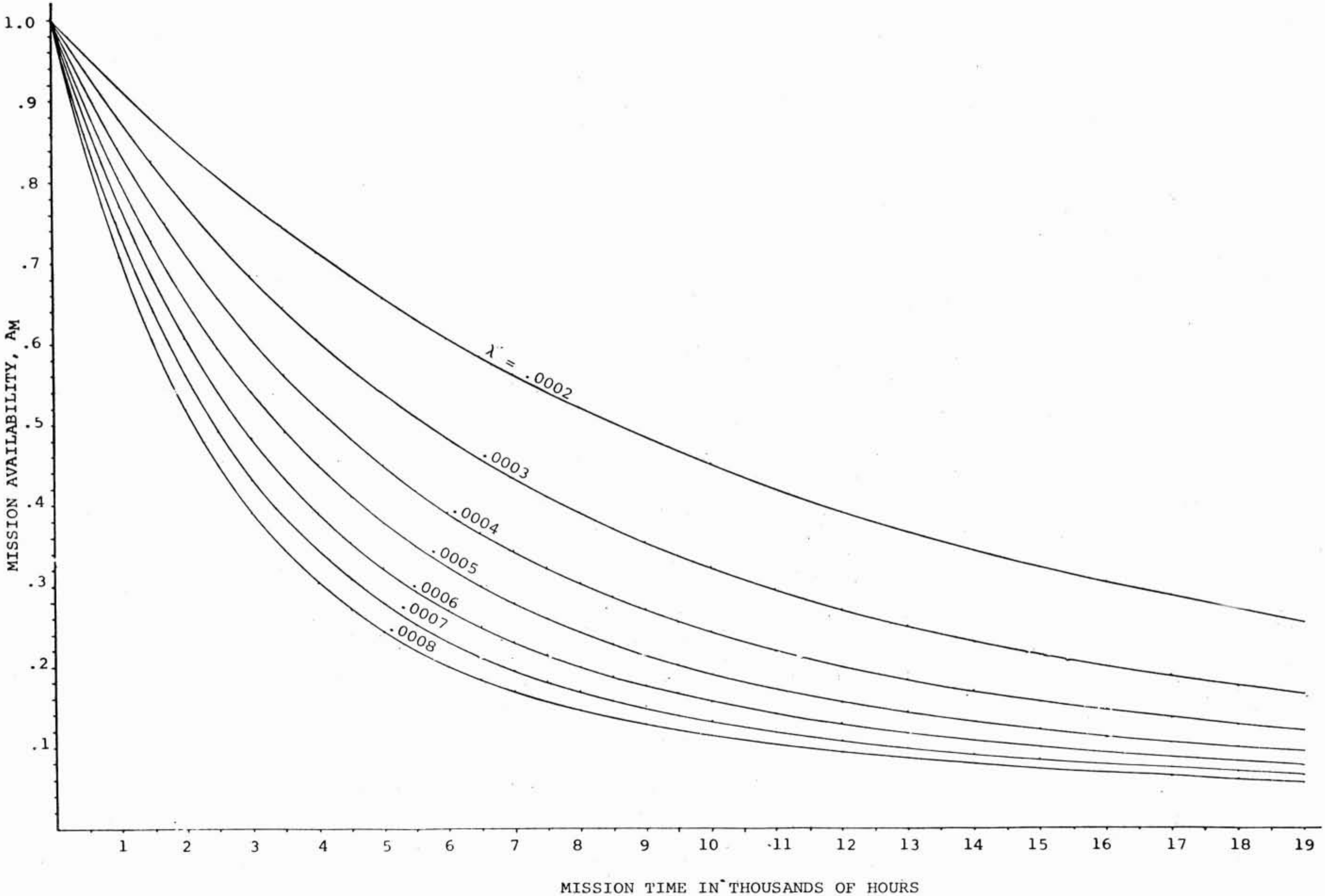

Fig. V

MISSION AVAILABILITY
for Repair Rate μ = 0.3333
and Time Constraint t_c = 2.5 Hrs.

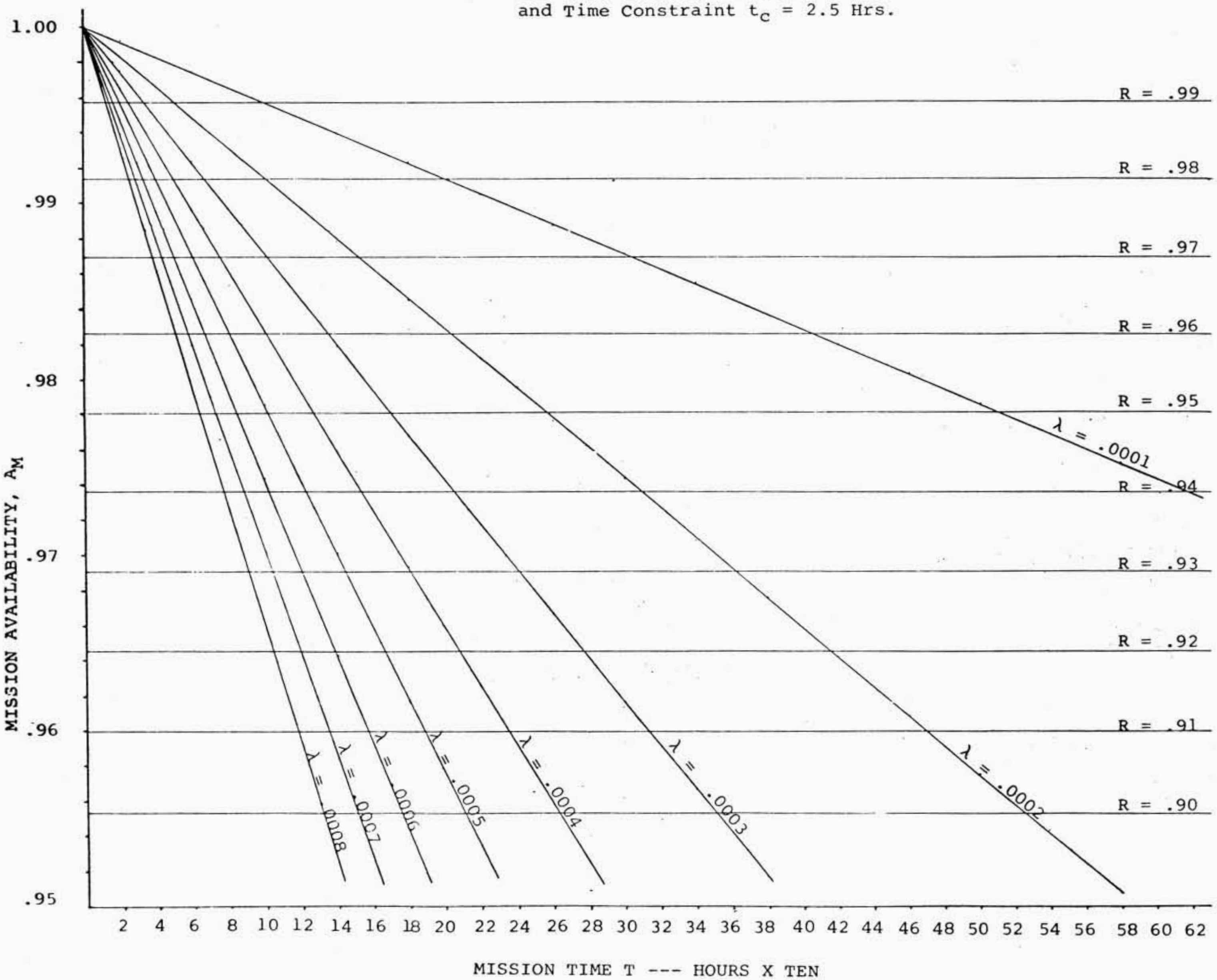

Fig. VI

COMPARISON OF

AVAILABILITY & RELIABILITY

Repair Rate μ = .333
Time Constraint t_c = 2.5 Hrs.

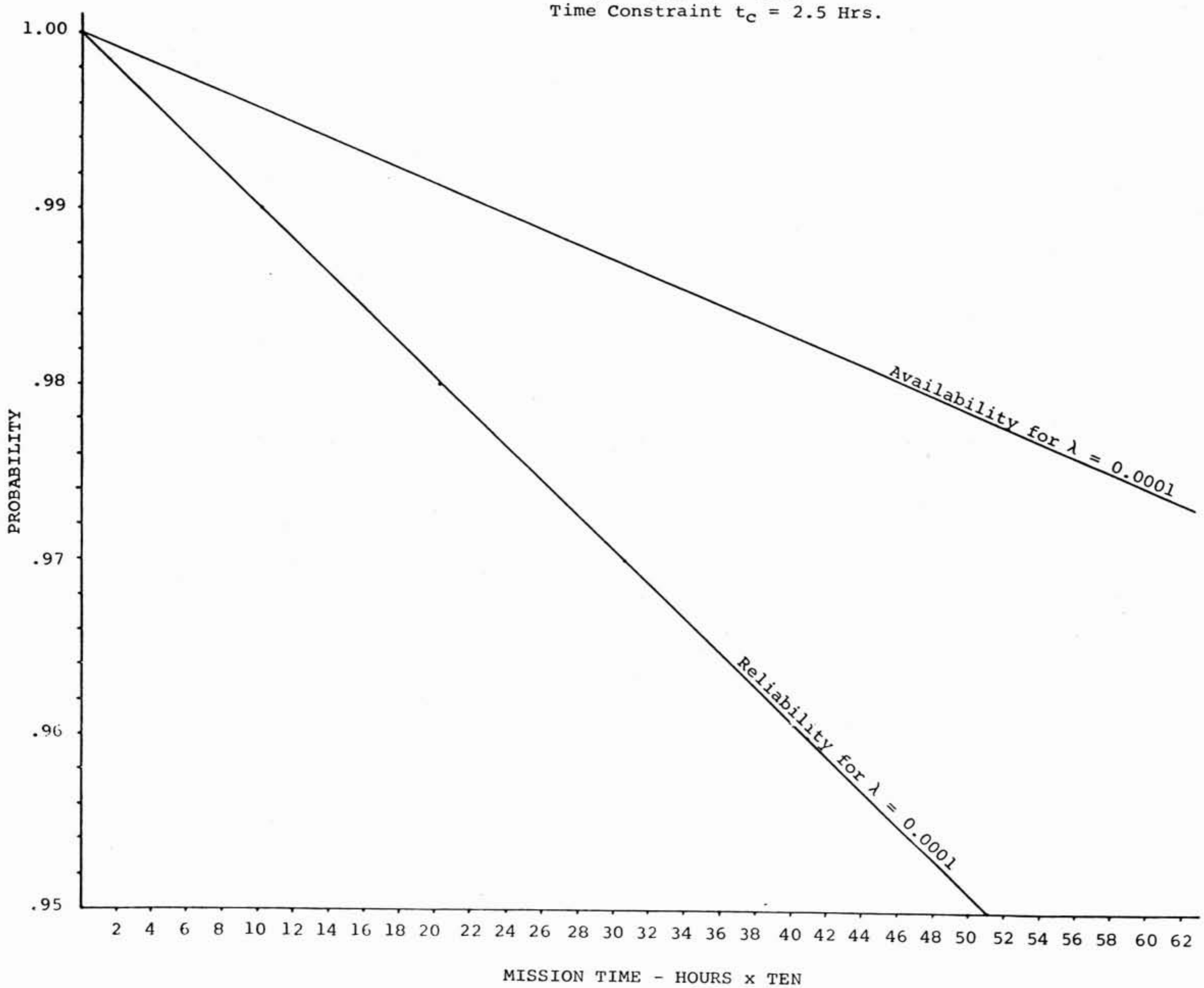

Fig. VII

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